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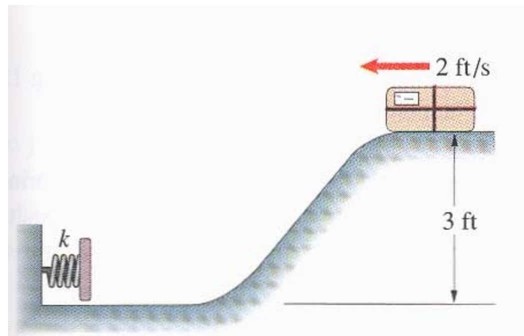
سایت آموزش مهندسی مکانیک ایران

Example 8.2

In a preliminary design for a mail sorting machine, parcels moving at 2 ft/s slide down a smooth ramp and are brought to rest by a linear spring. What should the spring constant be if it is required that the maximum acceleration of a 10-lb parcel not exceed 10g?



Solution:

Let the bottom of the ramp be the datum line for measuring gravitational potential energy and let the deflection of the spring when the package comes to rest be denoted by δ_{max}. The total energy of the package at the top of the ramp is

$$E_1 = T_1 + V_{g1} = \frac{1}{2}mv_1^2 + mgh$$
$$= \frac{1}{2}\left(\frac{10}{32.2}\right)(2)^2 + (10)(3) = 30.62 \text{ ft-lb}$$

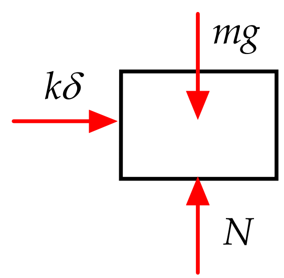
The total energy of the package when it is brought to rest by the spring is

$$E_2 = T_2 + V_{g2} + V_{s2} = \frac{1}{2} k \delta_{\max}^2$$

Since no energy is lost in this process energy is conserved and $E_1 = E_2$. We can therefore write

$$\frac{1}{2} k \delta_{\max}^2 = 30.62 \text{ ft-lb} \tag{i}$$

When the package is in contact with the spring at some general position when the spring deflection is δ, its free body diagram is as below.



The acceleration of the package in the horizontal direction at this general position is given by Newton's 2nd Law:

$$ma = k\delta \tag{ii}$$

This acceleration is to the right. However since the package is moving to the left throughout the time of contact with the spring the package is decelerating.

It is evident from (ii) that the deceleration of the particle will reach a maximum when the deflection in the spring reaches its maximum value. In other words,

$$ma_{\max} = k\delta_{\max} \tag{iii}$$

Since the maximum deceleration of the package is to be limited to 10g, we can determine the maximum deflection of the spring as

$$\delta_{\max} = \frac{m}{k}a_{\max} = \frac{10}{32.2k}(10)(32.2) = \frac{100}{k} \tag{iv}$$

Substituting this expression in (i) we obtain

$$\frac{1}{2}k\left(\frac{100}{k}\right)^2 = 30.62 \text{ ft-lb}$$

which can be solved for the desired k as

$$k = \frac{10000}{2(30.62)} = 163.29 \text{ lb/ft}$$