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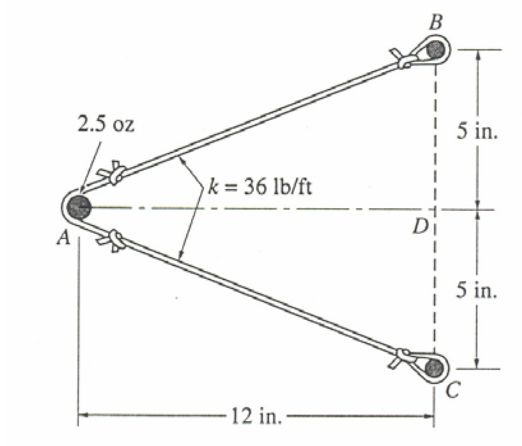
سایت آموزش مهندسی مکانیک ایران

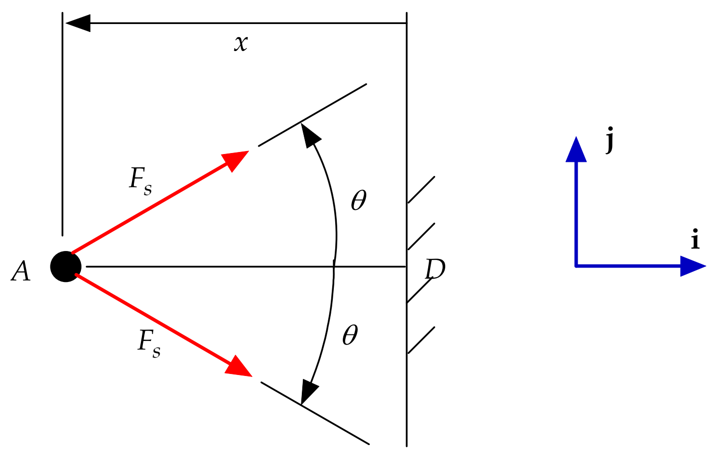
Example 7.4

A catapult is made of two elastic bands, each 5 in long when unstretched. Each band behaves like an ideal spring of stiffness 36 lb/ft. If a 2.5-oz rock is launched from the position shown, determine the speed of the rock when it leaves the catapult at D.



Solution:

Disregarding gravity the free-body diagram of the rock is as shown below.



The dynamic force balance equation in the horizontal direction is

$$m\ddot{x} = -2F_s \cos\theta$$

There is a negative sign on the right hand side because the spring forces act opposite to the direction of increasing x. The spring forces are given by

$$F_s = k\left(l - l_0\right) = \frac{36}{12}\left(\sqrt{x^2+25} - 5\right)\text{lb}$$

The angle that the spring forces make with the horizontal is given by

$$\cos\theta = \frac{x}{\sqrt{x^2+25}}$$

Thus the acceleration of the rock as a function of x is given by

$$\ddot{x} = -\frac{2k}{m}\left(\sqrt{x^2+25} - 5\right)\left(\frac{x}{\sqrt{x^2+25}}\right) = \frac{2k}{m}\left(\frac{5x}{\sqrt{x^2+25}} - x\right) \qquad \text{(i)}$$

Using Eqn. (2.7) in the class notes we can write (i) as:

$$\ddot{x} = \dot{x}\frac{d\dot{x}}{dx} = \frac{2k}{m}\left(\frac{5x}{\sqrt{x^2+25}} - x\right)$$

Separating the variables and integrating we obtain

$$\int_0^v \dot{x}\, d\dot{x} = \frac{2k}{m}\int_{12}^{0}\left(\frac{5x}{\sqrt{x^2+25}} - x\right)dx$$

Performing the integration and substituting the given values of the parameters we obtain

$$\frac{v^2}{2} = \frac{2(36/12)}{(2.5/16)/386.4}\left(5\sqrt{x^2+25} - \frac{x^2}{2}\right)\Bigg|_{12}^{0}$$

which results in

$$v = 974.5 \text{ in/s} = 81.21 \text{ ft/s}$$