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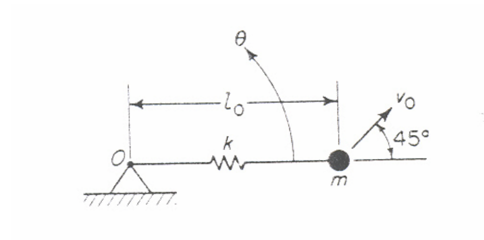
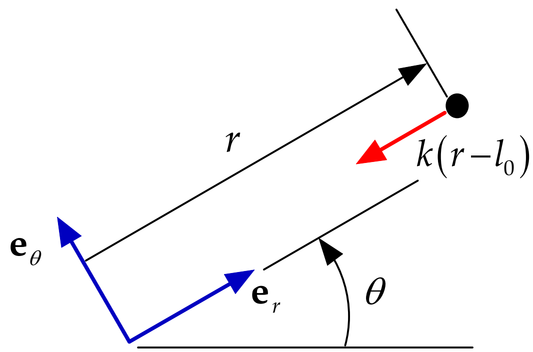
سایت آموزش مهندسی مکانیک ایران

Example 8.4

Initially the spring is unstretched with length l_0 and the particle of mass m is given a velocity v_0 in the direction shown. In the motion that follows the spring stretches to a maximum length of $4l_0/3$. Assuming no gravity solve for the spring stiffness k as a function of m, l_0, and v_0.





Solution:

Using polar coordinates the velocity of the particle is given by:

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

The angular momentum of the particle about O is given by:

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} = r\mathbf{e}_r \times m\left(\dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta\right) = mr^2\dot{\theta}\mathbf{e}_z$$

From the free-body diagram it is clear that the line of action of the only force acting on the particle passes through *O* and therefore has no moment about *O*. Thus $\mathbf{M}_O = \mathbf{0}$ which means $\mathbf{H}_O$ is constant. Thus we can write:

$$mr^2\dot{\theta} = mr_0^2\dot{\theta}_0$$

From the given initial conditions it is evident that $r_0 = l_0$ and $\dot{\theta}_0 = \dfrac{v_0}{\sqrt{2}l_0}$. Thus:

$$\dot{\theta} = \frac{v_0 l_0}{\sqrt{2}r^2} = \frac{h}{r^2} \tag{i}$$

where $h = \dfrac{v_0 l_0}{\sqrt{2}}$, a constant.

From the free-body diagram it is clear that the only force acting on the particle is a conservative force. Thus the principle of conservation of energy applies:

$$T + V = T_0 + V_0$$

From the velocity expression the kinetic energy at any time is obtained as:

$$T = \frac{1}{2}m\mathbf{v}\cdot\mathbf{v} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$$

and the potential energy at any time is:

$$V = \frac{1}{2}k\left(r - l_0\right)^2$$

From the initial conditions it is evident that $T_0 = \frac{1}{2} m v_0^2$ and $V_0 = 0$. The principle of conservation of energy yields:

$$\frac{1}{2} m\left(\dot{r}^2 + r\dot{\theta}^2\right) + \frac{1}{2} k\left(r - l_0\right)^2 = \frac{1}{2} m v_0^2$$

Substituting the relation described by (ii) and simplifying we obtain a relationship between r and $\dot{r}$:

$$m\left(\dot{r}^2 + \frac{h^2}{r^3}\right) + k\left(r - l_0\right)^2 = m v_0^2 \qquad \text{(ii)}$$

When $r = r_{\max} = \frac{4 l_0}{3}$ $\dot{r} = 0$. Substituting these values in (ii) we can solve for k in terms of the remaining given variables:

$$k = \frac{207}{32} \frac{m v_0^2}{l_0^2}$$