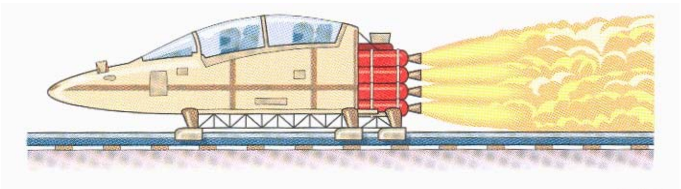
Example 2.4

A rocket sled starts from rest and accelerates at $a_1 = 3t^2$ m/s² until its velocity is 1000 m/s. It then hits a water brake which makes it decelerate at $a_2 = -0.001v^2$ m/s² until its velocity decreases to 500 m/s. What total distance does the sled travel during this maneuver?



Solution:

The velocity and displacement at any time in the first segment of the motion can be found by integrating the first acceleration expression with respect to time. From Eq. (2.11) we have:

$$v(t) = v_0 + \int_{t_0}^{t} a\,dt = 0 + \int_0^t \left(3t^2\right)dt = t^3 \text{ m/s} \qquad \text{(i)}$$

From Eqn. (2.14) we have:

$$s(t) = s_0 + \int_{t_0}^{t} v\,dt = 0 + \int_0^t \left(t^3\right)dt = \frac{t^4}{4} \text{ m} \qquad \text{(ii)}$$

Assume that the sled reaches the velocity $v_1 = 1000$ m/s at $t = t_1$. The value of t_1 can be computed from (i) above:

$$t_1 = \left(v\left(t_1\right)\right)^{\frac{1}{3}} = \left(v_1\right)^{\frac{1}{3}} = (1000)^{\frac{1}{3}} = 10 \text{ s}$$

The distance s_1 traveled in time t_1 can be computed from (ii):

$$s_1 = s\left(t_1\right) = \frac{t_1^4}{4} = \frac{(10)^4}{4} = 2{,}500 \text{ m}$$

The displacement of the sled in the second segment of the motion can be found using Eq. (2.27):

$$s = s_0 + \int_{v_0}^{v} \frac{vdv}{a(v)} = s_1 + \int_{v_1}^{v_2} -\frac{vdv}{0.001v^2}$$

$$= s_1 - \left(1000 \ln v\right)\Big|_{v_1}^{v_2} = s_1 + 1000 \ln\left(\frac{v_1}{v_2}\right)$$

With $v_1 = 1000$ m/s, $v_2 = 500$ m/s, and $s_1 = 2500$ m we find for s:

$$s = 2500 + 1000 \ln\left(\frac{1000}{500}\right) = 3{,}193 \text{ m}$$

The time at which $v_2 = 500$ m/s can be found from Eq. (2.23):

$$t_2 = t_1 + \int_{v_1}^{v_2} -\frac{dv}{0.001v^2} = t_1 + 1000\left(\frac{1}{v}\right)\Big|_{v_1}^{v_2}$$

$$= 10 + 1000\left(\frac{1}{500} - \frac{1}{1000}\right) = 11.0 \text{ s}$$

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