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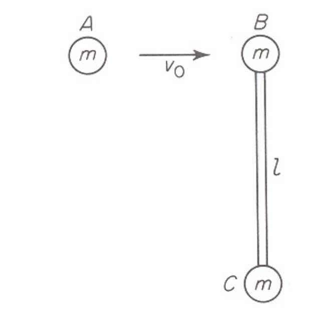
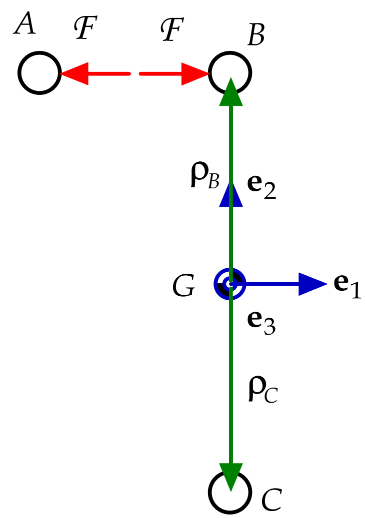
سایت آموزش مهندسی مکانیک ایران

Example 9.1

Particle A moving with velocity in a direction perpendicular to the motionless dumbbell BC, hits particle B squarely with a coefficient of restitution e.

a) Solve for the velocity of A and the translational and rotational velocities of the dumbbell after impact.
b) Assuming motion in free space find the value of e which will result in the collision of particles A and C as the motion proceeds.





a) Using a rotating reference frame whose origin is fixed at the center of mass of the dumbbell we can write for the velocity particle A after impact from the free-body impulse-momentum diagram:

$$-\mathcal{F}\mathbf{e}_1 = m_A\left(\mathbf{v}'_A - \mathbf{v}_A\right) = m\left(\mathbf{v}'_A - v_0\mathbf{e}_1\right)$$

or

$$\mathbf{v}'_A = \left(v_0 - \frac{\mathcal{F}}{m}\right)\mathbf{e}_1 \tag{1}$$

We can also write an impulse-momentum relationship for the center of mass G of the dumbbell:

$$\mathcal{F}\mathbf{e}_1 = (m_B + m_C)(\mathbf{v}'_G - \mathbf{v}_G) = 2m(\mathbf{v}'_G - 0) = 2m\mathbf{v}'_G$$

or

$$\mathbf{v}'_G = \frac{\mathcal{F}}{2m}\mathbf{e}_1 \tag{2}$$

The angular impulse-angular momentum relationship about the center of mass is given by:

$$\mathcal{M}_G = \mathbf{H}'_G - \mathbf{H}_G \tag{3}$$

Click here for derivation

where

$$\mathcal{M}_G = \left(\frac{l}{2}\mathbf{e}_2\right) \times \mathcal{F}\mathbf{e}_1 = -\frac{\mathcal{F}l}{2}\mathbf{e}_3 \tag{4}$$

Since the dumbbell is stationary before impact $\mathbf{H}_G = \mathbf{0}$. The angular momentum after impact can be computed from the definition of angular momentum:

$$\mathbf{H}'_G = \sum_{i=1}^{2}\left(\boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}'_i\right)$$

Since the particles are rigidly attached to each other

$$\dot{\boldsymbol{\rho}}'_i = \left(\dot{\boldsymbol{\rho}}'_i\right)_r + \boldsymbol{\omega}' \times \boldsymbol{\rho}_i = \boldsymbol{\omega}' \times \boldsymbol{\rho}_i$$

where $\boldsymbol{\omega}' = \omega' \mathbf{e}_3$ is the angular velocity of the rotating frame (and therefore the dumbbell) after impact. With $\boldsymbol{\rho}_B = \frac{l}{2}\mathbf{e}_2$ and $\boldsymbol{\rho}_C = -\frac{l}{2}\mathbf{e}_2$ the angular momentum of the dumbbell after impact can be obtained as:

$$\mathbf{H}'_G = \frac{ml^2}{2}\omega' \mathbf{e}_3 \qquad (5)$$

Substituting the relations from Eqs. (4) and (5) in Eq. (3) we obtain:

$$-\frac{\mathcal{F}l}{2} = \frac{ml^2}{2}\omega'$$

or

$$\omega' = -\frac{\mathcal{F}}{ml} \qquad (6)$$

Now we need to apply the impact condition between particles *A* and *B*. Using the definition of the coefficient of restitution:

$$v'_{nA} - v'_{nB} = e\left(v_{nB} - v_{nA}\right) \qquad (7)$$

At the instant of impact the normal direction is defined by $\mathbf{e}_1$. Before impact the velocity of particle *B* is zero $(v_{nB} = 0)$ and the velocity of particle *A* along the normal direction is v_0 $(v_{nA} = v_0)$. The velocity of particle *A* after impact is given by Eq. (3). The velocity of particle *B* after impact can be computed as:

$$\mathbf{v}'_B = \mathbf{v}'_G + \dot{\boldsymbol{\rho}}'_1 = \mathbf{v}'_G + \boldsymbol{\omega}' \times \boldsymbol{\rho}_1$$

Using Eqs. (2) and (6) the velocity of particle *B* after impact can written as:

$$\mathbf{v}'_B = \frac{\mathcal{F}}{2m}\mathbf{e}_1 + \left(-\frac{\mathcal{F}}{ml}\right)\mathbf{e}_3 \times \frac{l}{2}\mathbf{e}_2 = \frac{\mathcal{F}}{m}\mathbf{e}_1 \tag{8}$$

Thus

$$v'_{nB} = \frac{\mathcal{F}}{m} \tag{9}$$

Substituting relations from Eqs. (1) and (9) in Eq. (7) we obtain:

$$\left(v_0 - \frac{\mathcal{F}}{m}\right) - \frac{\mathcal{F}}{m} = e\left(0 - v_0\right)$$

which results in

$$\mathcal{F} = \frac{mv_0\left(1+e\right)}{2} \tag{10}$$

Substituting this result in Eqs. (1), (2), and (6) we obtain the motions of particle A and the dumbbell after impact:

$$\mathbf{v}'_A = \frac{v_0\left(1-e\right)}{2}\mathbf{e}_1 \tag{11}$$

$$\mathbf{v}'_G = \frac{v_0\left(1+e\right)}{4}\mathbf{e}_1 \tag{12}$$

$$\omega' = -\frac{v_0\left(1+e\right)}{2l} \tag{13}$$

b) Clearly after impact particle A and the center of mass G of the dumbbell will travel with constant velocity in the along the direction of impact. If the two velocities are different then particle A will have moved to a different location by the time particle C moves to its location at the time of impact due to the rotation of the dumbbell about its center of mass. Thus the only way that particle A and C can collide with each other is if the velocities of particle A and G are equal after impact. Applying this condition using Eqs. (11) and (12) we obtain a particular value of e:

$$\frac{v_0\left(1-e^*\right)}{2}=\frac{v_0\left(1+e^*\right)}{4}$$

or

$$e^*=\frac{1}{3}$$