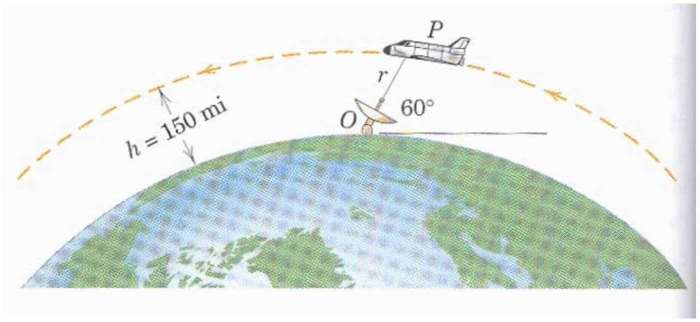
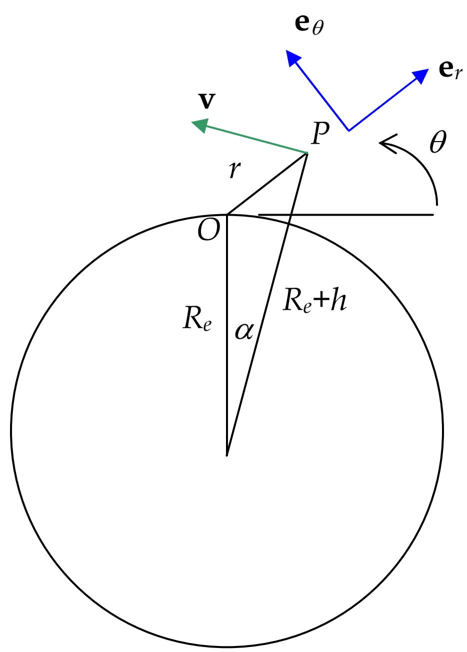
Example 6.2

At the instant depicted in the figure, the radar station at O measures the range rate of the space shuttle P to be $\dot{r} = -12,272$ ft/s, with O considered fixed. If it is known that the shuttle is in a circular orbit at an altitude $h = 150$ mi, determine the orbital speed of the shuttle from this information.



Solution:

To see the geometry of this problem consider the following figure where P locates the position of the shuttle and O locates the origin of the radar dish:



In the figure above R_e is the radius of the earth and is given by $R_e = 3959$ mi. The data given is $h = 150$ mi and $\theta = 60°$. From the given data it is also clear that:

$$r = \frac{h}{\sin\theta} = \frac{150}{\sin 60°} = 173.2 \text{ mi}$$

From this value the angle α can be calculated using the cosine rule:

$$r^2 = R_e^2 + (R_e + h)^2 - 2R_e(R_e + h)\cos\alpha$$

or

$$\alpha = \arccos\left[\frac{R_e^2 + (R_e + h)^2 - r^2}{2R_e(R_e + h)}\right] = 1.23°$$

Using polar coordinates the velocity of P is given by:

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

We only know the magnitude of the radial component ($v_r = \dot{r}$) of this velocity. We also know that the velocity must be tangent to the path of P (as shown by the green vector in the figure). Thus $\mathbf{v}$ is perpendicular to the line from the center of the earth to P. Thus we know the angle between the unit vector $\mathbf{e}_r$ and $\mathbf{v}$, say β. From the figure above the angle β can be calculated as:

$$\beta = 180 - (\alpha + \theta)$$

By taking the projection of $\mathbf{v}$ on the r direction we obtain:

$$v_r = v\cos\beta$$

where v is the magnitude of $\mathbf{v}$. Thus:

$$v = \frac{v_r}{\cos\beta} = \frac{\dot{r}}{\cos\beta} = \frac{-12272}{\cos(180 - 1.23 - 60)} = 25498 \text{ ft/s}$$