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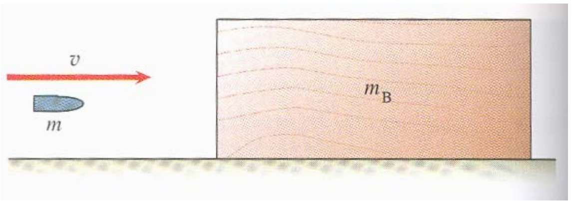
سایت آموزش مهندسی مکانیک ایران

Example 8.3

A bullet of mass m hits a stationary block of wood of mass m_B and becomes embedded in it. The coefficient of kinetic friction between the block and the floor is μ_k. Following the impact the block slides a distance d before stopping. What was the velocity v of the bullet?



Solution:

Since no external forces act on the system in the horizontal direction before motion begins (friction forces do not act until motion begins) the linear momentum of the system in the horizontal direction is conserved. This can be written as

$$(m + m_B)v' = mv \quad \text{(i)}$$

where v' is the velocity of the block and the bullet after impact.

The total energy of the system immediately after impact is the kinetic energy of the block with the bullet embedded in it. This energy is given by

$$T' = \frac{1}{2}(m + m_B)v'^2 = \frac{1}{2}\frac{m^2}{(m + m_B)}v^2 \quad \text{(ii)}$$

After impact the only force acting on the system in the horizontal direction is the friction force f_f which is given by

$$f_f = \mu_k \left(m + m_B\right) g$$

This force is not a conservative force and dissipates energy (consumes work) as the block with the embedded bullet moves against it. Since this force <u>is constant</u> (i.e. it does not depend on time or the displacement,or velocity of the block) the work it does (dissipates) as the block moves a distance d is given by

$$W = f_f d = \mu_k \left(m + m_B\right) g d \qquad \text{(iii)}$$

Since the block comes to rest this work equals the entire energy of the block plus the bullet. Specifically,

$$\mu_k \left(m + m_B\right) g d = \frac{1}{2} \frac{m^2}{\left(m + m_B\right)} v^2$$

which yields v in terms of the given quantities

$$v = \left(\frac{m + m_B}{m}\right) \sqrt{2 \mu_k g d}$$

<u>Effect of air resistance</u>

Suppose it is also desired to include air resistance in our calculations. Let the effect of the air resistance be a force $f_a = k\dot{s}$ that opposes the motion of the block. In this expression $\dot{s}$ is the velocity of the block plus the bullet at any time in the interval that begins immediately after impact and that ends when the block comes to rest. Now the total force resisting the motion of the block plus the bullet is

$$f_r = f_f + f_a = \mu_k\left(m + m_B\right)g + k\dot{s} = C + k\dot{s}$$

where $C = \mu_k\left(m + m_B\right)g$. In this case the resistive force is not conservative but it is also not constant. Specifically it depends on the velocity of the block. Since this is a one dimensional case Newton's law can easily be converted to a work-energy relationship. From

$$f_r = \left(m + m_B\right)\frac{d\dot{s}}{dt} = \left(m + m_B\right)\dot{s}\frac{d\dot{s}}{ds}$$

we can write

$$\left(C + k\dot{s}\right)ds = \left(m + m_B\right)\dot{s}d\dot{s}$$

Separating the variables and applying the given limits we obtain

$$\int_0^d ds = \left(m + m_B\right)\int_{v'}^0 \frac{\dot{s}d\dot{s}}{C + k\dot{s}}$$

Performing the integration we obtain

$$d = \left(m + m_B\right)\left[\frac{\dot{s}}{k} - \frac{C}{k^2}\ln\left(C + k\dot{s}\right)\right]\Bigg|_{v'}^0$$

or

$$d = \left(m + m_B\right)\left(\frac{C}{k^2}\ln\left(\frac{C + kv'}{C}\right) - \frac{v'}{k}\right)$$

This is a transcendental equation that cannot be inverted to express v' in terms of d.