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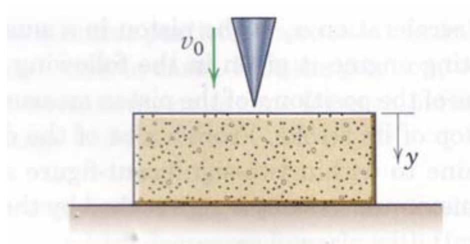
سایت آموزش مهندسی مکانیک ایران

Example 2.5

A cone falling with speed v_0 strikes and penetrates a block of packing material. The acceleration of the cone after impact is $a = g - cy^2$ where c is a positive constant and y is the penetration distance. If the maximum penetration depth is observed to be $y_{\max}$, determine the constant c.



Solution:

In this case the acceleration is given as a function of the distance traveled and is related to the initial velocity of the cone. Consequently we use Eqn. (2.25) which relates a displacement-dependent acceleration to velocity:

$$\int_{s_0}^{s} a(s)\, ds = \int_{v_0}^{v} v\, dv = \frac{1}{2}\left(v^2 - v_0^2\right)$$

For the given data $a(s) = g - cs^2$, $s = y_{\max}$, $s_0 = 0$, $v = 0$ (since the cone comes to rest at maximum penetration), and $v(0) = v_0$. Thus the equation above becomes:

$$\int_{0}^{y_{\max}} \left(g - cs^2\right) ds = \int_{v_0}^{0} v\, dv = -\frac{1}{2} v_0^2$$

which results in:

$$g y_{\max} - \frac{1}{3} c y_{\max}^{3} = -\frac{1}{2} v_0^2$$

Solving this relationship for c we obtain:

$$c = \frac{3 v_0^2 + 6 g y_{\max}}{2 y_{\max}^3}$$