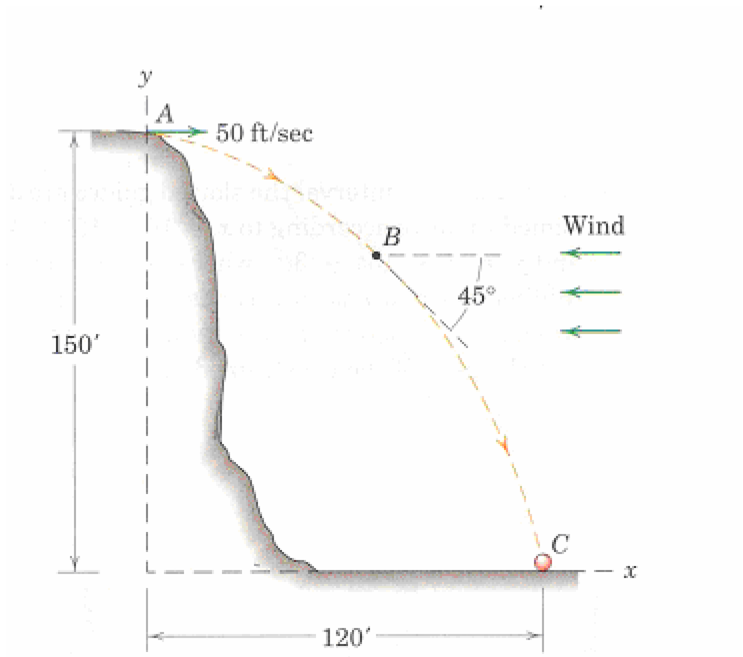
Example 5.2

A ball is thrown horizontally from the top of a 150-ft cliff at *A* with a speed of 50 ft/s and lands at point *C*. Because of the strong horizontal wind the ball has a constant acceleration in the negative *X*-direction. Determine the radius of curvature of the path of the ball at *B* where its trajectory makes an angle of 45° with the horizontal. Neglect any effect of air resistance in the vertical direction.



Solution:

First we need to compute the horizontal acceleration of the ball from the given data. Since the acceleration in the vertical direction is constant, the motion of the ball in the vertical direction is described by Eq. (3.10):

$$y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

In this case $y_0 = 150$ ft, $v_{y0} = 0$, and $y(t) = 0$. From this data the total time of travel can be obtained as:

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(150)}{32.2}} = 3.05 \text{ s}$$

Similarly, with constant acceleration in the horizontal direction the motion of the ball in the horizontal direction is described by:

$$x(t) = x_0 + v_{x0}t - \frac{1}{2}a_x t^2$$

where a_x is the constant acceleration due to the effect of the wind. In this equation $x_0 = 0$, $v_{x0} = 50$ ft/s, and when the ball reaches the bottom of the cliff $x(t) = 120$ ft, and $t = 3.05$ s. From this data, a_x can be obtained as:

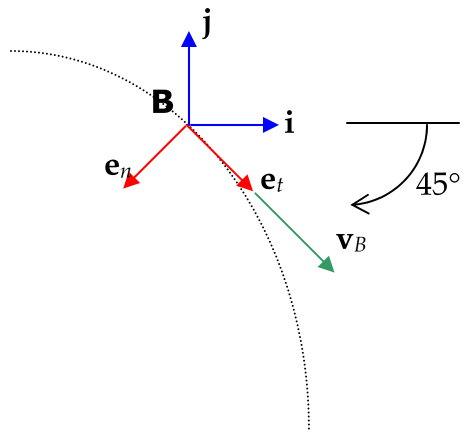
$$a_x = \frac{2}{t^2}\left(v_{x0}t - x(t)\right) = \frac{2}{(3.05)^2}\left((50)(3.05) - 120\right) = 7.00 \text{ ft/s}^2$$

Thus the acceleration of the ball can be written in vector form as:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = -7.00\mathbf{i} - 32.2\mathbf{j} \text{ ft/s}^2$$

Since both components of acceleration are constant this acceleration is valid at point B also.

To determine the radius of curvature of the path of the ball at point B we need to know the acceleration of the particle along the normal to the path of the particle at B (see Eq. (5.13)). To see the geometry, refer to the following figure:



From the figure above it is easy to see that:

$$\mathbf{e}_n = -\cos 45°\mathbf{i} - \sin 45°\mathbf{j}$$

The normal component of the acceleration a_n can be obtained by taking the projection of the total acceleration vector on $\mathbf{e}_n$. Thus:

$$a_n = \mathbf{a} \cdot \mathbf{e}_n = (-7.00\mathbf{i} - 32.2\mathbf{j}) \cdot (-\cos 45°\mathbf{i} - \sin 45°\mathbf{j}) = 27.71 \ \text{ft/s}^2$$

The normal component of acceleration is related to the radius of curvature ρ of the path by Eq. (5.13):

$$a_n = \frac{v^2}{\rho} \qquad \text{(i)}$$

Now that we know the magnitude of the normal component of acceleration, we need to compute the velocity of the ball at B, $\mathbf{v}_B$. Assume that the ball reaches B at time t_B. Then, using relations for constant acceleration, the components of $\mathbf{v}_B$ are given by:

$$v_{Bx} = v_{x0} - a_x t_B$$
$$v_{By} = v_{y0} - a_y t_B = -g t_B$$

We can compute t_B from the knowledge of the direction of $\mathbf{v}_B$ which is tangent to the path of the ball (the green arrow shown in the figure above) at B. Thus:

$$\frac{v_{By}}{v_{Bx}} = -\tan 45° = -1$$

This relation provides sufficient information to solve for t_B:

$$-g t_B = \left(v_{x0} - a_x t_B\right)(-1)$$

from which t_B can be found as:

$$t_B = \frac{v_{x0}}{\left(g + a_x\right)} = \frac{50}{(32.2 + 7.00)} = 1.28 \text{ s}$$

Thus the components of $\mathbf{v}_B$ are:

$$v_{Bx} = v_{x0} - a_x t_B = 41.1 \text{ ft/s}$$
$$v_{By} = v_{y0} - a_y t_B = -g t_B = -41.1 \text{ ft/s}$$

Now using (i) we can compute the radius of curvature of the path at B:

$$\rho_B = \frac{v_B^2}{a_n} = \frac{v_{Bx}^2 + v_{By}^2}{a_n} = \frac{(41.1)^2 + (-41.1)^2}{27.71} = 121.75 \text{ ft}$$