1.1 Position of a Particle

The position of a particle P relative to a reference frame can be described by a vector **r**. This vector, which extends from the origin of the frame O to P, is called the *position vector*. If the particle P is in motion relative to the reference frame, then **r** is a function of time (i.e. changes with time in both magnitude and direction). The figure below shows how **r** changes with time as P moves along a path in three-dimensional space.

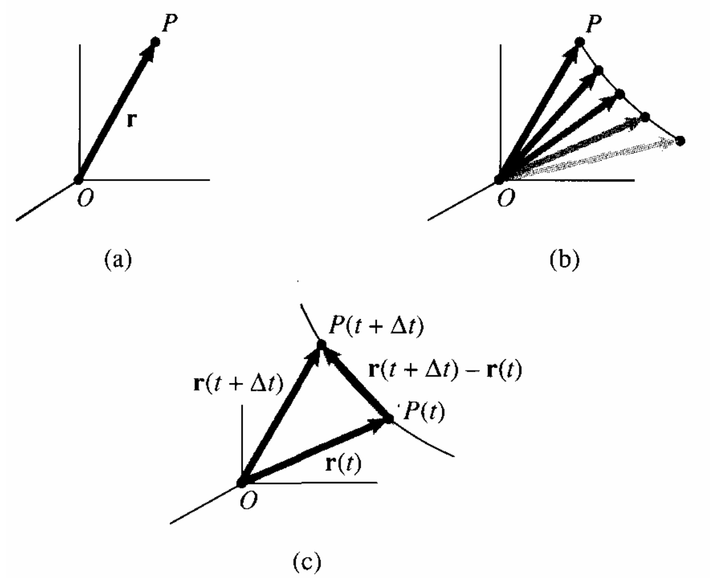

Figure 1. Position vector varying with time as P moves along a path

Click to see an animation of the vector as it changes in time

The time dependence of **r** is expressed as:

$$\mathbf{r} = \mathbf{r}(t) \tag{1.1}$$

The position vector may take different forms depending on the coordinates used to express it. For example if **r** is expressed in Cartesian coordinates and associated unit vectors it may be written as:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{1.2}$$

In cylindrical coordinates **r** is defined as:

$$\mathbf{r} = r\mathbf{e}_\rho + z\mathbf{e}_z \tag{1.3}$$

Click for notes on reference frames and coordinate systems

In spherical coordinates **r** is defined more simply as:

$$\mathbf{r} = r\mathbf{e}_r \tag{1.4}$$

1.2 Velocity of a Particle

The velocity **v** of *P* relative to a reference frame at time *t* is defined as:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t+\Delta t) - \mathbf{r}(t)}{\Delta t} \tag{1.5}$$

Click to see an animated sequence showing the change in the vector

where the vector $\Delta\mathbf{r} = \mathbf{r}(t+\Delta t) - \mathbf{r}(t)$ is the change in position or displacement of *P* during the time interval Δt (see Fig. 1).

Note:

a) **v** is the derivative of a vector with respect to a scalar
b) **v** represents the rate of change of **r** *in both magnitude and direction* with respect to time
c) all changes in **r** are measured within the same reference frame in which **r** was measured
d) **v** is always tangent to the path of *P*

1.3 Acceleration of a Particle

The acceleration **a** of P relative to a reference frame at time t is defined as:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \lim_{\Delta t \to 0} \frac{\mathbf{v}(t+\Delta t) - \mathbf{v}(t)}{\Delta t} \quad (1.6)$$

where the vector $\Delta\mathbf{v} = \mathbf{v}(t+\Delta t) - \mathbf{v}(t)$ is the change in the velocity of P during the time interval Δt.

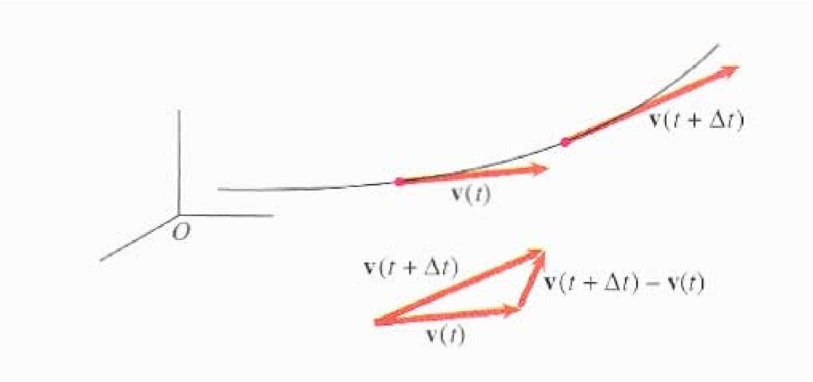

Figure 2. Change of the velocity of P as it moves along a path

Click to see an animation of the change of the velocity vector with time

Note:

a) **a** is the derivative of a vector with respect to a scalar
b) **a** represents the rate of change of **v** *in both magnitude and direction* with respect to time
c) all changes in **v** are measured within the same reference frame in which **v** was measured