

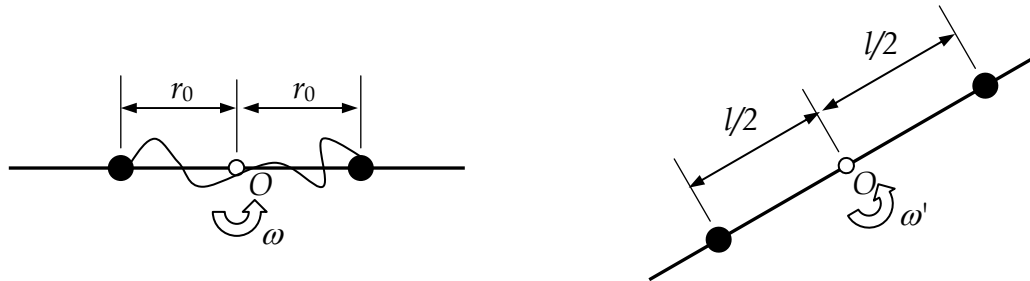
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سایت آموزش مهندسی مکانیک ایران

**Example 10.1**

Two particles of mass  $m$  are connected by a string of length  $l$  and are free to slide on a smooth massless rod which can rotate freely about the point  $O$ . Initially the two particles are latched to the rod at a distance of  $r_0$  from  $O$  and the rod rotates in the plane of the paper at the rate of  $\omega$ . At some instant the latches are released and the particles move outward as far as the string will permit them (i.e. to a distance of  $l/2$  from  $O$ ) and come to rest relative to the rod. Determine:

- the angular rate  $\omega'$  of the rod at the instant the particles reach their final positions;
- the tension in the string at that instant;
- the change in the kinetic energy of the particles.

**Solution:**

a) In the motion described angular momentum around  $O$  is conserved because there are no external forces that produce moments about  $O$ . Thus:

$$H_O = H'_O \quad (i)$$

From the definition of angular momentum we have for the two particles:

$$H_O = 2mr_0^2\omega$$

and

$$H'_O = 2m\left(\frac{l}{2}\right)^2\omega'$$

Substituting these relations in (i) we obtain:

$$\omega' = \frac{4r_0^2}{l^2}\omega$$

b) The acceleration of any one of the particles in the radial direction at a general position is:

$$a_r = \ddot{r} - r\dot{\theta}^2$$

At the instant the particles reach their final configuration  $\ddot{r} = 0$ ,  $r = \frac{l}{2}$ , and  $\dot{\theta} = \omega'$ . Thus the tension in the string, which is the only radial force acting on any one of the particles is:

$$P = \frac{ml\omega'^2}{2} = \frac{8mr_0^4}{l^3}\omega^2$$

c) The velocity of the particles at any time in their motion is:

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

It is clear from the statement of the problem that  $\dot{r} = 0$  both at the beginning and the end configurations. Thus kinetic energies of the particles at the beginning and end configurations are:

$$T_1 = \frac{1}{2}(2m)(r_0\omega)^2 = mr_0^2\omega^2$$

$$T_2 = \frac{1}{2}(2m)\left(\frac{l}{2}\omega'\right)^2 = m\frac{l^2}{4}\omega'^2 = \frac{4mr_0^4}{l^2}\omega^2$$

Thus the change in kinetic energy is:

$$\Delta T = T_2 - T_1 = m\omega^2\left(\frac{4r_0^4}{l^2} - r_0^2\right) = m\omega^2r_0^2\left(\frac{4r_0^2}{l^2} - 1\right)$$