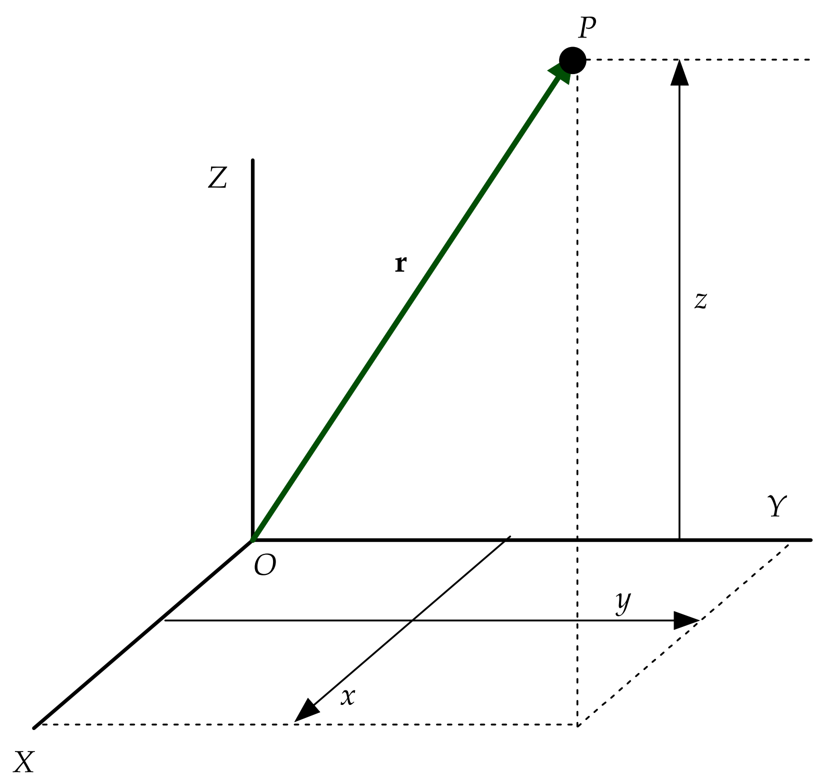
Coordinates and Coordinate Systems

A *reference frame* is a set of three mutually perpendicular axes with respect to which linear or angular measurements can be made.

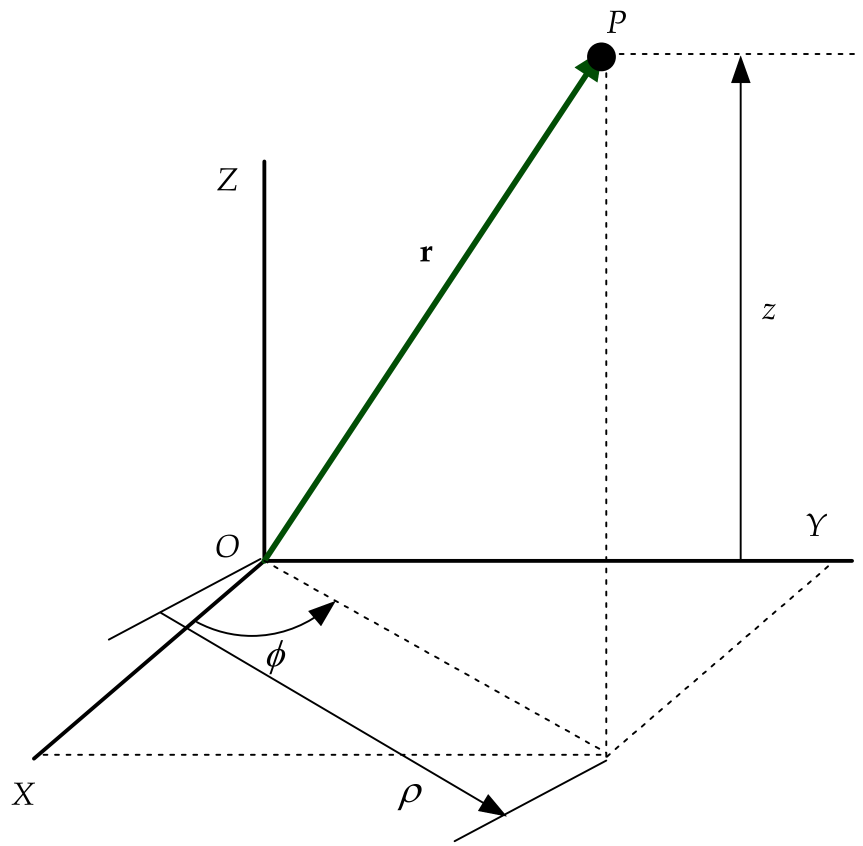
Three independent parameters (linear or angular measurements) are needed to specify the position of a particle in three-dimensional Cartesian space. These parameters are known as *coordinates*. Sets of coordinate triplets comprised of linear or angular measurements (at least one linear measurement) are known as *coordinate systems*.

Standard, commonly used coordinate systems are:

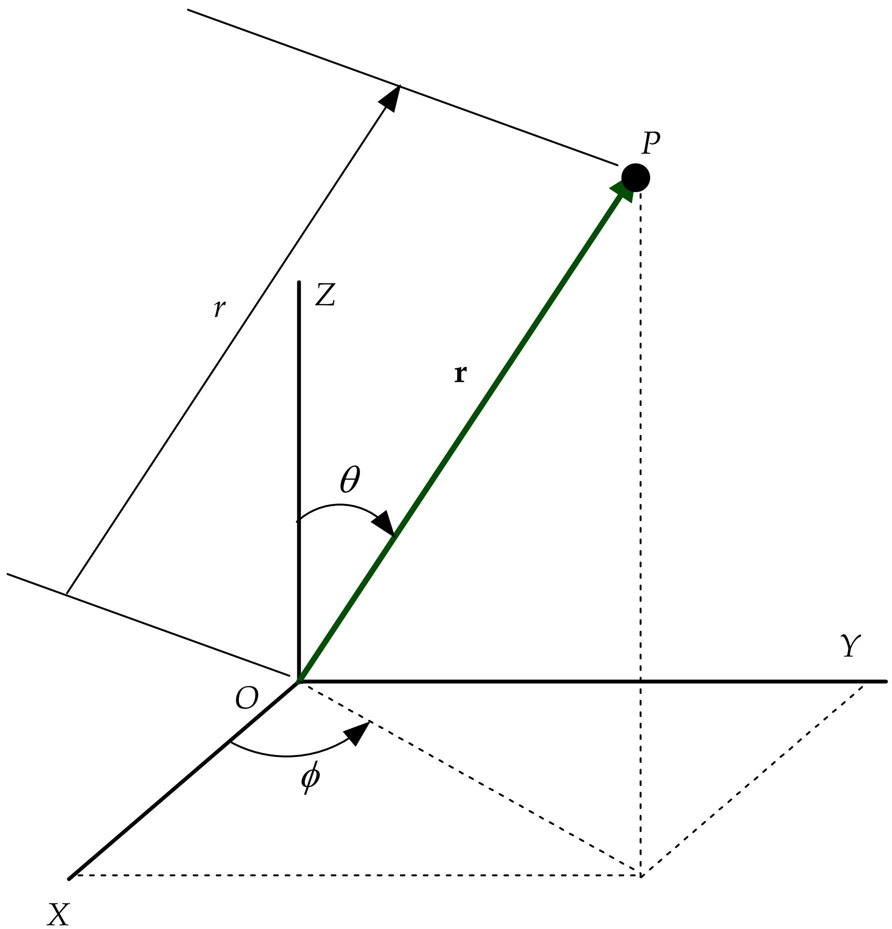
Cartesian (x, y, z):



Cylindrical (ρ, ϕ, z):



Spherical (r,θ,ϕ):



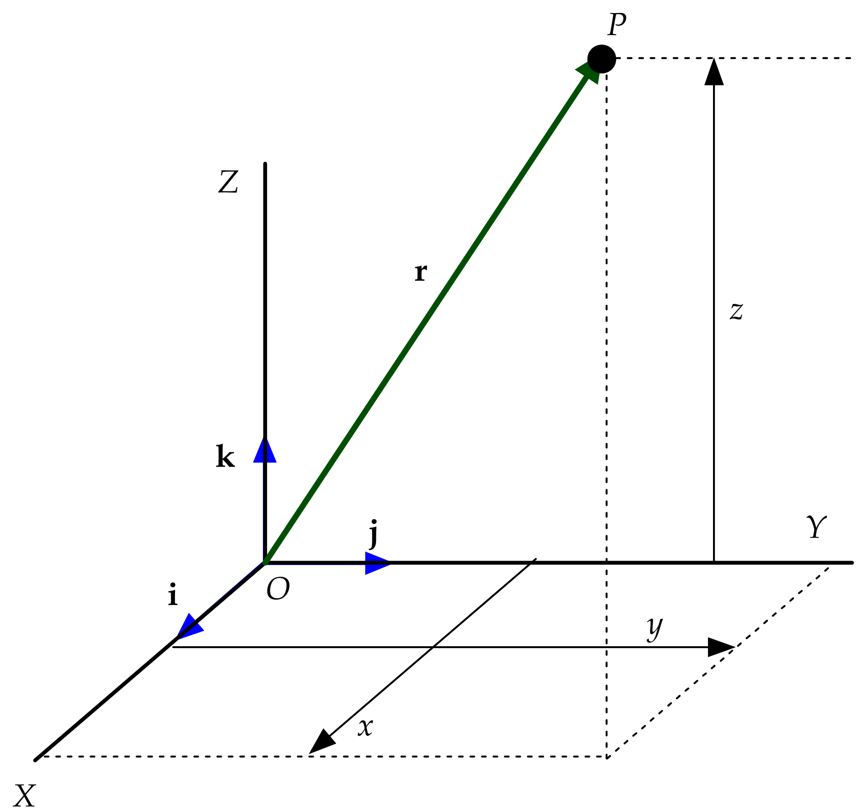
The $OXYZ$ frame in all the figures above is assumed to be the fixed or inertial reference frame.

Click for notes on inertial reference frames and inertial space

Reference Frames and Unit Vector Triads

Reference frames may be moving or fixed relative to inertial space. A convenient way of expressing reference frames mathematically is to attach a *triad of unit vectors* to the three mutually perpendicular axes. Depending on the problem it may be advantageous to choose moving reference frames whose motion depends on the motion of the particles of interest.

The triad of unit vectors commonly associated with Cartesian coordinates is assumed to be fixed to and aligned with the fixed reference frame.



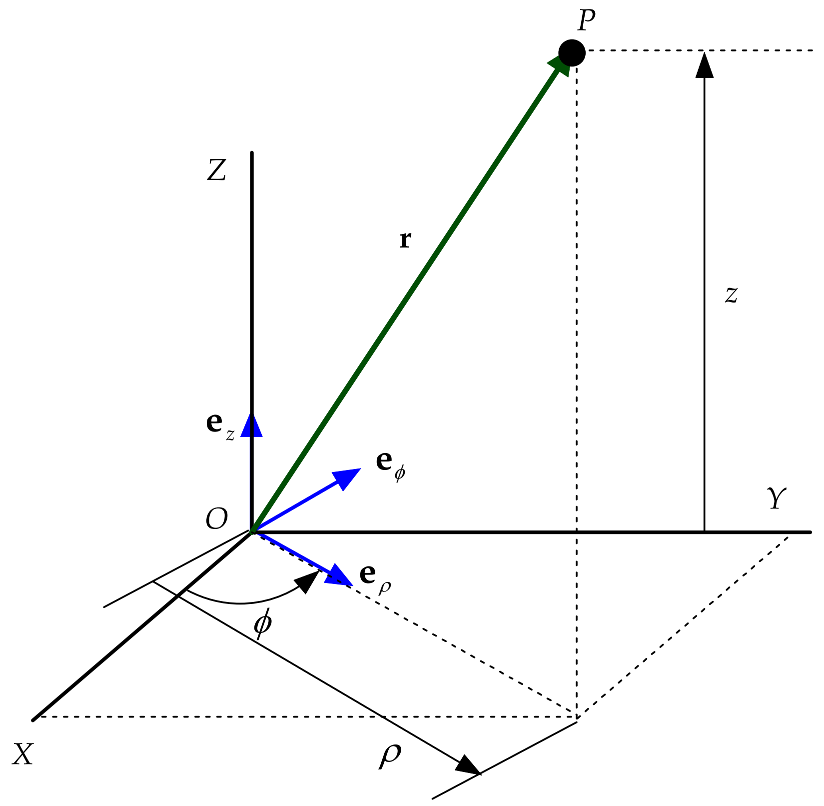
The relationship among the three unit vectors is defined as:

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}$$

The vector **r** is described in this frame as:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

The moving unit vector triad commonly associated with cylindrical coordinates is shown below:



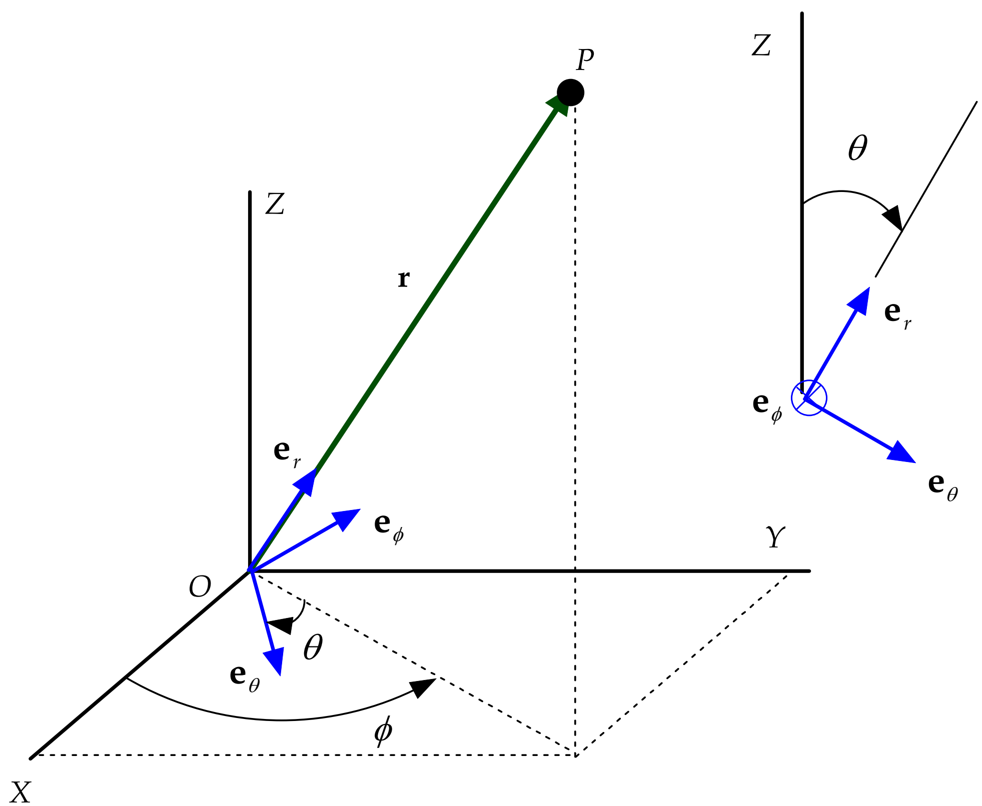
The relationship among the three unit vectors is defined as:

$$\mathbf{e}_\rho \times \mathbf{e}_\phi = \mathbf{e}_z$$

The vector $\mathbf{r}$ (which lies in the plane of $\mathbf{e}_\rho$ and $\mathbf{e}_z$) is described in this frame as:

$$\mathbf{r} = \rho \mathbf{e}_\rho + z \mathbf{e}_z$$

The moving unit vector triad commonly associated with spherical coordinates is shown below:



The relationship among the three unit vectors is defined as:

$$\mathbf{e}_r \times \mathbf{e}_\theta = \mathbf{e}_\phi$$

The vector **r** (which lies along $\mathbf{e}_r$ only) is described in this frame as:

$$\mathbf{r} = r\mathbf{e}_r$$