7. Kinetics (or Dynamics) of a Particle

The expressions derived in Sections 1 through 5 took into account only the motion of a particle and ignored the effects of forces acting on the particle. The science relating motion to forces is known *dynamics* or *kinetics* and the physical law that expresses the relationship between forces and motion is the Newton's Second Law of Motion. For a particle Newton Second Law states (in vector form);

$$\mathbf{F} = m\frac{d^2\mathbf{r}}{dt^2} \tag{7.1}$$

where $\mathbf{r}$ is the position of a particle, $\frac{d^2\mathbf{r}}{dt^2}$ is the acceleration of the particle measured in an *inertial reference frame*, m is the mass of the particle, and $\mathbf{F}$ is the resultant force acting on the particle.

Note:

a) the relationship expressed by Eq. (7.1) may be considered a definition of force or mass since either of them can only be defined by Newton's Second Law
b) the expression above is a vector expression and results in three scalar equations in three-dimensional Cartesian space
c) the scalar components of Eq. (7.1) can be expressed in any coordinate system

7.1 Notes on inertial reference frames

An inertial reference frame is a reference frame that is either at rest or in *uniform translation* relative to stationary space (in Newton's words, relative to "fixed stars"). This means that the origin of the reference frame in which the motion of the particle is measured must be at rest or traveling at constant speed along a straight line, and the three axes of the reference frame must not be changing direction (must be nonrotating) relative to stationary space.

Newton's Second Law for a particle is valid only if the acceleration of the particle is measured in or relative to an inertial frame. In other words, if, for example, the acceleration of a particle is measured inside an accelerating spaceship relative to some reference object attached to the spaceship "force" would *not* equal "mass times acceleration." As a result Newton's Second Law is not strictly valid for particles whose accelerations are measured relative to the earth because the earth is essentially a rotating reference frame whose origin accelerates (travels along a curved path) relative to stationary space. However because the earth's spin rate and speed of travel around the sun are both small the earth is used as an inertial frame for all practical purposes.

Although the acceleration of the particle must be measured *relative* to an inertial frame to be useable in Newton's Second Law the acceleration can be *expressed in* or *referred to* rotating and/or accelerating coordinate systems such polar, cylindrical, spherical, or path coordinates. Consequently the vector expression for Newton's Second Law is expressible in coordinate systems associated with rotating frames.

7.2 Notes on units

Because the mass of a particle is often associated with its weight in the earth's gravitational field some confusion exists about units in the use of Newton's Second Law. To avoid any confusion use the following table (here Newton's second law is expressed in scalar form):

Force (F)	Mass (m)	Acceleration (a)	Form of Newton's Second Law
Newtons (N)	Kilograms (kg)	m/s^2	$F = ma$
Pounds force (lbf)	Pounds mass (lbm)	ft/s^2	$F = \frac{m}{32.2}a$
Pounds force (lbf)	Pounds mass (lbm)	in/s^2	$F = \frac{m}{386.4}a$
Pounds force (lbf)	Slugs	ft/s^2	$F = ma$

7.3 Expressions for different coordinate systems

Using the results obtained in Modules 3, 4, and 5 explicit scalar expressions for Newton's Second Law can be obtained in different coordinate systems.

Components in Cartesian coordinates (spatial motion)

$$F_x = ma_x = m\ddot{x} \quad F_y = ma_y = m\ddot{y} \quad F_z = ma_z = m\ddot{z} \tag{7.2}$$

Components in path coordinates (planar motion)

$$F_t = ma_t = m\ddot{s} \quad F_n = ma_n = m\frac{\dot{s}^2}{\rho} \tag{7.3}$$

Components in polar coordinates (planar motion)

$$F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right) \quad F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \tag{7.4}$$

These scalar expressions can be used with the same mathematical methods as those developed in Section 1 for rectilinear motion.

Click here for Examples 1, 2, 3, 4, 5, and 6 on this topic