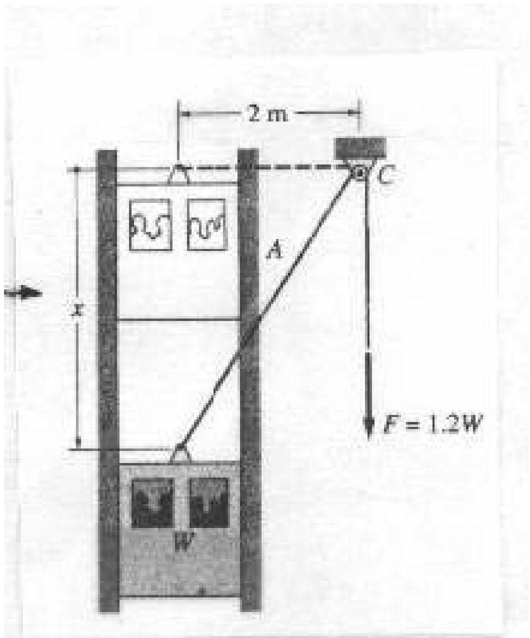
Example 7.6

An elevator of weight W rides in a vertical shaft with negligible friction operated by a cable that passes over the pulley C. The tension in the cable is kept constant at $F = 1.2W$. If the elevator starts from rest at position A, determine:

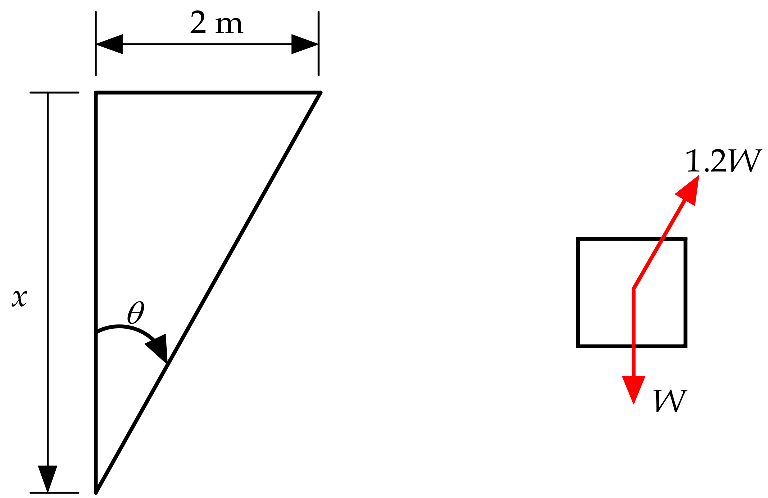
a) the acceleration of the elevator as a function of x;

b) the maximum speed of the elevator.



Solution:

The free-body diagram of the elevator is shown below.



a) Assuming that down is positive (since x is shown to be positive downward), from the free-body diagram of the elevator its acceleration in the vertical direction can be computed from:

$$ma = mg - F\cos\theta = W - 1.2W\cos\theta = W(1 - 1.2\cos\theta)$$

or

$$a = \frac{W(1-1.2\cos\theta)}{m} = \frac{W(1-1.2\cos\theta)}{W/g} = g(1-1.2\cos\theta)$$

Since the problem asks for the acceleration in terms of x we have to express the sinusoidal term in terms of x. From the geometry of the problem (see figure above):

$$\cos\theta = \frac{x}{\sqrt{2^2 + x^2}}$$

Thus the acceleration can be written as

$$a = g\left(1 - \frac{1.2x}{\sqrt{4 + x^2}}\right)\ \mathrm{m/s^2}$$

b) The velocity of the elevator is a maximum when the acceleration is zero. This occurs when

$$\frac{1.2x}{\sqrt{4 + x^2}} - 1 = 0$$

or when

$$x = 3.015\ \mathrm{m}$$

We now need an expression for the velocity of the elevator in terms of its displacement. From $a = v\dfrac{dv}{dx}$ we can write

$$\int_0^{v_{max}} v\,dv = \int_0^{3.015} g\left(1 - \frac{1.2x}{\sqrt{4 + x^2}}\right)dx$$

Carrying out the integration we obtain:

$$\frac{1}{2}v_{max}^2 = 1.073g$$

which results in

$$v_{\max} = 1.465\sqrt{g} \ \text{ m/s}$$