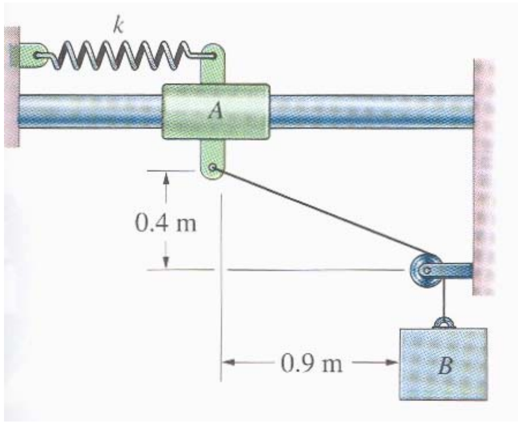
Example 8.1

The collar A slides on a smooth horizontal bar under the action of a spring and the weight B. The spring constant is $k = 850$ N/m, $m_A = 40$ kg, and $m_B = 60$ kg. The system is released from rest in the position shown with the spring unstretched. Determine the velocity of the collar when it has moved 0.5 m to the right.

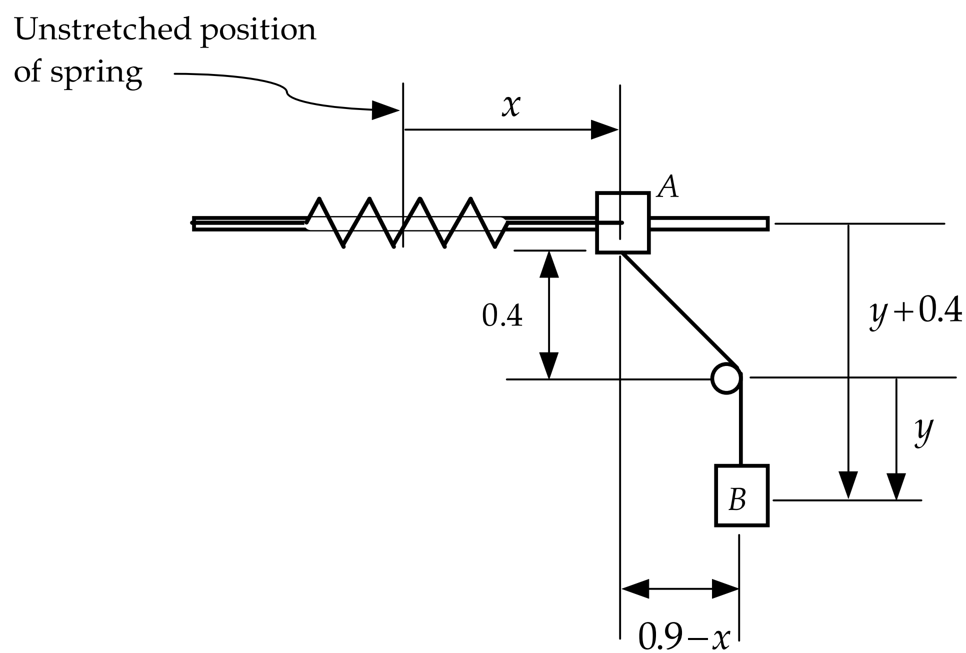


Solution:

This is a conservative systems since all forces acting on the particles are either conservative forces or workless constraints. Let the initial vertical position of the weight B be the datum line for measuring gravitational potential energy. Initially the total energy of the system is zero since both masses are at rest, the spring is unstretched, and weight B is at the datum line for gravitational potential energy:

$$E_1 = T_1 + V_{g1} + V_{s1} = 0 + 0 + 0 = 0$$

The configuration of the system when the collar *A* has moved by *x* m to the right is shown below.



From this figure the downward displacement of weight *B* can be written as

$$y = \sqrt{0.9^2 + 0.4^2} - \sqrt{0.4^2 + (0.9 - x)^2} \qquad \text{(i)}$$

From this equation the downward displacement of *B* when A has moved to the right by 0.5 m can be computed:

$$y_2 = \sqrt{0.9^2 + 0.4^2} - \sqrt{0.4^2 + 0.4^2} = 0.419 \text{ m}$$

The velocity of B for any position of A can also be computed by differentiating (i) with respect to time

$$\dot{y} = \frac{(0.9 - x)}{\sqrt{0.4^2 + (0.9 - x)^2}} \dot{x} \qquad \text{(ii)}$$

Thus the velocity of B when A has moved 0.5 m to the right is

$$\dot{y}_2 = \frac{(0.9 - 0.5)}{\sqrt{0.4^2 + (0.9 - 0.5)^2}} \dot{x}_2 = \frac{1}{\sqrt{2}} \dot{x}_2$$

Thus the total energy of the system when A has moved 0.5 m to the right is

$$\begin{aligned} E_2 &= T_2 + V_{g2} + V_{s2} = \frac{1}{2} m_A \dot{x}_2^2 + \frac{1}{2} m_B \dot{y}_2^2 - m_B g y_2 + \frac{1}{2} k x_2^2 \\ &= \frac{1}{2}\left(m_A + \frac{1}{2} m_B\right)\dot{x}_2^2 - m_B g y_2 + \frac{1}{2} k x_2^2 \end{aligned}$$

Since no energy is lost in this process energy is conserved and $E_1 = E_2$. We can therefore write

$$\frac{1}{2}\left(m_A + \frac{1}{2} m_B\right)\dot{x}_2^2 - m_B g y_2 + \frac{1}{2} k x_2^2 = 0$$

or

$$\frac{1}{2}\left(40 + \frac{1}{2}(60)\right)\dot{x}_2^2 = (60)(9.81)(0.419) - \frac{1}{2}(850)(0.5)^2$$

Solving for $\dot{x}$ we find the velocity of A when it has moved 0.5 m to the right

$$\dot{x}_2 = 2.003 \text{ m/s}$$