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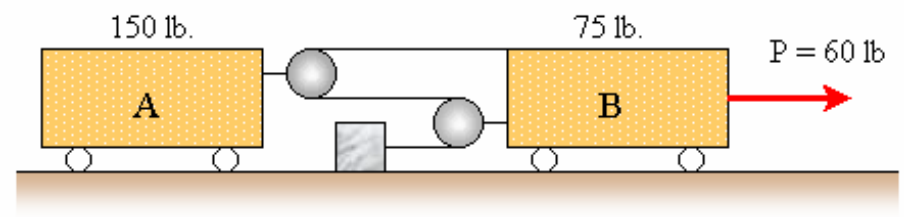
سایت آموزش مهندسی مکانیک ایران

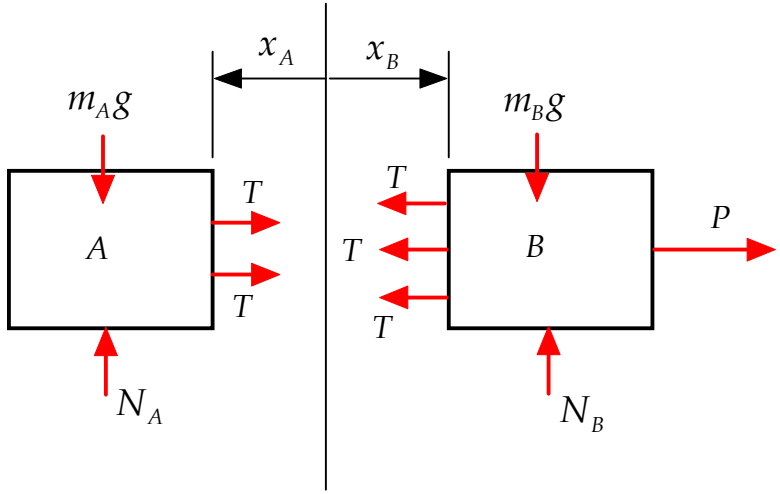
Example 7.1

Determine the accelerations of bodies *A* and *B* and the tension in the cable due to the application of the 60-lb force. Neglect all friction and the masses of the pulleys.



Solution:

The free body diagrams for the two masses and the kinematic parameters to be used in this problem are shown below. The tension in the string is denoted by *T*.



Using Newton's 2nd Law we can write:

$$m_A a_A = m_A \ddot{x}_A = -2T \tag{i}$$

$$m_B a_B = m_B \ddot{x}_B = P - 3T \tag{ii}$$

However the parameters x_A and x_B are not independent. Since the length of the string has to remain constant we can write:

$$2x_A + 3x_B = l \tag{iii}$$

where l is a constant that includes the total length of the string as well as the portions of the string that remain wrapped around the pulleys. Differentiating (iii) twice we obtain a relationship between accelerations of the two blocks:

$$2\ddot{x}_A + 3\ddot{x}_B = 0 \tag{iv}$$

Using this equation to eliminate $\ddot{x}_B$ from (ii) we obtain:

$$-\frac{2}{3} m_B \ddot{x}_A = P - 3T \tag{v}$$

Eliminating T from (i) and (v) we obtain:

$$\left(3m_A + \frac{4}{3} m_B\right)\ddot{x}_A = -2P$$

From this expression the acceleration of block A can be obtained as:

$$\ddot{x}_A = a_A = -\frac{2P}{\left(3m_A + \frac{4}{3} m_B\right)} = -\frac{2(60)}{\frac{1}{32.2}\left(3(150) + \frac{4}{3}(75)\right)} = -7.03 \text{ ft/s}^2 \rightarrow$$

The acceleration of block B can be obtained from (iv):

$$\ddot{x}_B = a_B = -\frac{2}{3}\ddot{x}_A = -\frac{2}{3}(-7.03) = 4.68 \text{ ft/s}^2 \rightarrow$$

The tension in the string can be obtained from (i):

$$T = -\frac{m_A \ddot{x}_A}{2} = -\frac{(150)(-7.03)}{(32.2)(2)} = 16.4 \text{ lbf}$$