EN0175

Advanced Mechanics of Solids

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http://www.engin.brown.edu/courses/en175/

Course Outline

1. Introduction
1.1 Scope of the course
1.2 Basic concepts of solid mechanics
1.3 Overview of finite element method in computational solid mechanics
2. Mathematical background
2.1 Vector algebra
2.2 Index notations
2.3 Matrices and tensors
2.4 Vector and tensor calculus
3. Stress in a solid
3.1 Body forces, surface forces and traction vector at a point on the surface
3.2 Stress tensor at a point
3.3 Principal stresses at a point
3.4 Balance of momentum and equilibrium equations
4. Strain in a solid
4.1 Displacement field in a deformed solid
4.2 Strain tensor in a Solid
4.3 Principal strains at a point
4.4 Compatibility conditions on a strain field
4.5 Principal strains at a point
5. Mechanical Behavior of Solids
5.1 Role of experiments in solid mechanics
5.2 Elastic material behavior
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5.4 Viscoelastic material behavior
6. Boundary value problems for linear elastic solids
6.1 Field equations for plane strain deformation
6.2 Thick walled pressure vessel
6.3 Field equations for plane stress deformation
6.3 Plate with hole in tension, stress concentration
7. Variational methods for elastic solids
7.1 Principle of virtual work
7.2 Variational statement of governing equations
7.3 Work and energy theorems in solid mechanics
7.4 Derivations of field equations for thin plate in bending
8. The finite element method for numerical analyses
8.1 Finite elements
8.2 Element interpolation functions
8.4 Element strains, stresses and strain energy density
8.5 Element Stiffness Matrix
8.6 Global Stiffness Matrix
8.7 Boundary Loading
9. Boundary value problems for elastic-plastic materials
9.1 Tension-torsion of thin walled tubes
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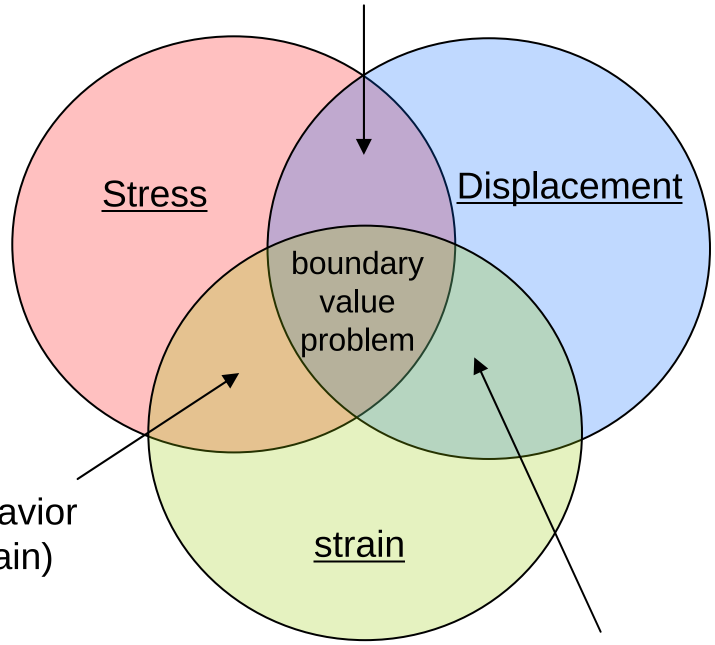
10. Failure modes in solid mechanics
10.1 Fracture
10.2 Fatigue
10.3 Buckling
10.4 Large deflections
10.5 Plastic collapse

Scope of the Course: Theory & computation in Mechanics of Deformable Solid



Momentum balance
(equilibrium)

Stress

Displacement

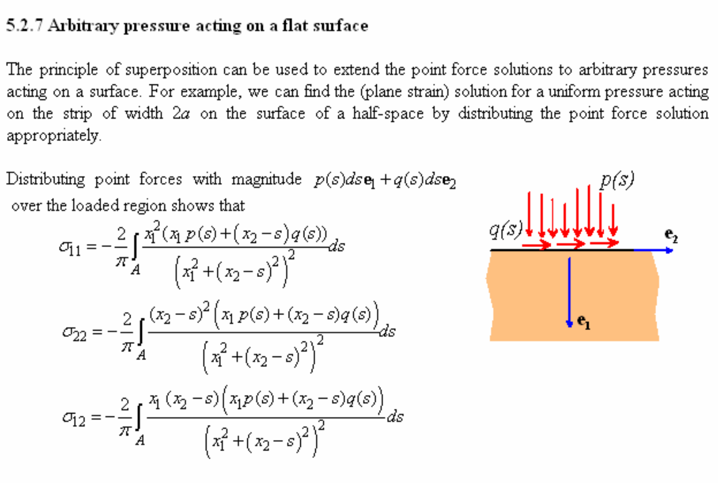
boundary
value
problem

strain

Material behavior
(stress-strain)

Compatibility
(strain-displacement)

Theory



5.2.7 Arbitrary pressure acting on a flat surface

The principle of superposition can be used to extend the point force solutions to arbitrary pressures acting on a surface. For example, we can find the (plane strain) solution for a uniform pressure acting on the strip of width $2a$ on the surface of a half-space by distributing the point force solution appropriately.

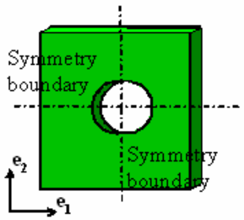
Distributing point forces with magnitude $p(s)ds\mathbf{e}_1 + q(s)ds\mathbf{e}_2$ over the loaded region shows that

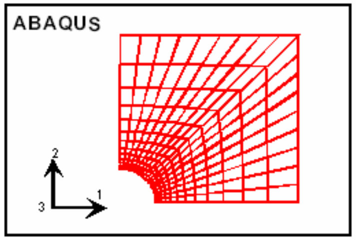
$$\sigma_{11} = -\frac{2}{\pi}\int_A \frac{x_1^2\left(x_1 p(s) + (x_2 - s)q(s)\right)}{\left(x_1^2 + (x_2 - s)^2\right)^2} ds$$

$$\sigma_{22} = -\frac{2}{\pi}\int_A \frac{(x_2 - s)^2\left(x_1 p(s) + (x_2 - s)q(s)\right)}{\left(x_1^2 + (x_2 - s)^2\right)^2} ds$$

$$\sigma_{12} = -\frac{2}{\pi}\int_A \frac{x_1 (x_2 - s)\left(x_1 p(s) + (x_2 - s)q(s)\right)}{\left(x_1^2 + (x_2 - s)^2\right)^2} ds$$

Computation





Prerequisites for the course

EN31 Mechanics of Solids and Structures
Mechanical behavior of materials and analysis of stress and deformation in engineering structures and continuous media. Topics include concepts of stress and strain; the elastic, plastic, and time-dependent response of materials; principles of structural analysis and application to simple bar structures, beam theory, instability and buckling, torsion of shafts; general three-dimensional states of stress; Mohr's circle; stress concentrations. Prerequisites: EN3; AM33.

EN3 Introduction to Engineering
An introduction to various engineering disciplines, thought processes, and issues. Topics include computing in engineering, engineering design, optimization, and estimation. Case studies in engineering are used to illustrate engineering fields and scientific principles, including in-depth studies of statics and optics. Laboratories and design projects are included.

AM33 Methods of Applied Mathematics
Mathematical techniques involving differential equations used in the analysis of physical, biological and economic phenomena. Emphasis on the use of established methods, rather than rigorous foundations. I: First and second order differential equations. II: Applications of linear algebra to systems of equations; numerical methods; nonlinear problems and stability; introduction to partial differential equations; ...

Exams

Midterm examination, to be held roughly at mid-semester, during class hrs. More details TBA.

Final examination, to be held as scheduled by the Registrar on 2 pm, Tues. 12/19/2006. Location TBA.

Computing Project

(http://www.engin.brown.edu/courses/en175/)

To provide experience with applying the principles covered in this course to practical engineering problems, and also to demonstrate your proficiency with finite element modeling, you will complete a detailed finite element analysis project.

The project will involve
1) Form a team (1-3 students).
2) Choose a problem to solve. One possibility is to apply FEA to help you with another design or independent study course, or a research project. Alternatively, you could consult the instructor for suggestions.
3) Write a short proposal that defines the problem to be solved, describes what will be calculated, outlines briefly how the calculation will be done.
4) Submit the proposal via email and obtain approval from the instructor.
5) Set up an FE model and perform the calculations.
6) Write a formal report no more than 10 pages for an individual report and no more than 15 pages for a team report that summarizes the results.
7) Give a short oral presentation to the rest of the class (10-20min depending on the class size and number of groups).

Some projects from previous years are available on course website.

Grading & Collaboration Policies

(http://www.engin.brown.edu/courses/en175/)

Grading Policy

Homework:	20%
Midterm exam:	20%
Project:	20%
Final:	35%

The lowest homework grade will not be counted.

Collaboration policy

We encourage discussions on homework and computer assignments: you can learn a lot from working with a group. This means that you are permitted to discuss homework problems and computer assignments with classmates, and are permitted to seek help from other students if you run into difficulties. However, material submitted for grading should represent independent work of its author. Any work done in collaboration should be clearly marked as such. It is not acceptable to copy the work of other students, and it is not acceptable for two students to submit identical copies of any part of an assignment.

Brown academic honor code

http://www.brown.edu/Administration/Dean_of_the_College/academic_code/

Class Meeting Times

(http://www.engin.brown.edu/courses/en175/)

Class Hours

Lectures: Tues & Thurs 2:30-3:50, BH 455. Teaching sessions on computational projects will be held in the Instructional Computer Facility (BH 191).

Sections: Sections will be provided on help with homework and computer assignments. They will be held in computer lab in weeks with ABAQUS assignments (i.e. usually!)

Text

(http://www.engin.brown.edu/courses/en175/)

There is no required text for this course.

Course notes, developed by Prof. Allan Bower (http://www.engin.brown.edu/courses/en175/)

Highly recommended textbooks:

“A first course in Continuum Mechanics" Y.C. Fung. 3rd Edition, Prentice-Hall, 1994.

“Classical and Computational Solid Mechanics" Y.C. Fung & Pin Tong. World Scientific, 2001.

Additional references:

``An Introduction to Continuum Mechanics, 3rd Edition," W. Lai, D. Rubin, and E. Krempl, Butterworth-Heinemann, 1995.

``Advanced Mechanics of Materials," W.B. Bickford, Addison-Wesley, 1998.

``Advanced Strength and Applied Stress Analysis," R.G. Budynas, McGraw-Hill, 1999.

``Advanced Strength of Materials," J.P. Den Hartog, Dover Publishing, 1996

``Introduction to the Mechanics of Continuous Media," L.E. Malvern, (recommended for advanced students only.

``Theory of Elasticity" S.P. Timoshenko and J.N. Goodier (Well written, and contains lots of useful solutions to elastic boundary value problems, but the notation is dated and the book does not cover plasticity or finite element analysis).

``Elasticity," J.R. Barber (A modern and well written introduction to linear elasticity). Kluwer, 2004.

Office Hours

Faculty:

Professor Huajian Gao
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Office hours: Drop by (open door) or email to schedule a time.

Graduate TA

Yuan Lin
Office: Barus & Holley 751
Email: yuan_lin@brown.edu

Office Hours: to be announced

Computing Resources

(http://www.engin.brown.edu/courses/en175/)

Finite element method: In this course, we will make extensive use of the commercial finite element code ABAQUS. (ABAQUS was founded by David Hibbitt, and Paul Sorensen, who are both Brown Alumni).

Running ABAQUS/CAE on the instructional computer facility:

ABAQUS/CAE comes with its own online tutorial. To access ABAQUS/CAE, you need to log into the Prince Lab computer facility, and from the Start menu on the bottom left hand corner of your screen, move to Programs and then to the Mech + Struct folder, and finally to ABAQUS/CAE. The program may take a few minutes to start up. Once it does, select the `tutorial' button from the startup window and follow instructions. If you'd like to make a desktop shortcut to CAE, right click on ABAQUS/CAE on the start menu and move down to create . This will work for any program in the start menu.

While you work, save your files on the default D:/ directory; this will minimize I/O and speed things up for you. But when you are done, copy your files (.cae, .odb, and any figures) to your file space on U:, or onto a zip or memory stick. The D: drive is a local disk, and the files on this disk may be deleted when you quit your session.

ABAQUS/CAE, like many complex programs, is prone to random crashes. To make sure you do not lose too much work on a crash, make sure you save the .cae file frequently.

If you do want to work from the U: drive, the full path no longer includes your name. For example, my file folder abaqus.cae on U: used to have the path U:\bower\abaqus.cae and now has U:\abaqus.cae

To print graphics directly from ABAQUS/CAE: On the File menu, select Print… Go to the Settings part of the dialog box, and for Destination, select the checkbox marked File, then select TIF format for the file type below, and finally enter a file name (either on your u:\ directory or on D:\) in the box provided. In the rendition area of the dialog box, select color. Then click OK. This will make a TIF file named filename in the appropriate directory. Double clicking on the TIF file in the directory will open the picture in some picture publishing program. Select Print... from the File... menu in that program and send the picture directly to the printer. You can also create postscript file in ABAQUS if you prefer; and use the PSPrint program to print it.

A quick way of printing ABAQUS/CAE graphics is to use the ALT+PRINT SCREEN key on the computer. This will copy an image of the active window to the Clipboard. You can then paste the clipboard contents into another file, such as a Word document, for printing. The PRINT SCREEN, key, when pressed without the ALT puts an image of the full screen on the clipboard. This image can be pasted into any program, such as Word.

Other Resources:

ABAQUS/Standard tutorial (for advanced users only - you only need to use ABAQUS/Standard to access features of ABAQUS that are not available in ABAQUS/CAE)

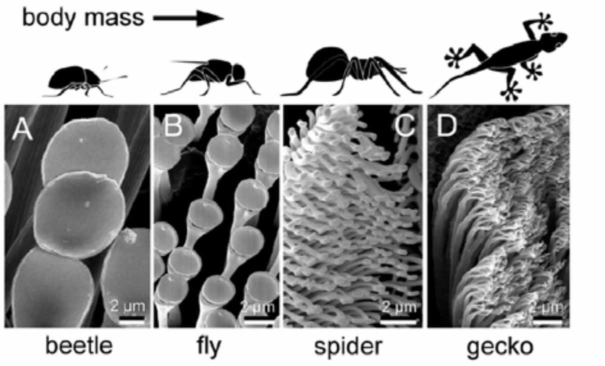
Maple tutorial (from your EN3 days)

Maple codes:

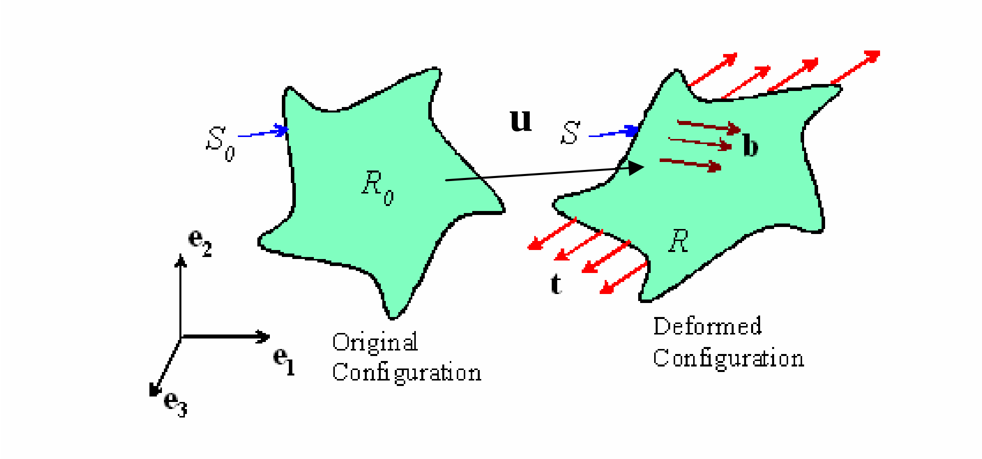
1. Simple 2D linear elastic code using constant strain triangles and sample input file
2. Advanced linear elastic code (both 2D and 3D)

Project ideas: be innovative/imaginative

(http://www.engin.brown.edu/courses/en175/)

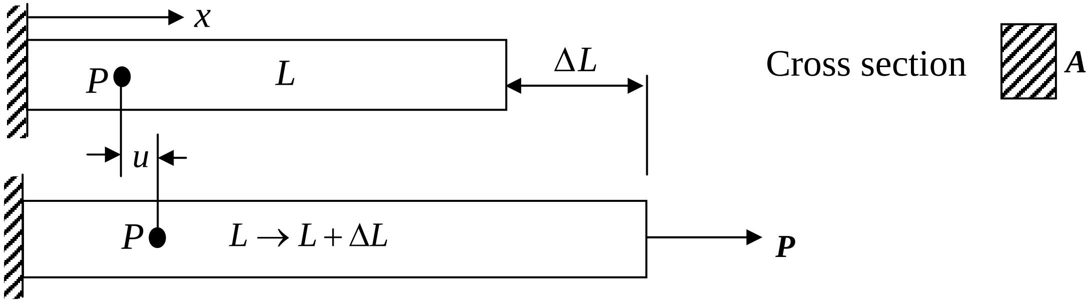


Review of Basic Concepts (EN31)

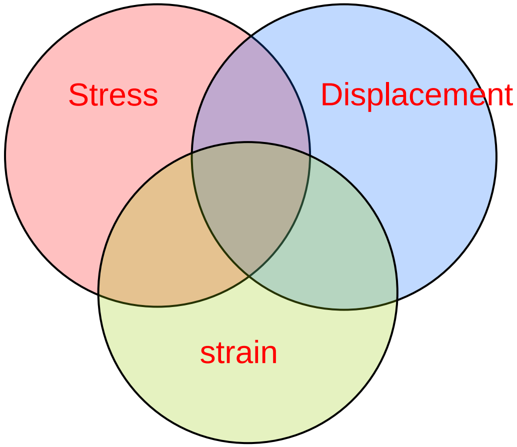


- Mechanics – branch of physics concerned with action of forces on materials systems.
- Solids – body of definite shape and volume; not liquid or gases
- Deformable – changes shape when stress is applied.
- Continuum – material of continuous extent, as oppose to discrete or atomistic.

Displacement, stress and strain



Solid mechanics


Stress

Displacement

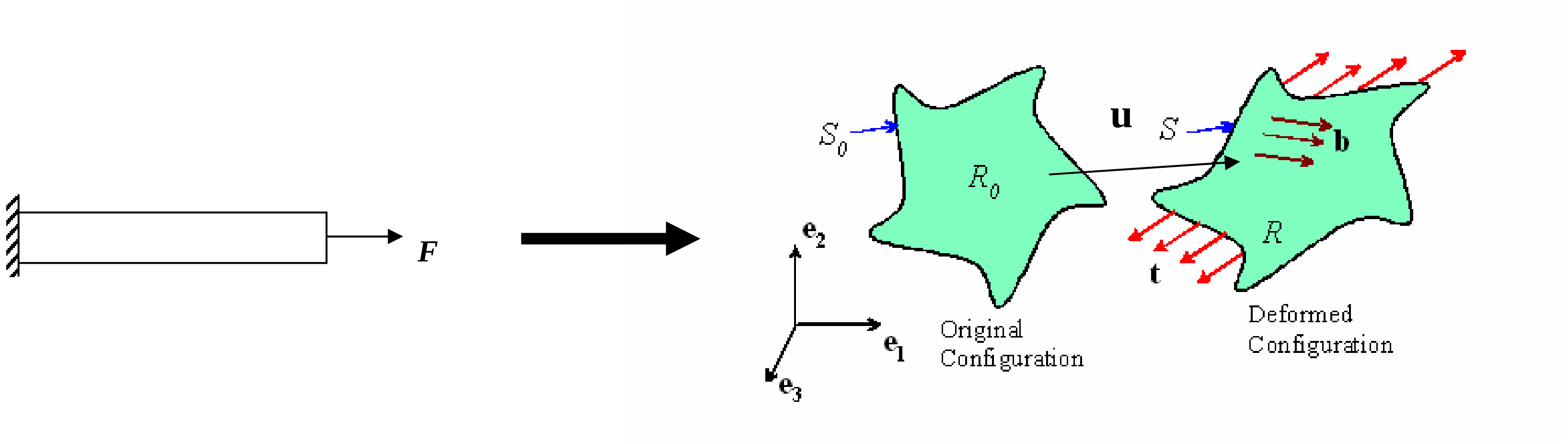
strain

Displacement: $u = u(x)$ = the distance over which a material point has moved during deformation

Stress: $\sigma = P / A$ = force per unit area

Strain: $\varepsilon = \Delta L / L$ =percent of elongation

What will we learn in EN175: Generalization of concepts to 3D

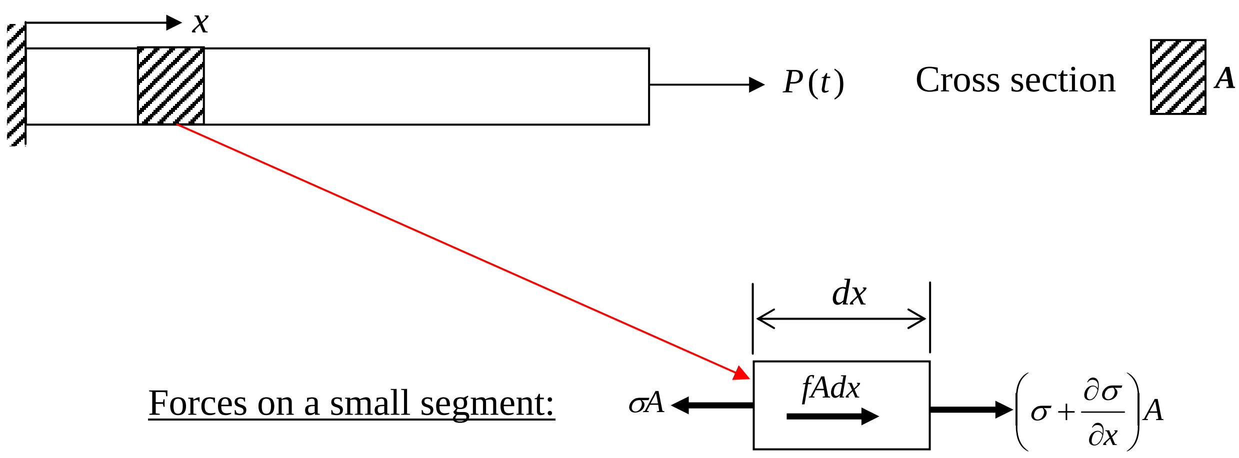


Displacement: vectors
Stress and strain: tensors
Governing equations
Material behaviors

We will learn the mathematical framework to understand the concepts of displacement, stress and strain in a fully three dimensional continuum solid body.

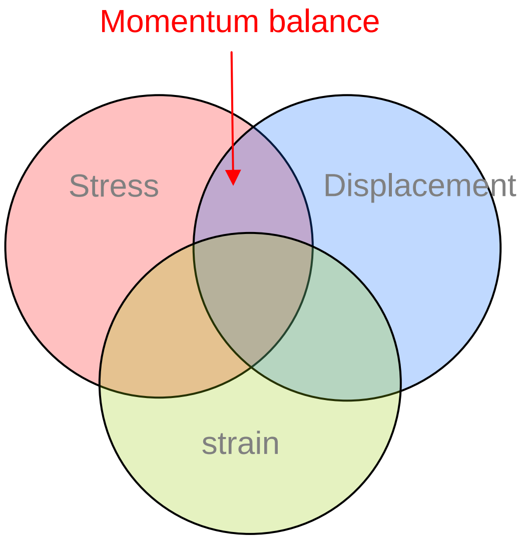
Equation of motion (equilibrium): momentum balance

Stress $\sigma = \sigma(x,t)$



Solid mechanics

Forces on a small segment:



Newton's law: $\sum F = ma$

$$\sum F = \left(\sigma + \frac{\partial \sigma}{\partial x}dx\right)A - \sigma A + fAdx = \left(\frac{\partial \sigma}{\partial x} + f\right)Adx$$

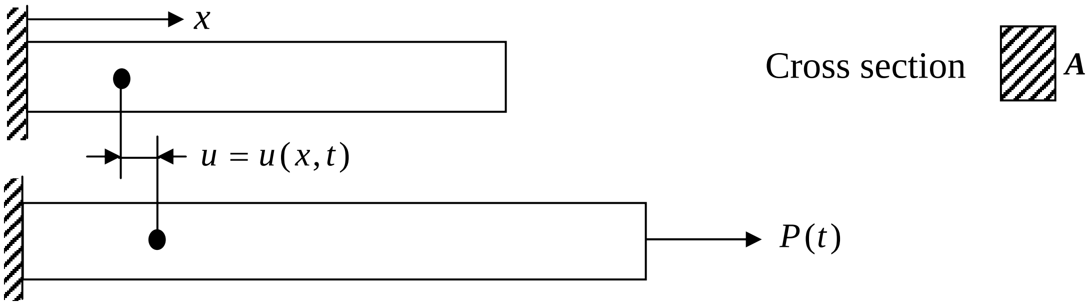
$$m = \rho A dx$$

$$a = \frac{\partial^2 u}{\partial t^2}$$

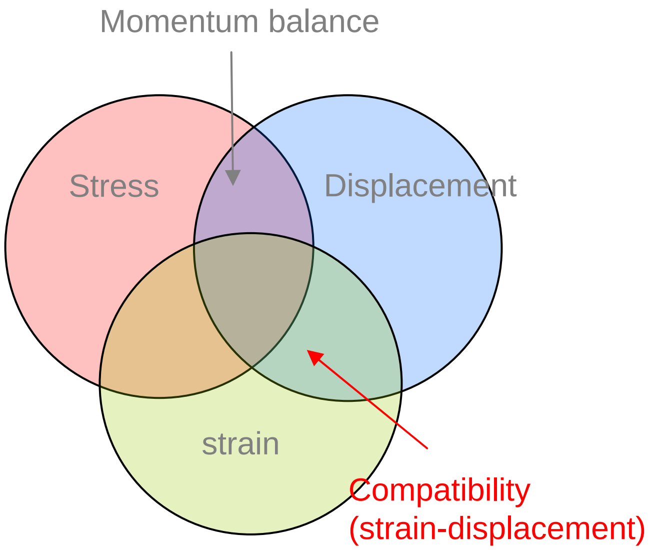
Equilibrium equation:

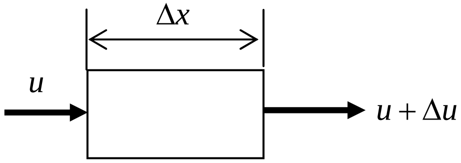
$$\sum F = ma \rightarrow \boxed{\frac{\partial \sigma}{\partial x} + f = \rho \frac{\partial^2 u}{\partial t^2}}$$

Strain-displacement relation: Kinematic equation



Solid mechanics



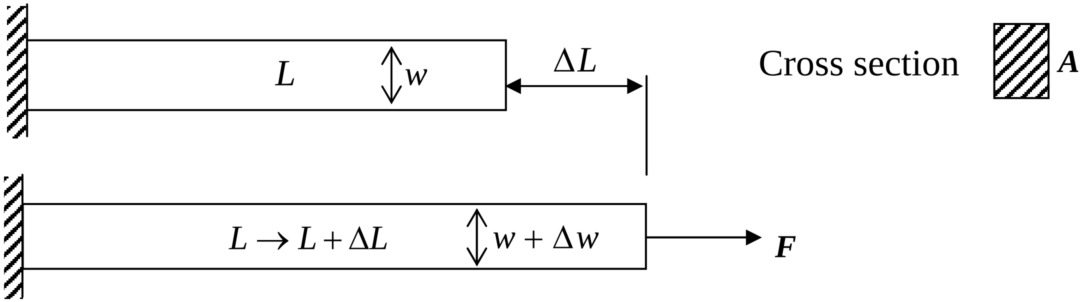


$$\varepsilon = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{\partial u}{\partial x}$$

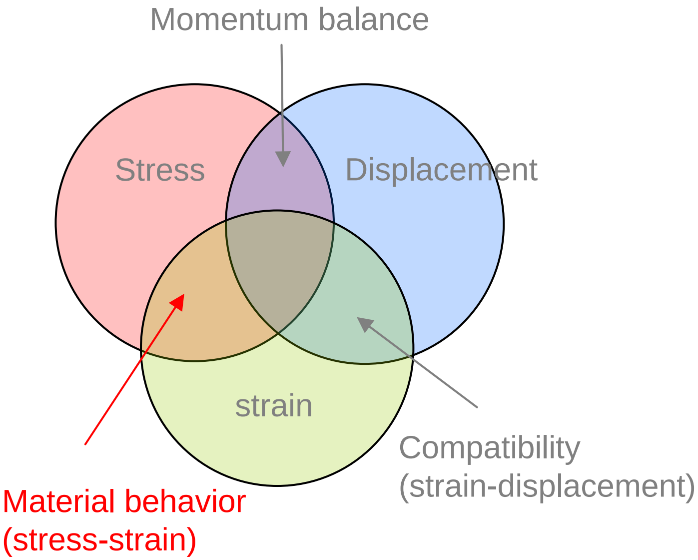
Strain is related to displacement gradient

$u = \text{constant}$ → rigid body motion

Stress-strain relation: material behavior



Solid mechanics



Momentum balance

Stress

Displacement

strain

Material behavior
(stress-strain)

Compatibility
(strain-displacement)

$\sigma = F / A \qquad \varepsilon = \Delta L / L \qquad \varepsilon' = \Delta w / w < 0$

Empirical/experimental observations for most solid materials under small deformation:

Hooke’s law: $\sigma = E\varepsilon$ or $\varepsilon = \dfrac{\sigma}{E}$

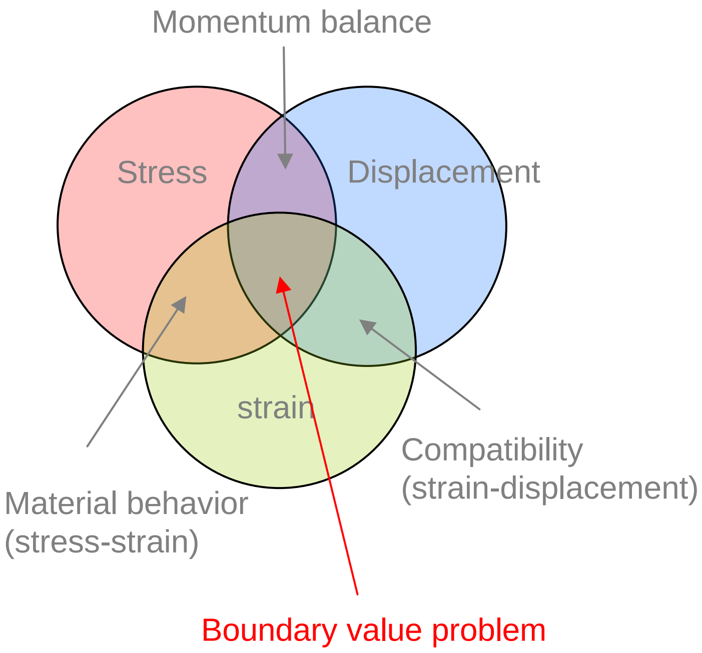
E =Young’s modulus

Poisson ratio: $\varepsilon' = -\nu\varepsilon$

Remark: When deformation increases, the material may undergo plastic deformation, creep, or even failure.

Boundary value problem in solid mechanics

Solid mechanics



Momentum balance

Stress

Displacement

strain

Material behavior
(stress-strain)

Compatibility
(strain-displacement)

Boundary value problem

Equilibrium equation: $\frac{\partial \sigma}{\partial x} + f = \rho \frac{\partial^2 u}{\partial t^2}$

Strain-displacement relation: $\varepsilon = \frac{\partial u}{\partial x}$

Hooke's law: $\sigma = E\varepsilon$

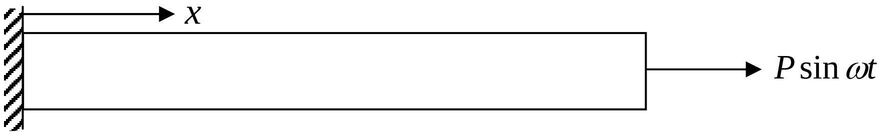
Unknowns: $u, \quad \varepsilon, \quad \sigma$

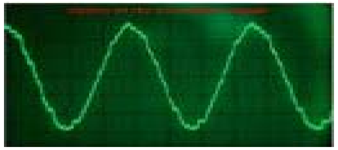
Boundary conditions:

Displacement BC: $u = \hat{u}$ @ boundaries

Traction BC: $\sigma = \hat{\sigma}$ @ boundaries

Example of BVP in Elasticity





$$\frac{\partial\sigma}{\partial x}+\cancel{f}=\rho\frac{\partial^2 u}{\partial t^2}\qquad \varepsilon=\frac{\partial u}{\partial x}\qquad \sigma=E\varepsilon=E\frac{\partial u}{\partial x}$$

$$\longrightarrow\quad \boxed{E\frac{\partial^2 u}{\partial x^2}=\rho\frac{\partial^2 u}{\partial t^2}}$$

BCs: $u=0$ @ $x=0$

$E\dfrac{\partial u}{\partial x}=P\sin\omega t$ @ $x=L$

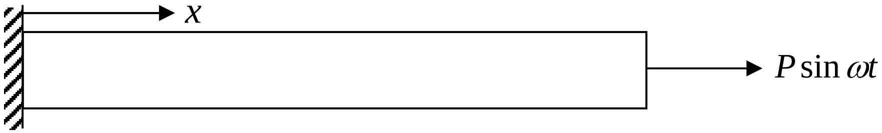
Method of solution: $u=U(x)\sin\omega t$ (separation of variable method in PDE)

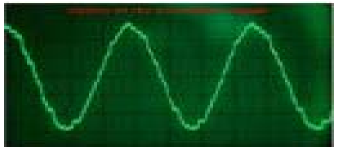
$\boxed{EU''(x)=-\rho\omega^2U(x)}$ BCs: $U(0)=0,\quad EU'(L)=P$

Solution: $U(x)=P\left(\sin\sqrt{\dfrac{\rho}{E}}\omega x\right)\Big/\left(\sqrt{\rho E}\,\omega\cos\sqrt{\dfrac{\rho}{E}}\omega L\right)$

$$u(x,t)=P\left(\sin\sqrt{\frac{\rho}{E}}\omega x\sin\omega t\right)\Big/\left(\sqrt{\rho E}\,\omega\cos\sqrt{\frac{\rho}{E}}\omega L\right),\qquad \sigma(x,t)=P\sin\omega t\left(\cos\sqrt{\frac{\rho}{E}}\omega x\Big/\cos\sqrt{\frac{\rho}{E}}\omega L\right)$$

Resonance





$$\sigma(x,t)=\frac{P\sin\omega t\cos\sqrt{\frac{\rho}{E}}\omega x}{\cos\sqrt{\frac{\rho}{E}}\omega L}$$

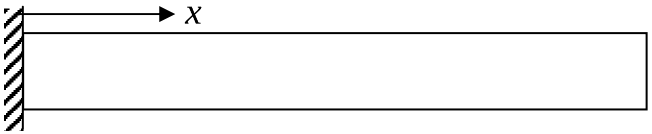
Large stresses develop when

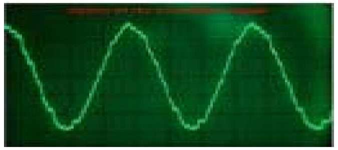
$$\cos\sqrt{\frac{\rho}{E}}\omega L=0 \qquad \sqrt{\frac{\rho}{E}}\omega L=\left(n+\frac{1}{2}\right)\pi \qquad n\ \text{=integer}$$

$$\omega=\left(n+\frac{1}{2}\right)\frac{\pi}{L}\sqrt{\frac{E}{\rho}} \longrightarrow \text{Structural failure}$$

Resonance phenomena can be widely observed: shattering of glass at certain high pitch; ultrasonic destruction of kidney stone; etc.

Natural Vibration of a Structure





$$\boxed{E\frac{\partial^2 u}{\partial x^2} = \rho\frac{\partial^2 u}{\partial t^2}}$$

BCs: $u = 0$ @ $x = 0$

$\sigma = E\dfrac{\partial u}{\partial x} = 0$ @ $x = L$

Assume: $u = U(x)\sin\omega t$

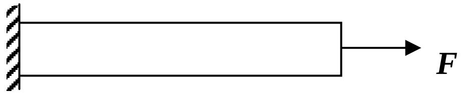
$\boxed{EU''(x) = -\rho\omega^2 U(x)}$ BCs: $U(0) = 0,\quad U'(L) = 0$

There are solutions of type: $U(x) = A\sin\sqrt{\dfrac{\rho}{E}}\omega x$ when $\omega_N = \left(n+\dfrac{1}{2}\right)\dfrac{\pi}{L}\sqrt{\dfrac{E}{\rho}}$ (natural frequencies of vibration of a structure)

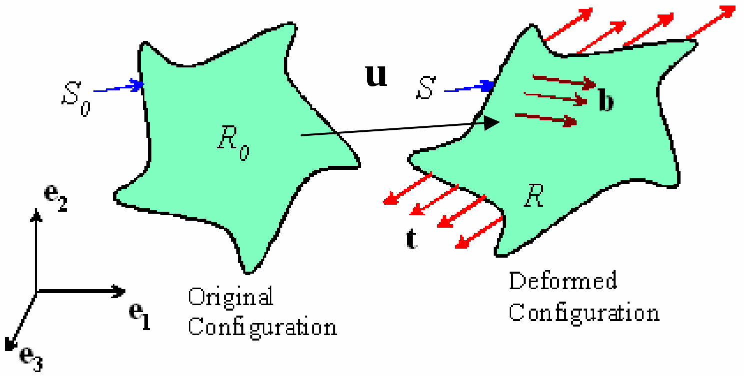
$$\boxed{u(x,t) = A\sin\frac{(2n+1)\pi x}{2L}\sin\omega t}$$

Resonance of a structure occurs when the frequency of applied force matches the natural vibrating frequency of the structure, i.e. $\omega = \omega_N$

What will we learn in EN175:
Formulation of BVPs of Solid Mechanics in 3D



$\longrightarrow$



$$\frac{\partial \sigma}{\partial x} + f = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\varepsilon = \frac{\partial u}{\partial x}$$

$$\sigma = E\varepsilon = E\frac{\partial u}{\partial x}$$

+ BCs $\longrightarrow$?

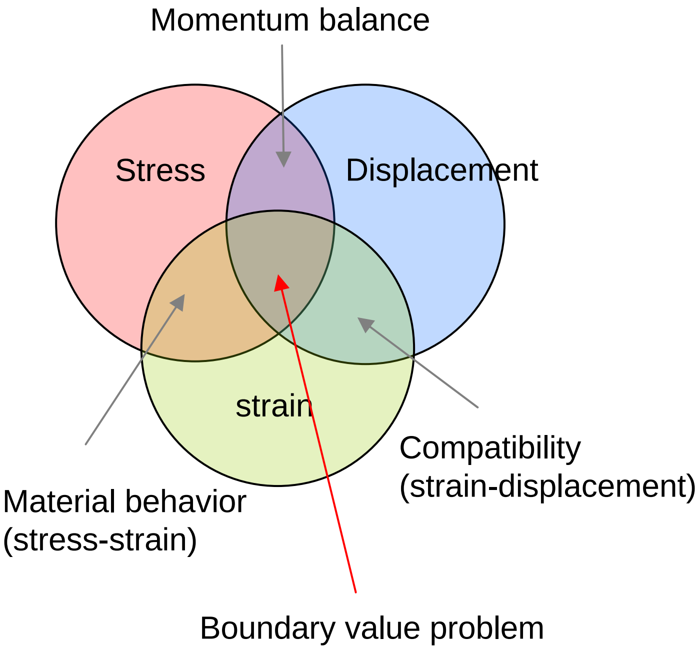
Governing principles and equations
Materials behaviors
Boundary conditions
Analytical solutions
Numerical analysis
Failure mechanisms

Applications of Solid Mechanics

The mechanics of deformable solids is an engineering science that underlies many engineering disciplines.

Mechanical engineering – designing load bearing components for vehicles; power generation and transmission; pressure vessels; engins and turbines
Civil engineering – designing foundations or structures (bridges, buildings);
Aeronautical engineering – aircraft; space shuttles;
Geomechanics – modeling the shape of planets; tectonics; and earthquake prediction;
Manufacturing engineering – designing metal and polymer forming processes; machining, etc;
Biomechanics – designing implants; bone mechanics; biomimetics; cellular and molecular processes;
Materials Science – designing composites; alloy microstructures, thin films, and developing materials processing
Microelectronics – designing failure resistant interconnects and packaging;
Nanotechnology – stress driven self-assembly on surfaces; manufacturing processes such as nano-imprinting; modeling atomic-force microscope/sample interactions.

Reading Assignment



Course notes section 1.1 (available on the course website)

- ***Recommended reading:***

Chapter 1 (especially 1.11), “A first course in Continuum Mechanics" Y.C. Fung. 3rd Edition, Prentice-Hall, 1994.