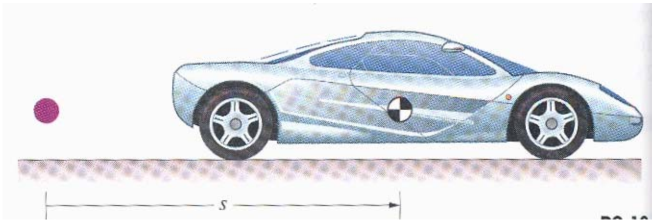
Example 2.1

In a test of a prototype car the driver starts the car from rest at t = 0 and then applies the brakes. Engineers measuring the position of the car find that from t = 0 to t = 18 it is approximated by $s = 5t^2 + \frac{1}{3}t^3 - \frac{1}{50}t^4$ ft.



a) What is the maximum velocity of the car, and at what time does it occur?
b) What is the maximum acceleration of the car and at what time does it occur?

Solution:

The expression for the velocity of the car can be found by differentiating the expression for s:

$$v = \frac{ds}{dt} = \dot{s} = 10t + t^2 - \frac{2}{25}t^3 \text{ ft/s}$$

Similarly the acceleration of the car is given by:

$$a = \frac{dv}{dt} = \dot{v} = 10 + 2t - \frac{6}{25}t^2 \text{ ft/s}^2$$

The velocity of the car is a maximum, or $v = v_{\max}$, when $\dfrac{dv}{dt} = 0$ which occurs at, say, $t = t^*$. Thus setting a to zero we find:

$$10 + 2t^* - \frac{6}{25}\left(t^*\right)^2 = 0$$

or

$$t^*_{1,2} = \frac{2 \pm \sqrt{4 + 4\left(\frac{6}{25}\right)10}}{12/25} = 11.85, -3.52 \text{ s}$$

Taking the positive value for t^* and substituting it in the equation for v we obtain:

$$v_{\max} = v\left(t^*\right) = 125.8 \text{ ft/s}$$

The acceleration of the car is a maximum, or $a = a_{\max}$, when $\dfrac{da}{dt} = 0$ which occurs at, say, $t = t^\dagger$. Differentiating the expression for a and setting it to zero we find:

$$\frac{da}{dt} = 2 - \frac{12}{25}t^\dagger = 0$$

or

$$t^\dagger = 4.17 \text{ s}$$

Substituting this value in the expression for acceleration we obtain:

$$a_{\max} = a\left(t^\dagger\right) = 14.17 \text{ ft/s}^2$$

Click to see an animation and plots for the motion of the car