A
Comments on Units

This text omits units on quantities wherever convenient in order to shorten the presentation and to allow the reader to focus on the development of ideas and principles. However, in an application of these principles the proper use of units becomes critical. Both design and measurement results will be erroneous and could lead to disastrous engineering decisions if they are based on improper consideration of units. This appendix supplies some basic information regarding units, which will allow the reader to apply the techniques presented in the text to real problems.

Beginning physics texts define three fundamental units of length, mass, and time and consider all other units to be derived from these. In the *International System* of units, denoted by SI (from the French 'Système International'), the fundamental units are as follows:

Quantity	Unit name	Unit abbreviation
Length	meter	m
Mass	kilogram	kg
Time	second	s

These units are chosen as fundamental units because they are permanent and reproducible quantities.

The United States has historically used length (in inches), time (in seconds), and force (in pounds) as the fundamental units. The conversion between the US Customary system of units, as it is called, and the SI units is provided as a standard feature of most scientific calculators and in many text and notebook covers. Some simple conversions are

$$1\,\text{kg} = 2.204622622\,\text{lb}$$

$$4.5\,\text{N} \cong 1\,\text{lbf}$$

$$2.54 \times 10^{-2}\,\text{m} = 1\,\text{in}$$

As most equipment and machines used in the United States in vibration design, control, and measurement are manufactured in US Customary units, it is important to be able to convert between the two systems. This is discussed in detail, with examples, by Thomson (1988).

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In vibration analysis, position is measured in units of length in meters (m), velocity in meters per second (m/s), and acceleration in meters per second squared (m/s^2). Since the basic equation of motion comes from a balance of force, each term in the equation

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{A.1}$$

must be in units of force. Force is mass times acceleration, $kg\,m/s^2$. This unit is given the special name of Newton, abbreviated N (i.e., $1\,N = 1\,kg\,m/s^2$). The coefficients in equation (A.1) then must have the following units:

Quantity	Unit symbol	Unit abbreviation
Mass	m	kg
Damping	c	kg/s
Stiffness	k	kg/s^2

Note that stiffness may also be expressed in terms of N/m, and damping (viscous friction) in terms of N m/s, and these units are normally used. Using other formulae and definitions in the text, the following units and quantities can be derived (some are given special names because of their usefulness in mechanics):

Quantity	Unit name	Abbreviation	Definition
Force	newton	N	$kg\,m/s^2$
Velocity	—	—	m/s
Acceleration	—	—	m/s^2
Frequency	hertz	Hz	1/s
Stress	pascal	Pa	N/m^2
Work	joule	J	N m
Power	watt	W	J/s
Area moment of inertia	—	—	m^4
Mass moment of inertia	—	—	kg/m^2
Density	—	—	kg/m^3
Torque	—	—	N/m
Elastic modulus	—	—	Pa

Because vibration measurement instruments and control actuators are often electromechanical transducers, it is useful to recall some electrical quantities. The fundamental electrical unit is often taken as the ampere, denoted by A. Units often encountered with transducer specification are as follows.

Quantity	Unit name	Abbreviation	Definition
Electrical potential	volt	V	W/A
Electrical resistance	ohm	Ω	V/A
Capacitance	farad	F	A s/V
Magnetic flux	weber	Wb	V s
Inductance	henry	H	V s/N

The gains used in the control formulation for vibration control problems have the units required to satisfy the units of force when multiplied by the appropriate velocity or displacement term. For instance, the elements of the feedback matrix G_f of Equation (7.18) must have units of stiffness, i.e., N/m.

Often these quantities are too large or too small to be convenient for numerical and computer work. For instance, a newton, which is about equal to the force exerted in moving an apple, would be inappropriately small if the vibration problem under study is that of a large building. The meter, on the other hand, is too large to be used when discussing the vibration of a compact disc in a stereo system. Hence, it is common to use units with prefixes, such as the millimeter, which is 10^3 m, or the gigapascal, which is 10^9 Pa. Of course, the fundamental unit kilogram is 10^3 g. The following table lists some commonly used prefixes:

Factor	10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^3	10^6	10^9
Prefix	pico-	nano-	micro-	milli-	centi-	kilo-	mega-	giga-
Abbreviation	P	n	μ	m	c	k	M	G

For example, a common rating for a capacitor is microfarads, abbreviated μF.

Many of the experimental results given in the text are discussed in the frequency domain in terms of magnitude and phase plots. Magnitude plots are often given in logarithmic coordinates. In this situation, the *decibel*, abbreviated db, is used as the unit of measure. The decibel is defined in terms of a power ratio of an electrical signal. Power is proportional to the square of the signal voltage, so that the decibel can be defined as

$$1\,\text{db} = 20\log_{10}\frac{V_1}{V_2}$$

where V_1 and V_2 represent different values of the voltage signal (from an accelerometer, for instance).

The phase is given in terms of either degrees (°) or radians (rad).

REFERENCE

Thomson, W.T. (1988) *Theory of Vibration with Applications*, 3rd ed, Prentice-Hall, Englewood Cliffs, New Jersey.