

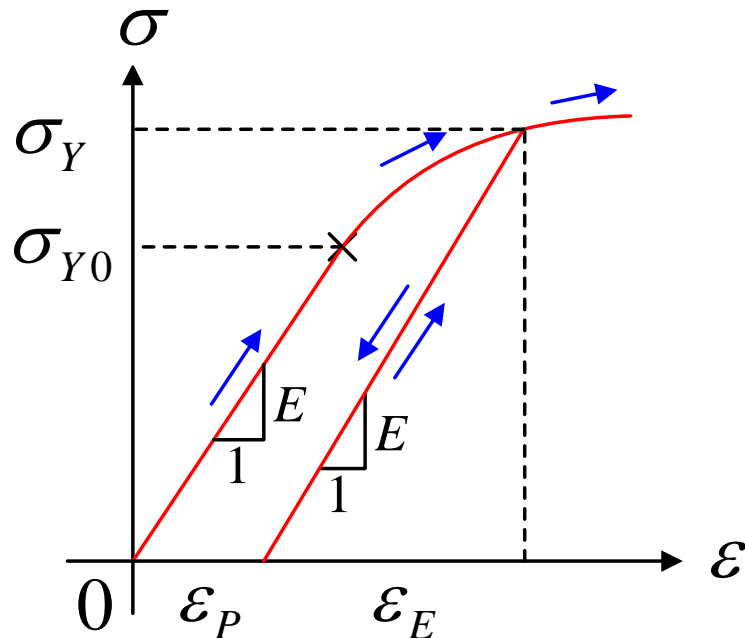
<http://www.Drshokuhi.com>

سایت آموزش مهندسی مکانیک ایران

Plastic material behavior

Yield condition: $\sigma = \sigma_Y$

Plastic loading: $\sigma = \sigma_Y, d\sigma > 0$



We will now denote the initial yield stress as σ_{Y0} and the current yield stress as σ_Y ; see above figure.

Decompose strain into elastic & plastic parts

$$\varepsilon = \varepsilon_E + \varepsilon_P$$

In incremental form:

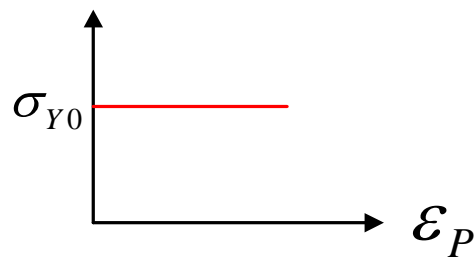
$$d\varepsilon_E = \frac{d\sigma}{E}$$

$$d\varepsilon_P = \begin{cases} \frac{d\sigma}{h}, & \text{if } \sigma = \sigma_Y, d\sigma > 0 \\ 0, & \text{otherwise} \end{cases}$$

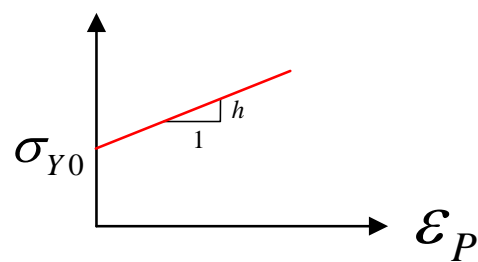
$h = \frac{d\sigma}{d\varepsilon_P}$ is the tangent modulus of $\sigma - \varepsilon_P$ curve (which is obtained from experiments or

fitting an assumed mathematical curve to experimental data).

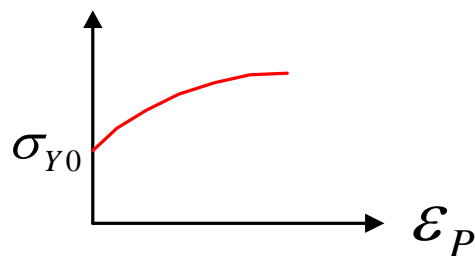
Perfectly plastic: $h = 0$



Linear work hardening: $h = \text{const.}$



Power law hardening: $\sigma_Y = \sigma_{Y0} \left(1 + \frac{E \varepsilon_P}{\sigma_{Y0}} \right)^N$



How to generalize this idea to 3D?

For elastic part,

$$\varepsilon_E = \frac{\sigma}{E} \quad \Rightarrow \quad \varepsilon_{ij}^E = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij} = \frac{\sigma'_{ij}}{2\mu} + \frac{\sigma_{kk}}{9K} \delta_{ij}$$

$$d\varepsilon_{ij} = d\varepsilon_{ij}^E + d\varepsilon_{ij}^P$$

$$d\varepsilon_{ij}^E = \frac{1}{2\mu} d\sigma'_{ij} + \frac{1}{9K} \delta_{ij} d\sigma_{kk}$$

The plastic strain is typically modeled by the Levy-Mises theory,

$$d\varepsilon_{ij}^P = \sigma'_{ij} d\lambda$$

The yield condition:

$$\sigma_e = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}} = \sigma_Y \quad (\text{von Mises stress})$$

shows that σ_e generalizes the 1D stress to an effective stress in 3D.

To utilize the measured plastic stress-strain behavior in 1D, we also need an effective measure of plastic strain in 3D. To see how this is generalized, consider strain energy,

$$dw = \sigma'_{ij} d\varepsilon_{ij} = \sigma'_{ij} (d\varepsilon_{ij}^E + d\varepsilon_{ij}^P) = dw^E + \sigma'_{ij} d\varepsilon_{ij}^P = dw^E + dw^P$$

$$dw^P = \sigma'_{ij} d\varepsilon_{ij}^P = \sigma_e d\varepsilon_P = \sigma'_{ij} \sigma'_{ij} d\lambda = \frac{2}{3} \sigma_e^2 d\lambda \quad \Rightarrow \quad d\lambda = \frac{3}{2} \frac{d\varepsilon_P}{\sigma_e}$$

$$d\varepsilon_{ij}^P = \frac{3}{2} \sigma'_{ij} \frac{d\varepsilon_P}{\sigma_e}$$

Self consistency requires,

$$d\varepsilon_{ij}^P d\varepsilon_{ij}^P = \frac{3}{2} \sigma'_{ij} \frac{d\varepsilon_P}{\sigma_e} \cdot \frac{3}{2} \sigma'_{ij} \frac{d\varepsilon_P}{\sigma_e} = \frac{3}{2} d\varepsilon_P^2$$

Therefore,

$$\varepsilon_P = \sqrt{\frac{2}{3} d\varepsilon_{ij}^P d\varepsilon_{ij}^P}$$

Summary:

Generalization from 1D to 3D,

$$\sigma \rightarrow \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}}, \quad d\varepsilon_P \rightarrow \sqrt{\frac{2}{3} d\varepsilon_{ij}^P d\varepsilon_{ij}^P}$$

1D plastic stress-strain law

$$d\varepsilon = d\varepsilon_E + d\varepsilon_P$$

$$d\varepsilon_E = \frac{d\sigma}{E}$$

$$d\varepsilon_P = \begin{cases} \frac{d\sigma}{h}, & \text{if } \sigma = \sigma_Y, d\sigma > 0 \\ 0, & \text{otherwise} \end{cases}$$

is generalized to 3D plastic stress-strain law

$$d\varepsilon_{ij} = d\varepsilon_{ij}^E + d\varepsilon_{ij}^P$$

$$d\varepsilon_{ij}^E = \frac{1}{2\mu} d\sigma'_{ij} + \frac{1}{9K} \delta_{ij} d\sigma_{kk}$$

$$d\varepsilon_{ij}^P = \begin{cases} \frac{3}{2} \sigma'_{ij} \frac{d\sigma_e}{h\sigma_e}, & \text{if } \sigma_e = \sigma_Y, d\sigma_e \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

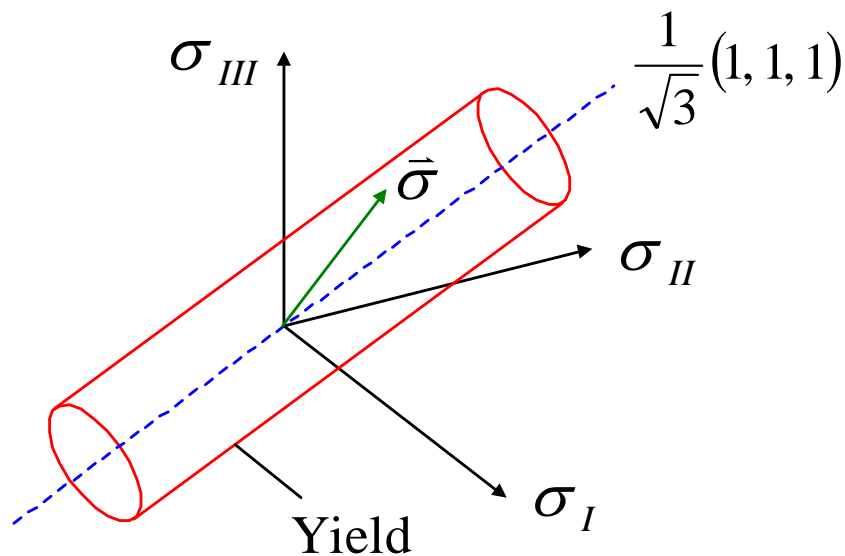
A few remarks:

1) Yield conditions:

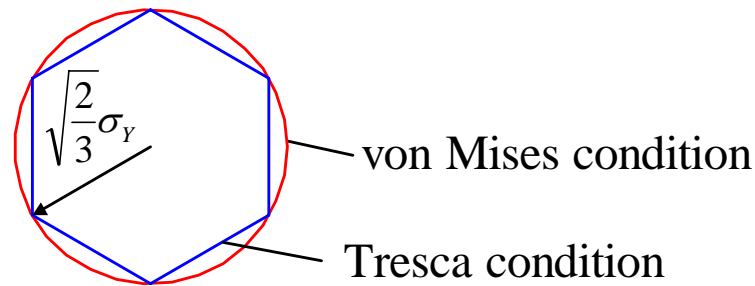
Mises condition: $\sigma_e = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}} = \sigma_Y$

Tresca condition: $\sigma_I, \sigma_{II}, \sigma_{III}, \text{Max}(|\sigma_I - \sigma_{II}|, |\sigma_I - \sigma_{III}|, |\sigma_{II} - \sigma_{III}|) = \sigma_Y$

Representation of yield conditions in stress space:



This is called the yield surface. A perspective view along the $\frac{1}{\sqrt{3}}(1, 1, 1)$ direction would show the projection of the von Mises yield surface as a circle



The Tresca yield condition corresponds to the inscribed hexagon inside the von Mises circle. The two yield conditions are actually quite close to each other.

2) J_2 -flow theory

The three invariants of stress σ_{ij} are sometimes denoted as

$$I_1 = \sigma_{kk}, \quad I_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij}, \quad I_3 = \det(\sigma_{ij}) \quad (3 \text{ invariants of } \sigma_{ij})$$

The three invariants of deviatoric stress σ'_{ij} are sometimes denoted as:

$$J_1 = \sigma'_{kk} = 0, \quad J_2 = \frac{1}{2} \sigma'_{ij} \sigma'_{ij}, \quad J_3 = \det(\sigma'_{ij}) \quad (3 \text{ invariants of } \sigma'_{ij})$$

The Levy-Mises flow rule can be written as

$$d\varepsilon_{ij}^P = \sigma'_{ij} d\lambda = \frac{\partial J_2}{\partial \sigma'_{ij}} d\lambda$$

where J_2 acts as a potential for plastic deformation. Therefore, this theory is also called the J_2 -flow theory.