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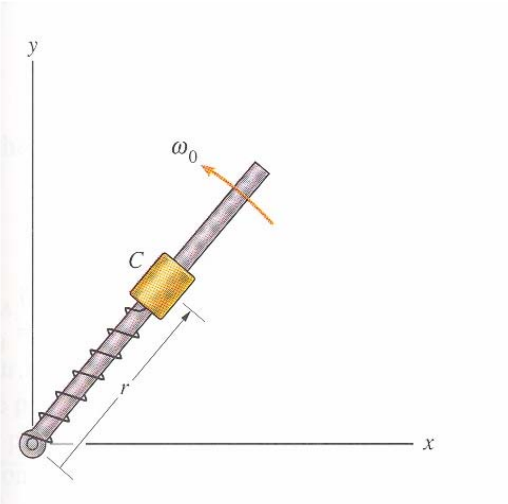
سایت آموزش مهندسی مکانیک ایران

Example 6.1

The bar shown rotates in the *xy* plane with constant angular velocity ω_0. The spring attached to the collar *C* causes the radial component of its acceleration to vary as $a_r = -Kr$ where *K* is a constant. When $r = r_0$ the radial component of the velocity of *C* is v_0. Determine the radial and transverse components of the velocity of *C* as functions of *r*.



Solution:

The radial and transverse components of the velocity and acceleration of a particle is given in polar coordinates by Eqs. (6.5) and (6.10):

$$v_r = \frac{dr}{dt} = \dot{r} \qquad v_\theta = r\frac{d\theta}{dt} = r\dot{\theta}$$
$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

In this case $\dot{\theta} = \omega_0 = \text{constant}$. Thus $\ddot{\theta} = \frac{d\omega_0}{dt} = 0$. Then the velocity and acceleration components become:

$$v_r = \dot{r} \qquad v_\theta = r\omega_0$$
$$a_r = \ddot{r} - r\omega_0^2 \qquad a_\theta = 2\dot{r}\omega_0$$

Using the given data the radial component of the acceleration can be written as:

$$\ddot{r} - r\omega_0^2 = -Kr \quad \text{or} \quad \ddot{r} = \left(\omega_0^2 - K\right)r \tag{i}$$

From this expression $\dot{r}$ can be obtained as a function of r by:

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d\dot{r}}{dr}\frac{dr}{dt} = \dot{r}\frac{d\dot{r}}{dr} = \left(\omega_0^2 - K\right)r$$
$$\dot{r}\,d\dot{r} = \left(\omega_0^2 - K\right)r\,dr$$

Now the two sides of this equation can be integrated using the initial values given ($v = v_0$ when $r = r_0$)

$$\int_{v_0}^{\dot{r}} \dot{r}\,d\dot{r} = \int_{r_0}^{r} \left(\omega_0^2 - K\right)r\,dr$$

which results in

$$\dot{r}^2 - v_0^2 = \left(\omega_0^2 - K\right)\left(r^2 - r_0^2\right)$$

or

$$\dot{r} = \left[v_0^2 + \left(\omega_0^2 - K \right)\left(r^2 - r_0^2 \right) \right]^{1/2}$$

Thus the radial and transverse components of the velocity of C as functions of r are:

$$v_r = \left[v_0^2 + \left(\omega_0^2 - K \right)\left(r^2 - r_0^2 \right) \right]^{1/2}$$
$$v_\theta = r\omega_0$$

In this case r can be solved for fully as a function of time from the relations given. Rearranging (i) we can write:

$$\ddot{r} + \left(K - \omega_0^2 \right) r = 0$$

This is a linear homogeneous second-order differential equation whose solution is given by:

$$r(t) = R\sin\left(\omega_n t + \phi \right) \qquad \text{for} \quad K > \omega_0^2$$

where $\omega_n = \sqrt{K - \omega_0^2}$.

The values of, R and ϕ can be determined from initial conditions as:

$$R = \sqrt{r_0^2 + \left(\frac{v_0}{\omega_n} \right)^2}$$
$$\phi = \arctan\left(\frac{\omega_n r_0}{v_0} \right)$$