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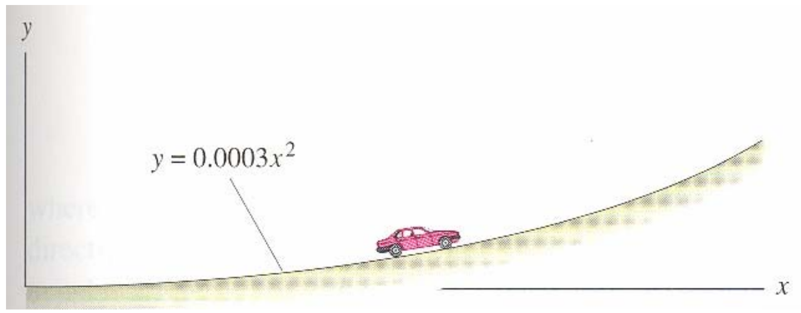
سایت آموزش مهندسی مکانیک ایران

Example 5.1

A car travels at a constant speed of 100 km/hr along a curved road of increasing grade whose vertical profile is approximated by the equation $y = 0.0003x^2$. What are the tangential and normal components of the acceleration of the car when its horizontal coordinate is $x = 400$ m?



Solution:

This problem is identical to Example 3.2 except that the tangential and normal (as opposed to horizontal and vertical) components of acceleration are required.

According to Eqs. (5.11) and (5.12) the tangential acceleration of a point moving along a curve is given by:

$$a_t = \frac{dv}{dt}$$

where, by definition, v is along the curve. In this case the velocity of the car (along the curve) is constant. Thus:

$$a_t = \frac{dv}{dt} = 0 \text{ m/s}^2$$

The normal acceleration of a point moving along a curve is given by:

$$a_n = \frac{v^2}{\rho}$$

where ρ is the radius of curvature of the path at a given point. The definition of ρ is given by Eqn. (5.14):

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left(\frac{d^2y}{dx^2}\right)}$$

In this case $\frac{dy}{dx} = 0.0006x$ and $\frac{d^2y}{dx^2} = 0.0006$. Substituting these expressions and $x = 400$ in the relation above we obtain:

$$\rho = \frac{\left[1 + \left(0.0006(400)\right)^2\right]^{\frac{3}{2}}}{0.0006} = 1813 \text{ m}$$

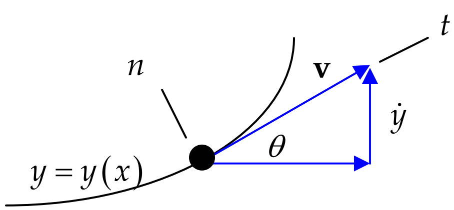
This results in a normal acceleration of:

$$a_n = \frac{\left(\frac{(100)(1000)}{3600}\right)^2}{1813} = 0.426 \text{ m/s}^2$$

These same results can also be obtained from the results obtained in Example 3.2 by a transformation of coordinates. Resolving the x and y components of the acceleration found in that example along the tangential and normal directions we obtain:

$$\begin{aligned} a_t &= \ddot{x}\cos\theta + \ddot{y}\sin\theta \\ a_n &= -\ddot{x}\sin\theta + \ddot{y}\cos\theta \end{aligned} \qquad \text{(i)}$$

where the angle θ is defined by the figure below:



As noted in Example 3.2 the angle θ can be determined from the curve parameters by:

$$\theta = \arctan\left(\frac{dy}{dx}\right) = \arctan(y')$$

In this case the value of θ at $x = 400$ m is:

$$\theta = \arctan\big((0.0006)(400)\big) = 13.50°$$

Substituting this value together with values for $\ddot{x}$ and $\ddot{y}$ obtained in Example 3.2 into the relations (i) we get:

$$a_t = 0 \text{ m/s}^2$$
$$a_n = 0.426 \text{ m/s}^2$$