Example 4.1

A bar begins to rotate from rest at $t = 0$ with linearly varying angular acceleration $\alpha = 6t$ rad/s^2 for 3 s. It then slows down with constant angular acceleration of $\alpha = -3$ rad/s^2 until it stops.

a) What is the maximum angular velocity the bar attains?
b) Through what total angle does it turn?

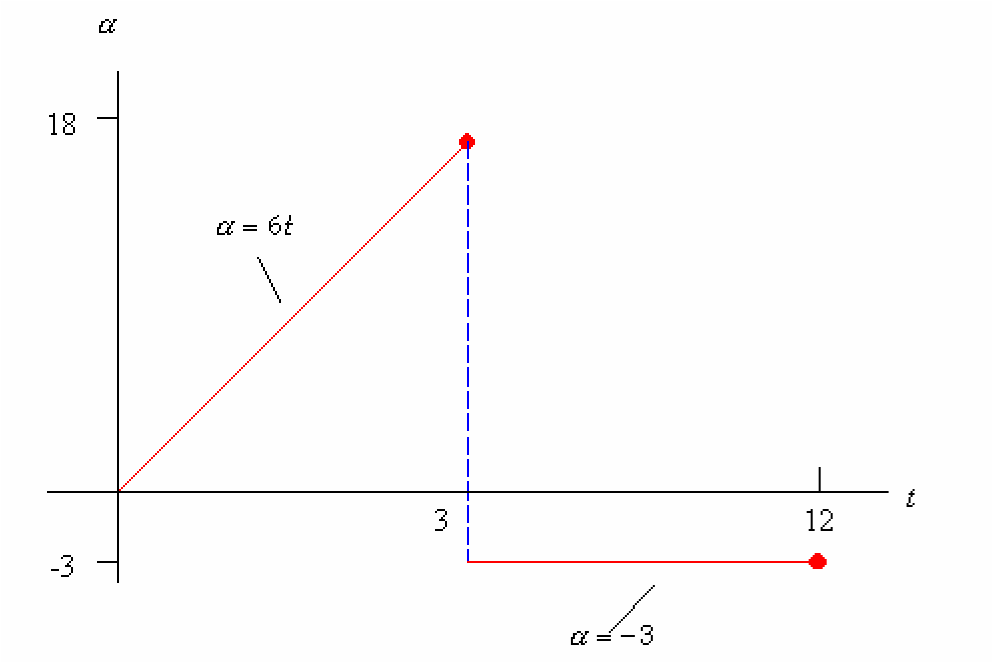

Figure 1 Angular acceleration profile of bar

Solution:

a) Applying Eq. (4.14) to the expressions given for α we obtain:

$$\omega(t) = \omega_0 + \int_{t_0}^{t} \alpha(t)\,dt = 0 + \int_0^t 6t\,dt = 3t^2 \quad 0 \le t \le 3 \text{ s}$$

The same equation can be used to compute the angular velocity for $t > 3$ s but the angular velocity at the beginning of this phase is not zero. Instead it is the angular velocity attained at the end of the first phase (at $t = 3$ s):

$$\omega_0' = \omega(3) = 3(3)^2 = 27 \text{ rad/s}$$

Thus the angular velocity for the second phase of motion is:

$$\omega'(t) = \omega_0' + \int_{t_0}^{t} \alpha'(t)\,dt = 27 + \int_3^t (-3)\,dt = 27 - 3(t-3) \quad t > 3$$

In more compact form the angular velocity of the bar at any time during the entire interval can be written as:

$$\omega(t) = \begin{cases} 3t^2 & 0 \le t \le 3 \\ 27 - 3(t-3) & t > 3 \end{cases} \qquad \text{(i)}$$

The angular velocity is a maximum or minimum when

$$\frac{d\omega}{dt} = \alpha = 0$$

As seen in Fig. 1 this happens when $t = 0$ (when ω is a minimum) and when $t = 3$ (at which point α changes abruptly from 18 rad/s^2 to -3 rad/s^2 going through zero). Thus the angular velocity attains its maximum value when $t = 3$ s. Substituting this value in the expression for ω above we obtain:

$$\omega_{\max} = 3(3)^2 = 27 \text{ rad/s}$$

b) To determine the total angular displacement first we need to determine when the bar stops rotating. This can easily be found from the second of Eqs. (i) by setting ω to zero:

$$27 - 3\left(t_{stop} - 3\right) = 0$$

which results in:

$$t_{stop} = 12 \text{ s}$$

Now using Eq. (4.15) we can find the total angular displacement by integrating the expressions for ω over the total time interval:

$$\theta_{total} = 0 + \int_0^3 3t^2\,dt + \int_3^{12} \left[27 - 3(t-3)\right] dt = 148.5 \text{ rad}$$