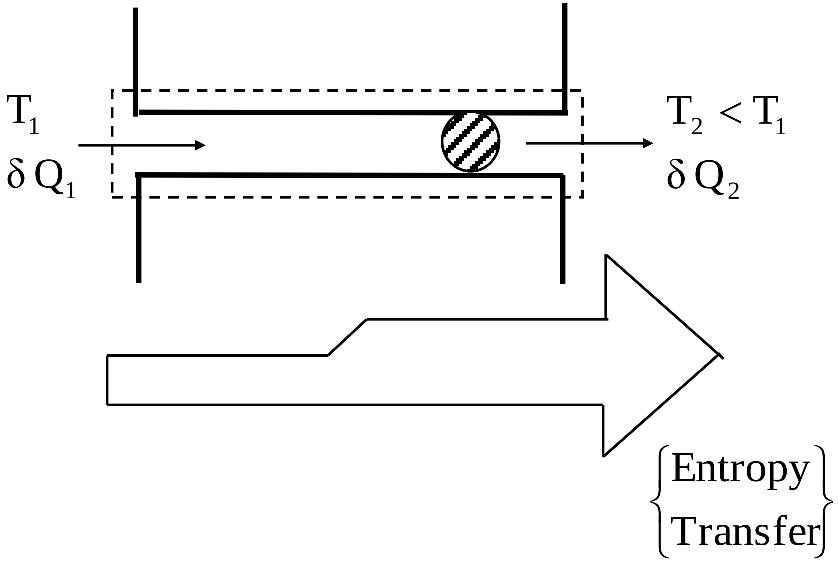
Example

__________Steady Conduction through a Rod

Example: Steady Conduction through a Rod



E.B. $$(\delta Q_1 - \delta Q_2) - 0 = 0$$

$$\delta Q_1 = \delta Q_2 = \delta Q$$

2nd Law $$\delta S_{gen} = dS - \left(\frac{\delta Q_2}{T_2} - \frac{\delta Q_1}{T_1} \right) \geq 0$$

But the process is <u>steady</u>, dS = 0.

$\therefore$ $$\delta S_{gen} = \delta Q \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

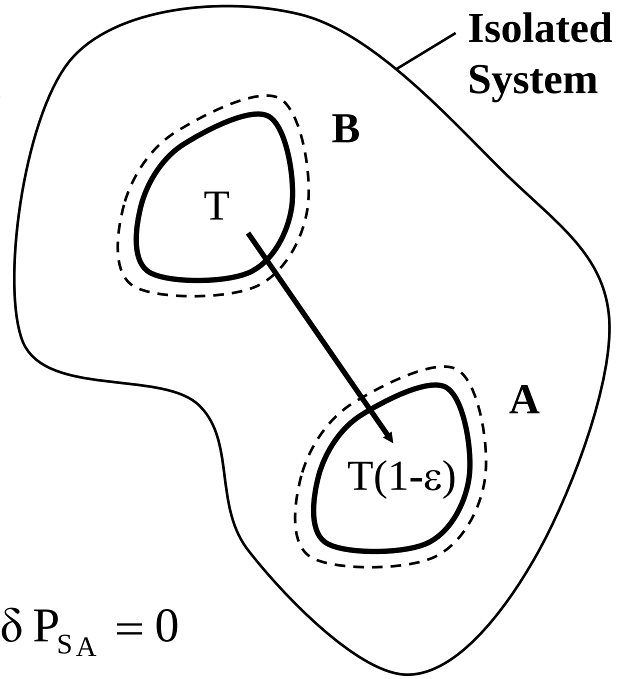
or $$S_{gen} = Q \left(\frac{1}{T_2} - \frac{1}{T_1} \right) = \frac{Q}{T_1 T_2} \left(T_1 - T_2 \right)$$

Under what conditions is entropy not generated?

$$T_1 = T_2 \qquad \text{i.e. } \Delta T \rightarrow 0$$

Example: A Reversible Process Involving Heat Transfer

Note: Heat transfer will not take place "up the temperature hill". Therefore, heat transfer "down the temperature hill" is irreversible.



Consider two TERs: Internally Reversible

One at temp T, the other at temp T(1-ε)

$$\delta P_S = dS_A + dS_B$$

$$dS_A = \frac{\delta Q}{T(1-\varepsilon)} = \frac{\delta Q}{T}(1+\varepsilon+\varepsilon^2+\cdots) \cong (1+\varepsilon)$$

$$\therefore \quad \left.\begin{aligned} dS_A &= \frac{\delta Q}{T}(1+\varepsilon) \\ dS_B &= -\frac{\delta Q}{T} \end{aligned}\right\} \begin{aligned} &\text{Internally Reversible; } \delta P_{SA} = 0 \\ &\text{Internally Reversible; } \delta P_{SB} = 0 \end{aligned}$$

$$\delta P_S = dS_A + dS_B = \frac{\delta Q}{T}(1+\varepsilon) - \frac{\delta Q}{T} \cong \underbrace{\varepsilon\frac{\delta Q}{T}}_{\substack{\text{due solely}\\ \text{to heat transfer}}}$$

Thus we see:

$$\delta P_S \to 0 \quad \text{as} \quad \varepsilon \to 0$$

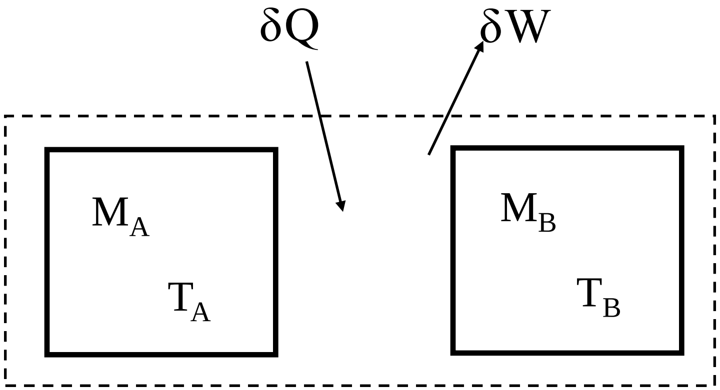
Implication: Reversible Heat Transfer can only be accomplished with zero temperature difference.

- Heat transfer across a finite temperature difference is irreversible
- Note each reservoir was internally reversible
- Heat transfer was external to CM_A , CM_B

Closed System Example

Example: (see Bejan, Entrpy Generation….Prob. 1.1)

Consider the Process from ① → ② due to the direct contact between two resevoirs with masses M_A and M_B and $T_{Ai} > T_{Bi}$; $M_A = M_B$



Find: (a) the final temperature T_∞
(b) S_{gen} for the process

(a) E.B. $\delta Q - \delta W = dU$

$\therefore$ $dU = dU_A + dU_B = 0$

For an ICL: $dU = Mc\,dT$

$\therefore$ $(Mc)_A\,(T_{Af} - T_{Ai}) + (Mc)_B\,(T_{Bf} - T_{Bi}) = 0$

But $T_{Af} = T_{Bf} = T_\infty$

$\therefore$ $2T_f = T_{Ai} + T_{Bi}$

$$\left\{ T_f = \frac{T_{Ai} + T_{Bi}}{2} \right\} \text{ answer(a)}$$

For a Process:

$$S_{gen} = (S_2 - S_1) - \int_1^2 \frac{\delta Q}{T} \geq 0$$

For this problem: $\int_1^2 \frac{\delta Q}{T} \rightarrow 0$, all internal

$$\therefore \qquad S_{gen} = (S_{Af} - S_{Ai}) + (S_{Bf} - S_{Bi})$$

Now, the combined first and second Laws for a process will show: TdS = dU +PdV Gibbs First Equation or the First TdS equation…..

$$\text{then:} \qquad dS = \frac{dU}{T} = Mc\frac{dT}{T}$$

$$\text{thus:} \qquad \Delta S = Mc\ln\left(\frac{T_f}{T_i}\right)$$

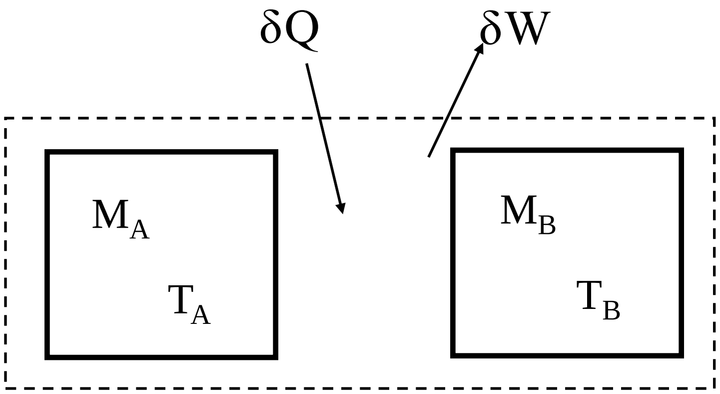
$$\text{Finally:} \quad S_{gen} = (Mc)_A \ln\left(\frac{T_\infty}{T_{Ai}}\right) + (Mc)_B \ln\left(\frac{T_\infty}{T_{Bi}}\right)$$

Which can be simplified to:

$$\left\{S_{gen} = M\,c\ln\left[\frac{{T_\infty}^2}{T_{Ai}T_{Bi}}\right]\right\}$$

Example: 2nd Law Analysis (see Bejan Ex. 2.1)

- Two bodies of water of masses m_1, m_2 and temps T_1, T_2
- The water reservoirs are instantaneous thermal energy reservoirs for a heat engines that operates reversibly.



Find: Final equlibrium temperature: T_∞

Total work delivered: W

$$-W_{12} = (U_f - U_i)$$

$$S_{gen12} = S_f - S_i = 0$$

Now : $(U_f - U_i) = (U_f - U_i)_{m1} + (U_f - U_i)_{engine} + (U_f - U_i)_{m2}$

Equation of State: du = c dT ; ICL model

$$\therefore \quad (U_f - U_i) = m_1\, c\,(T_\infty - T_1) + (0) + m_2\, c\,(T_\infty - T_2)$$

$$S_2 - S_1 = (S_2 - S_1)_{m1} + (S_2 - S_1)_{engine} + (S_2 - S_1)_{m2}$$

$$S_2 - S_1 = m_1\, c \ln\left(\frac{T_\infty}{T_1}\right) + 0 + m_2\, c \ln\left(\frac{T_\infty}{T_2}\right)$$

$$\text{Note:}\quad du = c\,dT \qquad ds = c\frac{dT}{T}$$

$$S_2 - S_1 = 0 \quad \therefore\ m_1\, c \ln\left(\frac{T_\infty}{T_1}\right) + m_2\, c \ln\left(\frac{T_\infty}{T_2}\right) = 0$$

$$\text{Then: } T_\infty = T_1^{\,a}\, T_2^{1-a} \qquad \text{where: } a = \frac{m_1}{m_1 + m_2}$$

Then substituting the result T_∞ into the first law equation:

$$W_{12} = U_1 - U_2 = m_1\, c\, T_1\left[1 - \left(\frac{T_2}{T_1}\right)^{1-a}\right] + m_2\, c\, T_2\left[1 - \left(\frac{T_1}{T_2}\right)^{a}\right]$$

$$\text{where} \quad a \rightarrow 0 \qquad T_\infty = T_2, \quad \text{doesn't change}$$

$$m_1 << m_2 \qquad W_{12} = \underbrace{m_1\, c\,(T_1 - T_2)}_{\Delta U \text{ of } m_1}$$

If the reservoir m_1 is much less than m_2, then, the amount of work that can be performed is due to the energy that can be “absorbed” by m_1, in changing its temperature from T_{1i} to $T_1=T_2$.

Thus, regardless of the energy content of m_2, we are limited by m_2.

when $m_1 = m_2$ $a = 1/2$

$$T_\infty = \sqrt{T_1 T_2}$$

thus, when the masses are equal, $T_\infty \neq \left(\frac{T_1 + T_2}{2} \right)$

$$W_{12} = m_1 c_1 T_1 \left(1 - \sqrt{\frac{T_2}{T_1}} \right) + m_2 c_2 T_2 \left(1 - \sqrt{\frac{T_1}{T_2}} \right)$$

$$W_{12} = m_1 c_1 T_1 + m_2 c_2 T_2 - m_1 c_1 \sqrt{T_1 T_2} - m_2 c_2 \sqrt{T_1 T_2}$$

$$W_{12} = m_1 c_1 (T_1 - T_\infty) + m_2 c_2 (T_2 - T_\infty)$$

$$W_{12} = Q_1 - Q_2$$

Question: If there were no internal irreversibilities, what does this imply about the heat transfer?

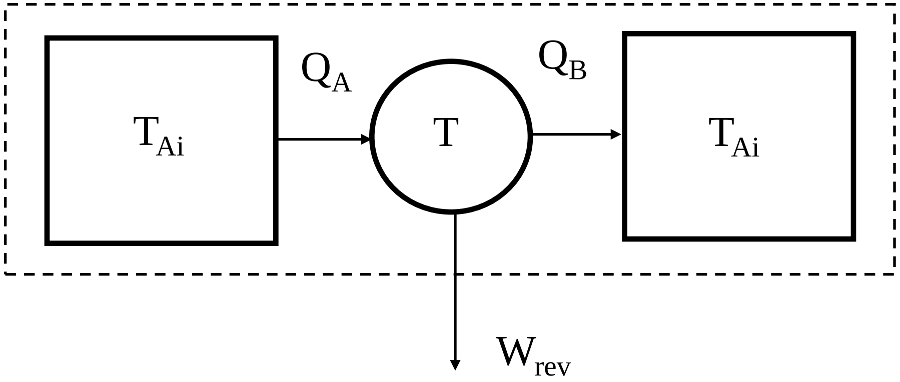
$$\dot{Q} = h A_S (T_S - T_\infty)$$

For finite $\dot{Q}$, $(h A_S) \to \infty$ $(\Delta T) \to 0$

(b) Now consider that a reversible heat engine operates between the reservoirs A, B:

After a sufficient number of cycles the two pools achieve a temperature $T_{f,\ rev}$

- Find $T_{f,\ rev}$
- Find W_{rev}



E.B. $$dU + \delta W_{rev} = 0$$

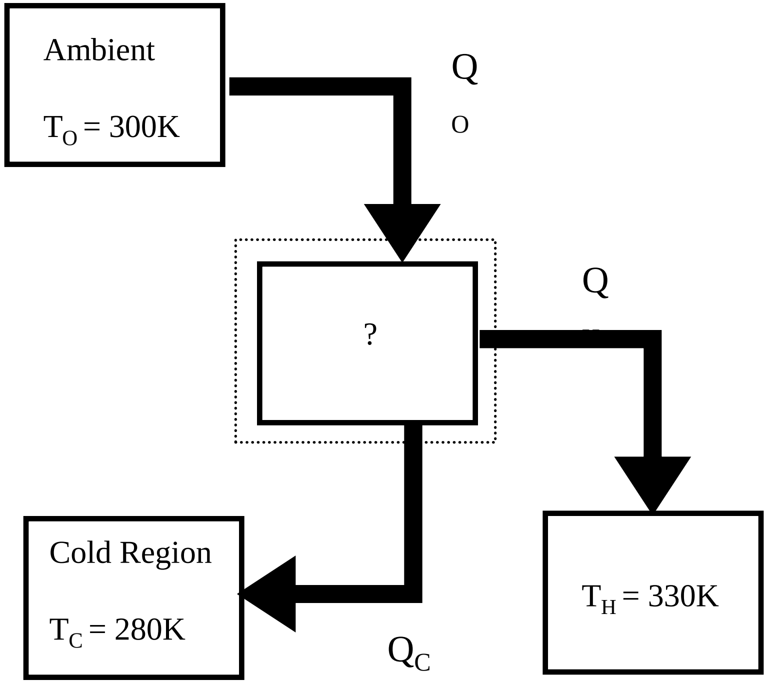
$$\delta W_{rev} = -dU = -dU_A - dU_B \;; \qquad \text{unknown} \quad W_{rev}, T_f$$

2nd Law $$dS_{gen} = dS$$

$$dS_{gen} = dS_A + dS_{engine} + dS_B \;; \qquad \text{entire CV}$$

A 2T Heating Device

2nd Law for CM; Applications



Example 2: (see WCR notes)

Consider a device that accepts energy as heat from a source at $T_o = 300K$, and rejects heat to a sink at $280K = T_C$. The function of the device is to supply heat to a space at $T_H = 330K$.

For a given T_o, T_H, T_C, what is $\left(\frac{Q_H}{Q_O}\right)_{\text{maximum}}$?

First Law $$Q_O = Q_H + Q_C + \cancelto{0}{\Delta E} \quad \text{(cycle or S.S.)}$$

2nd Law $$P_S + \frac{Q_O}{T_O} = \cancelto{0}{\Delta S} + \frac{Q_C}{T_C} + \frac{Q_H}{T_H}$$

$$T_C\left(P_S + \frac{Q_O}{T_O} - \frac{Q_H}{T_H}\right) = Q_C$$

$$\frac{Q_H}{Q_O} = 1 - \frac{Q_C}{Q_O} = 1 - \frac{T_C}{Q_O}\left(P_S + \frac{Q_O}{T_O} - \frac{Q_H}{T_H}\right)$$

$$\frac{Q_H}{Q_O} = 1 - \frac{P_S T_C}{Q_O} - \frac{T_C}{T_O} + \frac{Q_H}{Q_O}\frac{T_C}{T_H}$$

$$\frac{Q_H}{Q_O}\left[1 - \frac{T_C}{T_H}\right] = 1 - \frac{P_S T_C}{Q_O} - \frac{T_C}{T_O}$$

Finally $$\frac{Q_H}{Q_O} = \left[\frac{1 - \frac{T_C}{T_O} - \frac{P_S T_C}{Q_O}}{1 - \frac{T_C}{T_H}}\right]$$

Divide right side top and bottom by T_C :

$$\frac{Q_H}{Q_O} = \frac{\left(\frac{1}{T_C} - \frac{1}{T_O}\right) - \frac{P_S}{Q_O}}{\left(\frac{1}{T_C} - \frac{1}{T_H}\right)}$$

For a given $T_C, T_O,$ and T_H :

$$\left(\frac{Q_H}{Q_O}\right)_{\max} = \left(\frac{\frac{1}{T_C} - \frac{1}{T_O}}{\frac{1}{T_C} - \frac{1}{T_H}}\right) \qquad P_S = 0,\ \text{reversible device}$$

$$\left(\frac{Q_H}{Q_O}\right)_{\max} = 0.44 \quad \text{for given numbers}$$

Let's pose it a different way:

Given Q_O, Q_H, Q_C, T_O, T_H and T_C, is the process possible?

$$P_S = \frac{Q_C}{T_C} + \frac{Q_H}{T_H} - \frac{Q_O}{T_O}$$

but $Q_C = Q_O - Q_H$

$$P_S = \frac{Q_O}{T_C} - \frac{Q_H}{T_C} + \frac{Q_H}{T_H} - \frac{Q_O}{T_O}$$

$$P_S = Q_O\left[\frac{1}{T_C} - \frac{1}{T_O}\right] + Q_H\left[\frac{1}{T_H} - \frac{1}{T_C}\right]$$

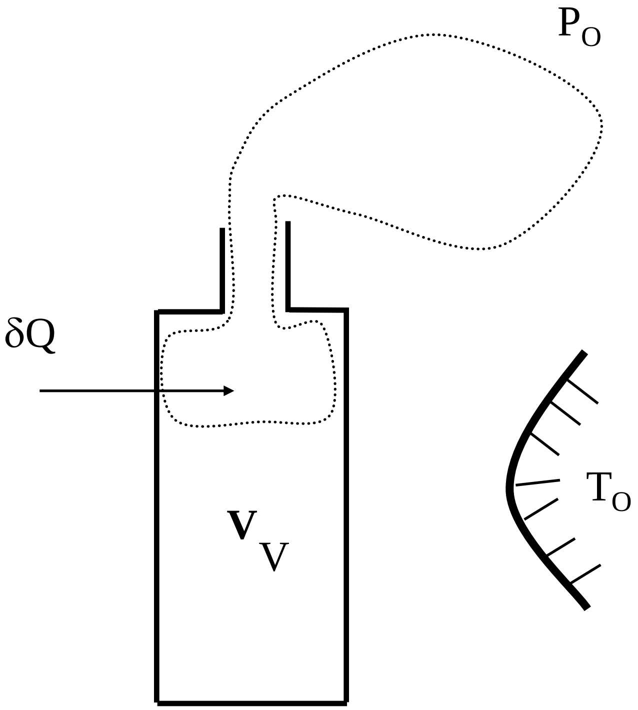
In order for this process to be achievable:

$$P_S \geq 0 \qquad \therefore Q_O\left[\frac{1}{T_C} - \frac{1}{T_O}\right] \geq Q_H\left[\frac{1}{T_H} - \frac{1}{T_C}\right]$$

Filling of a Cylinder

Example 3: 2nd Law for Closed Systems

Recall the filling of an evacuated bottle:



$$Q_{1\,2} = -P_O V$$

2nd Law

$$\delta S_{gen} + \frac{\delta Q}{T} = dS$$

$$S_{gen} = (S_2 - S_1) - \int_1^2 \frac{\delta Q}{T}$$

$$S_{gen} = M(s_2 - s_1) - \frac{Q_{1\,2}}{T_O}$$

but $s_2 = s_1$ because started and ended at same state

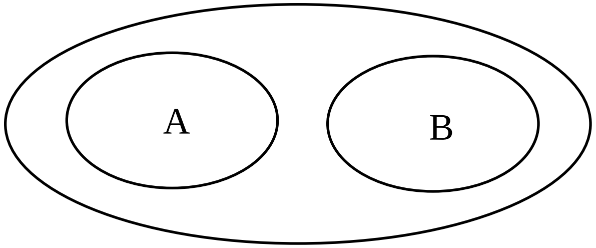
$$S_{gen} = \frac{P_O V}{T_O}; \qquad W_{lost} = T_O\, S_{gen}$$

Second Law of Thermodynamics

General Statements:

1. Every system has entropy. It is a property of the system.
   It is a measure of microscopic chaos.

2. It is an extensive property. Depends on the extent of the system:



$S_{A+B} = S_A + S_B$

3. Entropy can be produced but not destroyed. The entropy of an isolated system can never decrease. $\delta P_S \geq 0$

4. Work transports no entropy.
   Q transports entropy.
   The second law is the only way in which we can distiguish between heat and work.

5. Absolute entropy
   S = 0 for a system with no microscopic chaos (T = 0K for a pure substance).

Examples:

2nd Law Open Systems

The 2nd Law for a Control Volume (CV)

Second Law for a Control Volume

use the CM to CV transformation used before

$$\delta P_S = dS_{CM} + \left(\frac{\delta Q}{T}\right)_{out} - \left(\frac{\delta Q}{T}\right)_{in} \ge 0$$

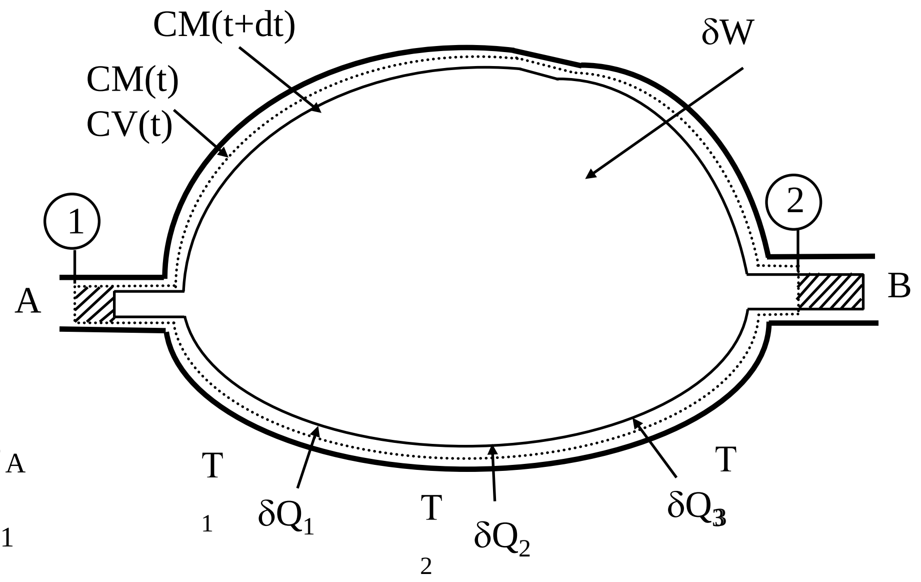
$$dS_{CM} = S_{CM}(t+dt) - S_{CM}(t)$$

Since $CM(t) = CV(t), S_{CM}(t) = S_{CV}(t)$

and from inspection:

$$S_{CM}(t+dt) = S_{CV}(t+dt) + S_B - S_A$$

where $\quad S_B = \delta M_B\, s_2 \qquad S_A = \delta M_A\, s_1$



$$\therefore \quad dS_{CM} = S_{CV}(t+dt) - S_{CV}(t) + \delta M_B\, s_2 - \delta M_A\, s_1$$

$$dS_{CM} = dS_{CV} + (s\,\delta M)_{out} - (s\,\delta M)_{in}$$

2nd Law for CM:

$$\delta P_{S_{CM}} = dS_{CM} + \sum_{out}\frac{\delta Q}{T} - \sum_{in}\frac{\delta Q}{T} \ge 0$$

If we write this in terms of the properties of the CV, we get
2nd Law for CV:

$$\delta P_{S_{CV}} = dS_{CV} + \left[\sum_{out} s\,\delta M + \sum_{out} \frac{\delta Q}{T} \right] - \left[\sum_{in} s\,\delta M + \sum_{in} \frac{\delta Q}{T} \right] \geq 0$$

on a rate basis:

$$\dot{P}_S = \frac{dS}{dt} + \left[\sum_{out} \left(s\,\dot{m} \right) + \sum_{out} \frac{\dot{Q}}{T} \right] - \left[\sum_{in} \left(s\,\dot{m} \right) + \sum_{in} \frac{\dot{Q}}{T} \right] \geq 0$$

$\dot{P}_S$ = rate of entropy production (generation) within CV

$s\,\dot{m}$ = mass-associated (convection) rate of transport of entropy

$\frac{\dot{Q}}{T}$ = rate of entropy transport with heat

Second Law for Open Systems

$$\delta S_{gen} = dS - \left(\frac{\delta Q}{T}\right) + (\delta m\, s)_{outflow} - (\delta m\, s)_{inflow} \geq 0$$

on a rate basis:

$$\dot{S}_{gen} = \frac{dS}{dt} - \sum \frac{\dot{Q}}{T} + \left(s\,\dot{m}\right)_{outflow} - \left(s\,\dot{m}\right)_{inflow} \geq 0$$

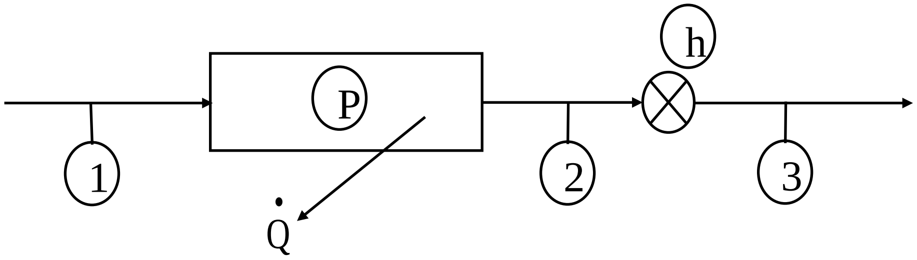
$$\sum \frac{\dot{Q}}{T} = \text{entropy transfer via heat transfer}$$

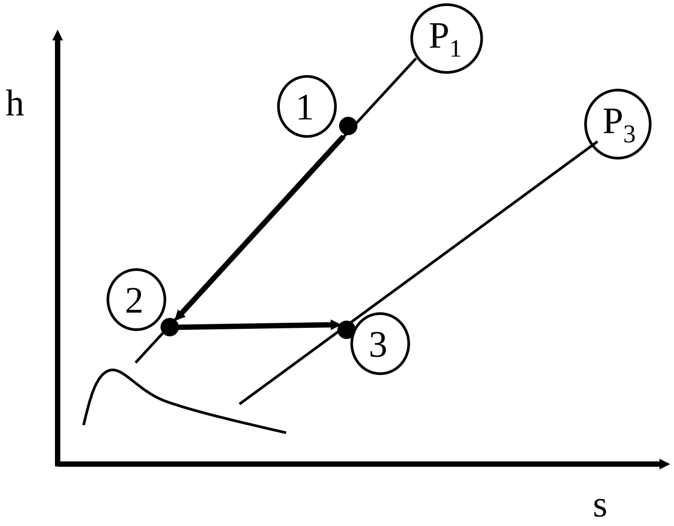
Example

A steady flow device operating between fixed states

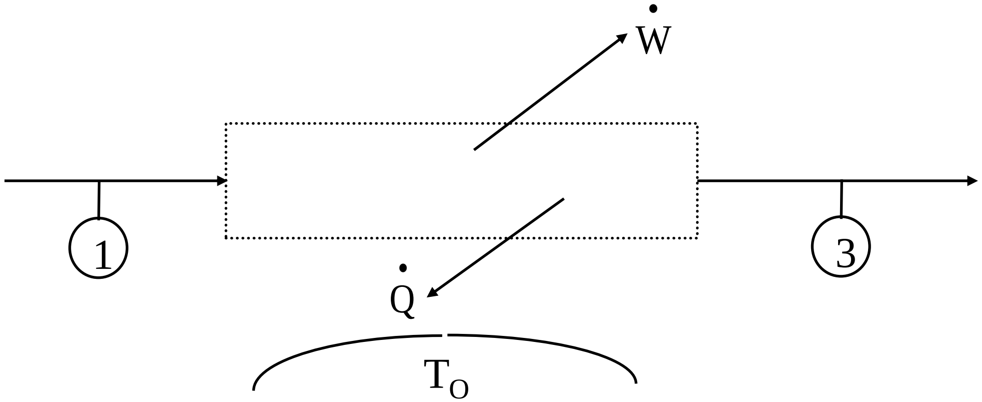
Analysis of Irreversibilities in CVs

Example: A steady flow device operating between fixed states.





Question: Can we get useful work out of this process?
What would the system look like?



E.B.

$$+\left(-\dot{Q}_O\right)-\left(\dot{W}\right)=\frac{dU}{dt}+\dot{m}_3\left(h+\frac{v^2}{2g_c}+\frac{g}{g_c}z\right)_3-\dot{m}_1\left(h+\frac{v^2}{2g_c}+\frac{g}{g_c}z\right)_1$$

or $$\dot{Q}_O=\dot{m}_1(h_1)-\dot{m}_3(h_3)-\dot{W}$$

2nd Law :

$$\dot{S}_{gen}=\frac{dS}{dt}-\frac{\left(-\dot{Q}_O\right)}{T_O}+\dot{m}_3 s_3-\dot{m}_1 s_1$$

or

$$\dot{Q}_O=T_O\dot{S}_{gen}+\left(\dot{m}_1 s_1-\dot{m}_3 s_3\right)T_O$$

$$\dot{m}(h_1-h_3)-\dot{W}=T_O\dot{S}_{gen}+T_O\dot{m}s_1-T_O\dot{m}s_3$$

$$\dot{W}=\dot{m}(h_1-h_3)-T_O\dot{m}(s_1-s_3)-T_O\dot{S}_{gen}$$

Let $b = h - T_O S \equiv$ <u>SSSF Availability Function</u>

then $\dot{W} = \dot{m}(b_1 - b_3) - T_O \dot{S}_{gen}$

$$\dot{W}_{max} = \dot{m}(b_1 - b_3)$$

$$\dot{W}_{lost} = T_O \dot{S}_{gen}$$

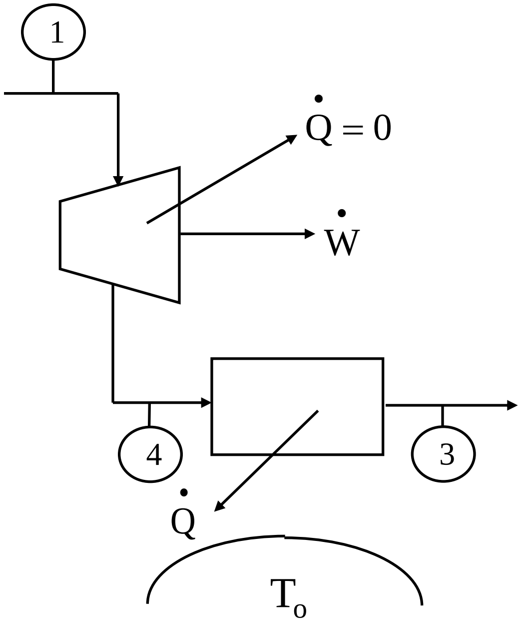
$$\dot{W}_{lost} = \dot{W}_{max} - \dot{W}$$

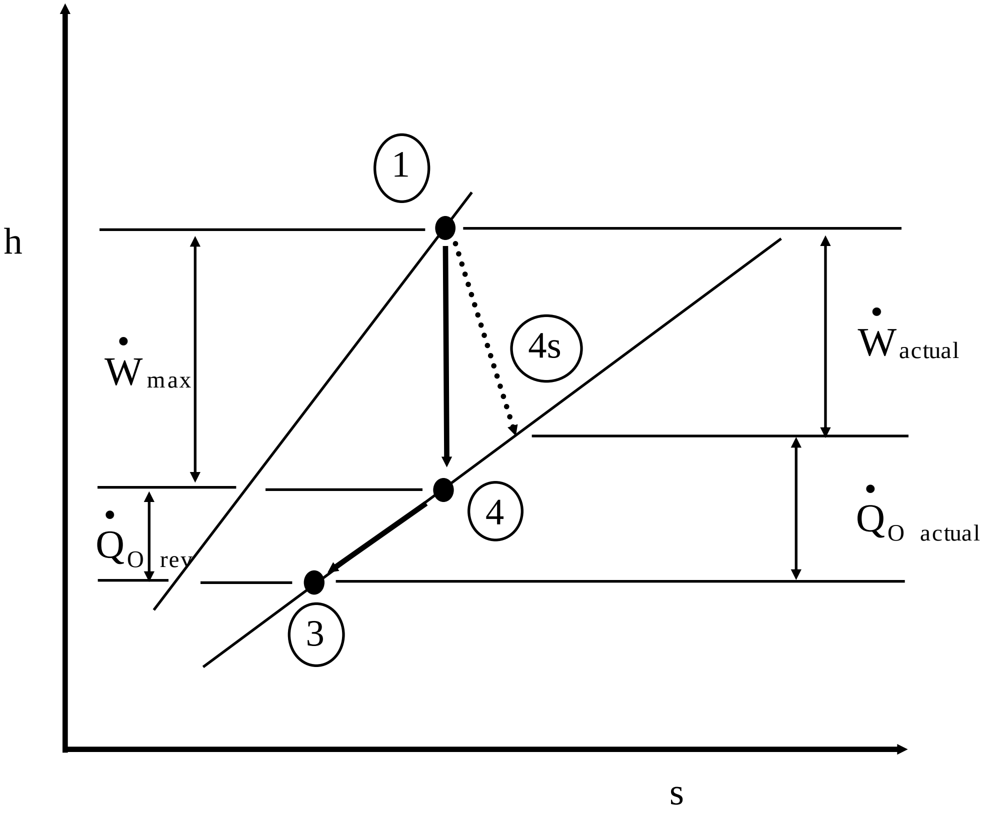
Guy Stoudola Theorem: $\dot{W}_{lost} = T_O \dot{S}_{gen}$

What are the implications of using T_o?

Possible Configurations

Case (1):



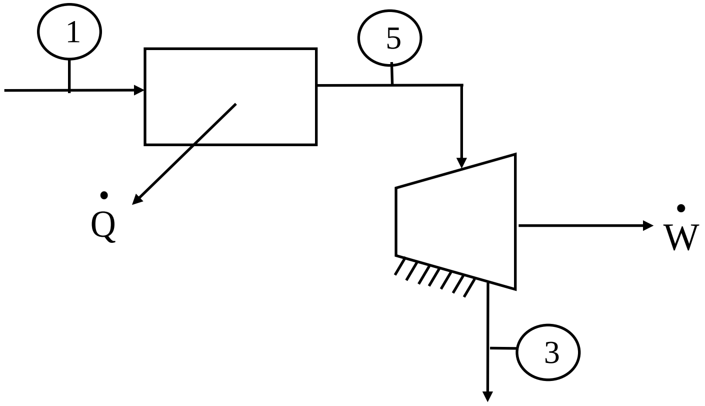


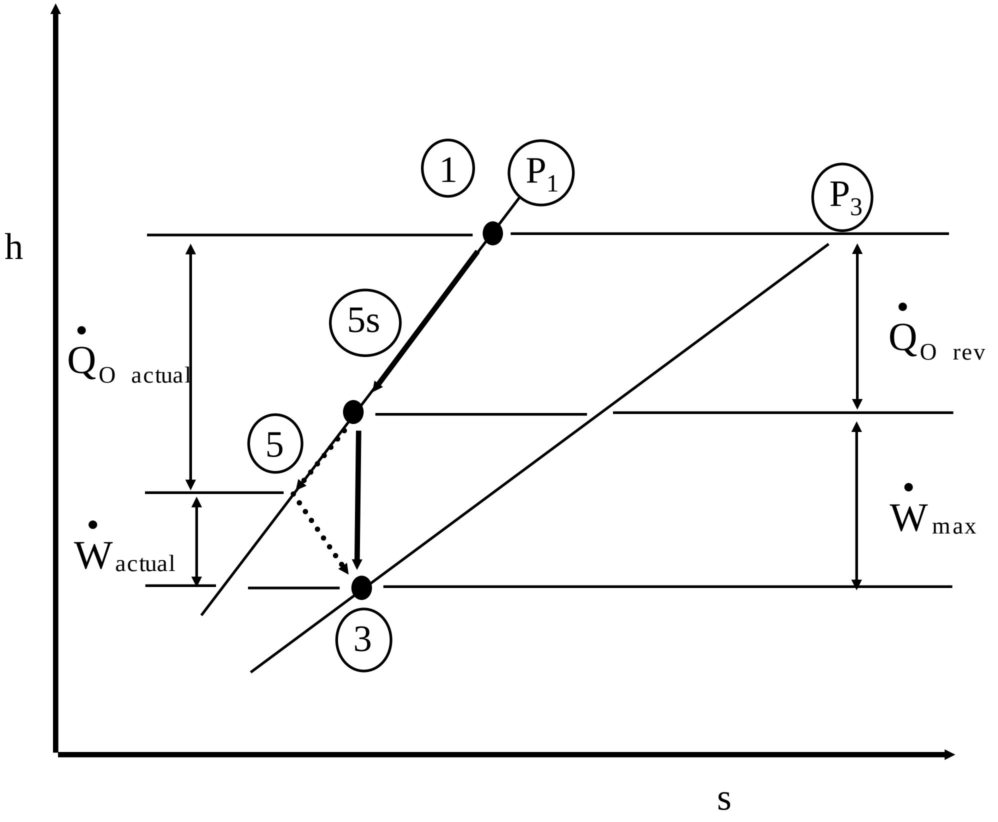
Real Process: 1 - 4 - 3

Ideal Reversible Process: 1 - 4s - 3

$$\eta_S = \frac{\dot{W}_{actual}}{\dot{W}_{max}} = \frac{h_1 - h_4}{h_1 - h_{4S}}$$

Case (2):





Which would you choose?

Case (1) or Case (2)

(1) Design (1) rejects heat at lower temperature. Therefore, may need larger heat exchanger, far more surface area.

(2) On the other hand (h_1-h_4) is probably greater than (h_5-h_3) because the pressure lines diverge hence for the same ΔP and η_S

$$\left(\frac{\dot{W}_1}{\dot{m}}\right) > \left(\frac{\dot{W}_2}{\dot{m}}\right)$$

Example

Steady Compression

Example - Steady Compression of a gas

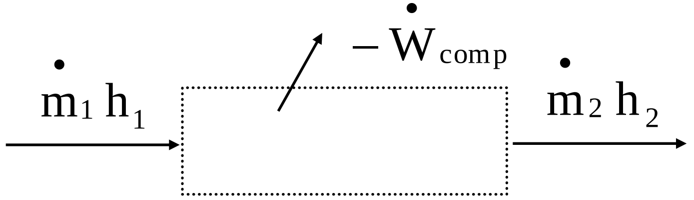
Case 1: Adiabatic Compression

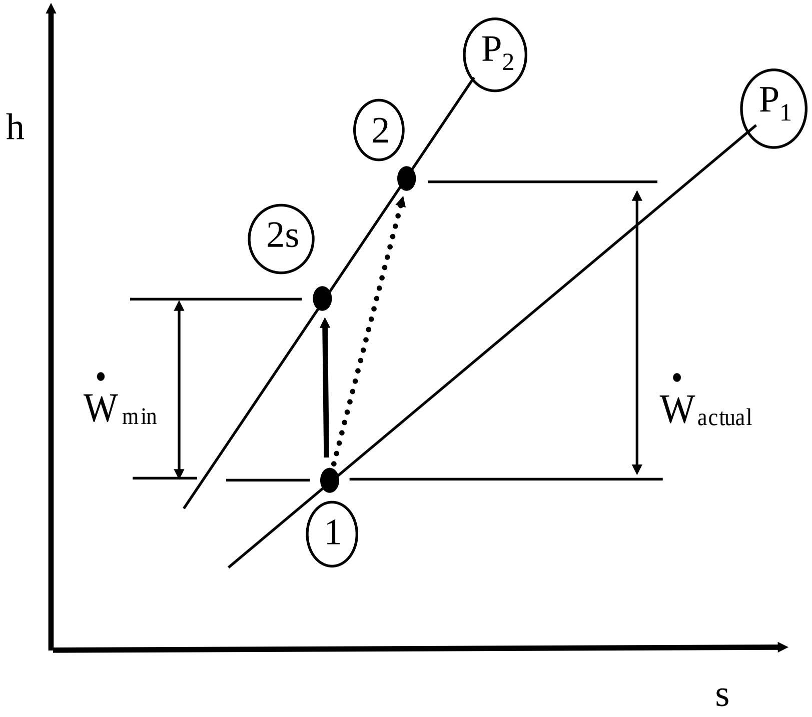
Given: (1) $\dot{Q} = 0$

(2) fixed inlet state

(3) fixed P_2 (but not fixed T_2)

Question: What is the minimum work of compression?





E.B.: $$(0) - \left(-\dot{W}_{comp}\right) = \dot{m}_2 h_2 - \dot{m}_1 h_1$$

2nd Law: $$\dot{S}_{gen} = \dot{m}_2 s_2 - \dot{m}_1 s_1 \geq 0$$

Clearly: $$\dot{W}_{comp,min} = (h_{2S} - h_1)$$

$$\eta_S = \frac{\dot{W}_{min}}{\dot{W}_{actual}} = \frac{h_{2S} - h_1}{h_2 - h_1}$$

Assuming Ideal Gas Behavior, Constant Cp:

Also note:

$$\left.\frac{\dot{W}_{comp,\min}}{\dot{m}}\right)_{S=const} = (h_{2S} - h_1) = C_P T_1 \left(\frac{T_{2S}}{T_1} - 1\right)$$

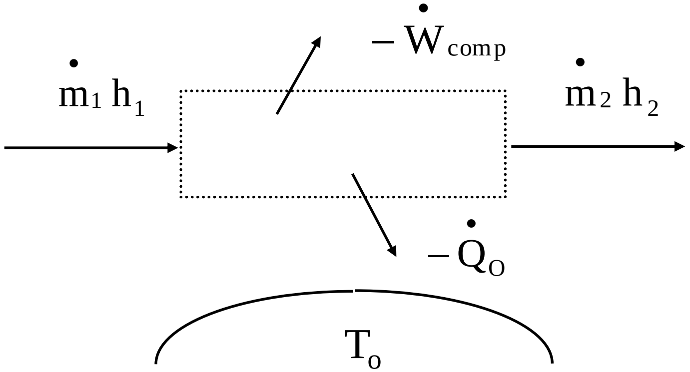
$$\left.\frac{\dot{W}_{comp,\min}}{\dot{m}}\right)_{S=const} \cong C_P T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$

Case 2: Non-Adiabatic Compression

Given: (1) fixed state 1, $T_1 \neq T_O$

(2) fixed P_2

(3) $\dot{Q} \neq 0$, T_o dead state



What is $\dot{W}_{min}$? Allowing for heat transfer:

E.B.: $$(-\dot{Q}_O) - \left(-\dot{W}_{comp}\right) = \frac{dU}{dt} + \dot{m}_2 h_2 - \dot{m}_1 h_1$$

2nd Law: $$\dot{S}_{gen} = \left(\frac{dS}{dt}\right) - \left(\frac{-\dot{Q}_O}{T_O}\right) + \dot{m}_2 s_2 - \dot{m}_1 s_1 \geq 0$$

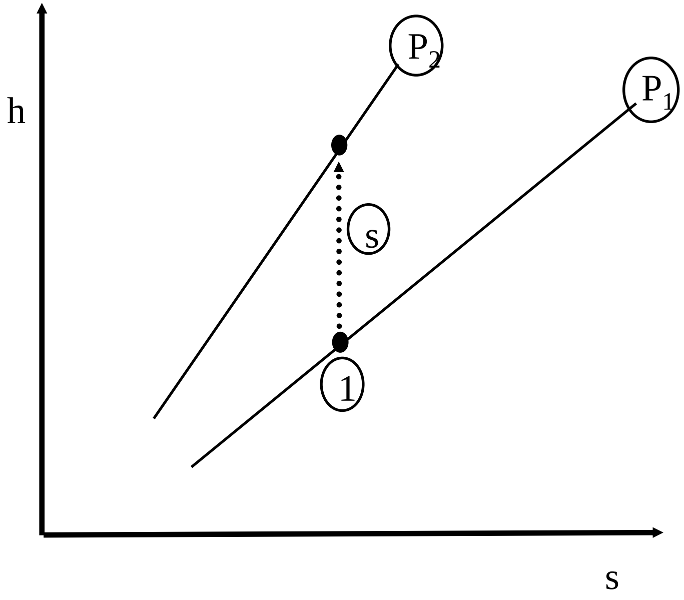
Solve for $\dot{Q}_O$:

$$\dot{Q}_O = T_O \dot{S}_{gen} + T_O \dot{m}(s_1 - s_2)$$

$$\dot{W} = \underbrace{\dot{m}\left[(h_2 - T_O s_2) - (h_1 - T_O s_1)\right]}_{\dot{W}_{min}} + \underbrace{T_O \dot{S}_{gen}}_{\dot{W}_{lost} \text{ or } \dot{I}}$$

Thus:

$$\dot{W}_{min} = \dot{m}(b_2 - b_1)$$



Now, since we can allow T_2 to vary, let's find variation of $\dot{W}_{min}$ with respect to T_2 for a fixed P_2:

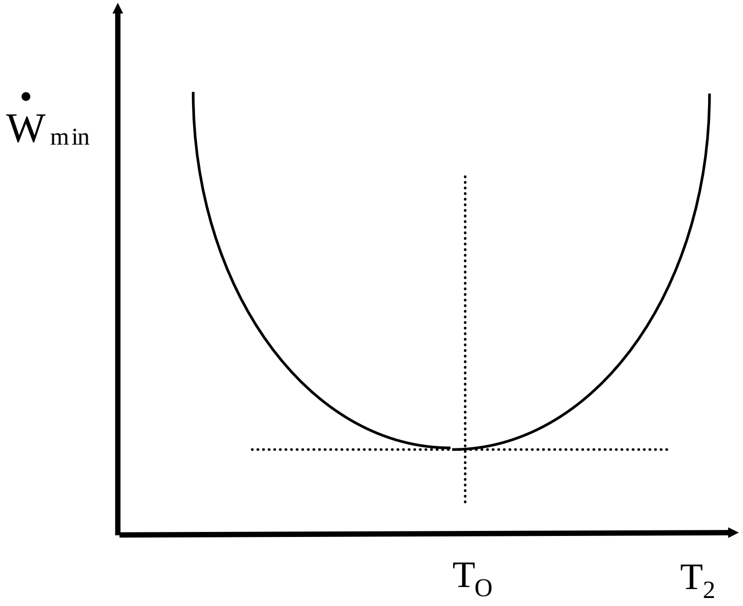
$$\frac{d\left(\dot{W}_{min}\right)}{dT_2} = \left.\frac{dh_2}{dT_2}\right|_{P_2} - T_O \left.\frac{dS_2}{dT_2}\right|_{P_2}$$

$$= \left(\frac{\delta h_2}{\delta T_2}\right)_{P_2} - T_O \left(\frac{\delta S_2}{\delta T_2}\right)_{P_2}$$

$$= C_P(T_2) - T_O \underbrace{\left[\frac{C_P(T_2)}{T_2}\right]}_{\text{can be proven}}$$

Thus:

$$\frac{d\left(\dot{W}_{min}\right)}{dT_2} = C_P(T_2)\left[1 - \frac{T_O}{T_2}\right]$$



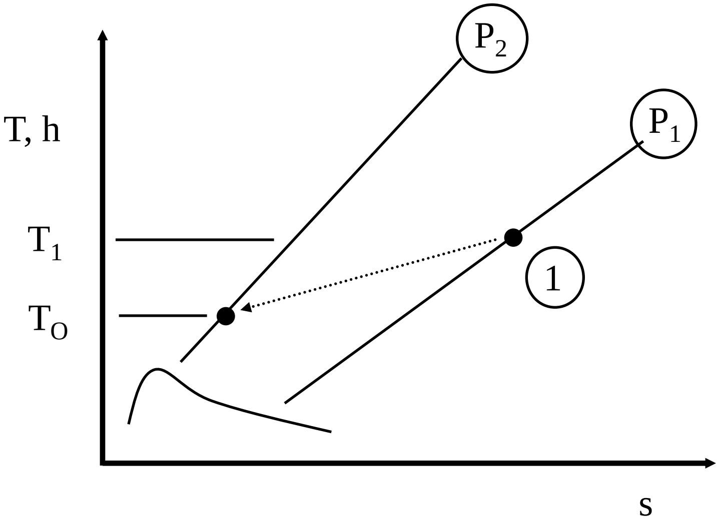
Note: for gases $C_P(T_2) > 0$

monotonically increases

$$\text{Hence:} \quad \frac{d\left(\dot{W}_{min}\right)}{dT_2} = 0 \qquad \text{when} \quad T_2 = T_O \;\; !$$

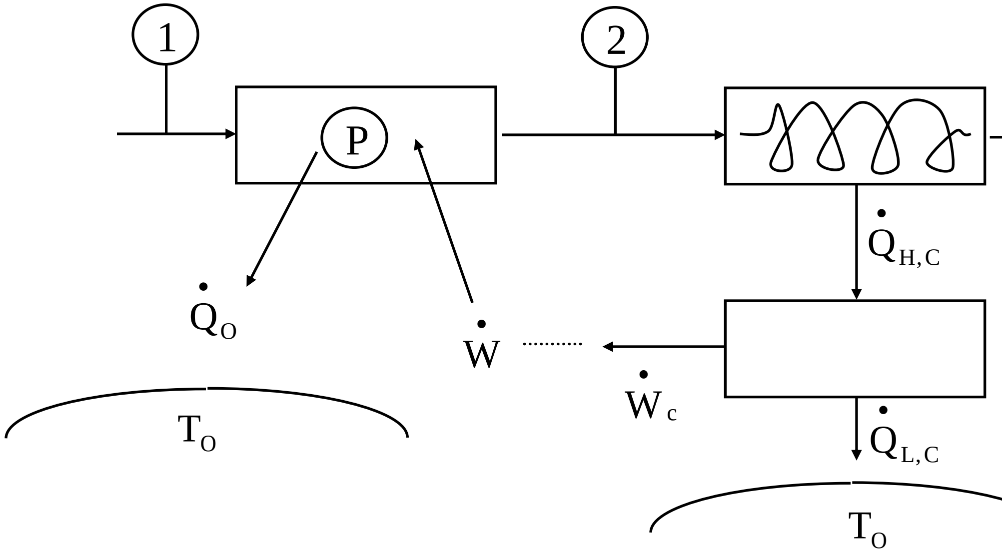
$$\text{thus:} \quad \dot{W}_{min,min} \quad \text{when} \quad T_2 = T_O$$

<u>given</u> T_1, P_1, P_2



<u>Why is this</u>?

<u>Consider the following</u>?



Does this help explain what is the minimum, minimum?

Case 3: Reversible Isothermal Compression

Given: Inlet state 1, P_2

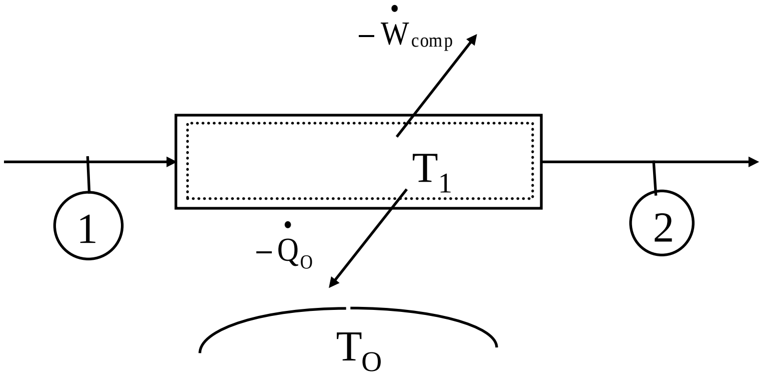
$$\text{E.B.}: \qquad (-\dot{Q}_O) - \left(-\dot{W}_{comp}\right) = \frac{dE}{dt} + \dot{m}_2 h_2 - \dot{m}_1 h_1$$

$$\text{2nd Law}: \quad \dot{S}_{gen} = \left(\frac{dS}{dt}\right) - \left(\frac{-\dot{Q}_O}{T_1}\right) + \dot{m}_2 s_2 - \dot{m}_1 s_1$$

Consider only the reversible case: $\dot{S}_{gen} = 0$

$$\dot{Q}_O = T_1 \dot{m}(s_1 - s_2)$$

$$\frac{\dot{W}_{comp}}{\dot{m}} = (h_2 - h_1) - T_1(s_2 - s_1)$$



Thus for an ideal gas with constant C_P :

$$\frac{\dot{W}_{comp,\,rev}}{\dot{m}} = C_P(T_2 - T_1) - T_1\left\{C_P \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)\right\}$$

$$\frac{\dot{W}_{comp,\,rev}}{\dot{m}} = T_1 R \ln\left(\frac{P_2}{P_1}\right) \qquad \text{Since } T_2 = T_1$$

This may also be written:

$$\left.\frac{\dot{W}_{comp,rev}}{\dot{m}}\right|_{T=const} = T_1 C_P \left(\frac{k-1}{k}\right) \ln\left(\frac{P_2}{P_1}\right)$$

$$\left.\frac{\dot{W}_{comp,rev}}{\dot{m}}\right|_{T=const} = T_1 C_P \ \ln\left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)}$$

Compare to reversible adiabatic :

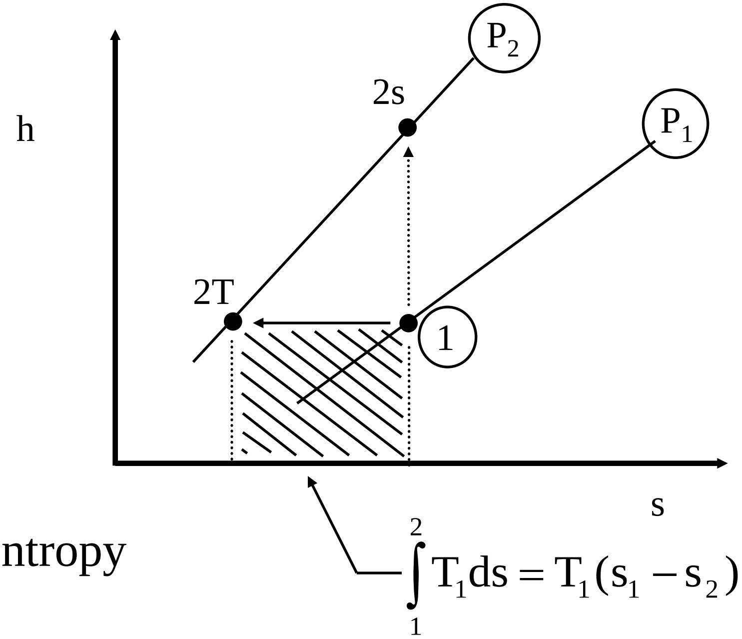
$$\left.\frac{\dot{W}_{comp,rev}}{\dot{m}}\right|_{S=const} = T_1 C_P \left[\left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)} - 1\right]$$

Furthermore, note:

$$(h_2 - h_1) = C_P (T_2 - T_1) \neq 0 \qquad \frac{\dot{Q}_O}{\dot{m}} = \left.\frac{\dot{W}_{comp,rev}}{\dot{m}}\right|_{T=const}$$

Process Plane:

$$\frac{\dot{Q}_O}{\dot{m}} = T_1(s_1 - s_2)$$



$$\int_1^2 T_1 ds = T_1(s_1 - s_2)$$

Why did the entropy drop?
Did this violate the 2nd Law?

Compare constant temperature and constant entropy compressions

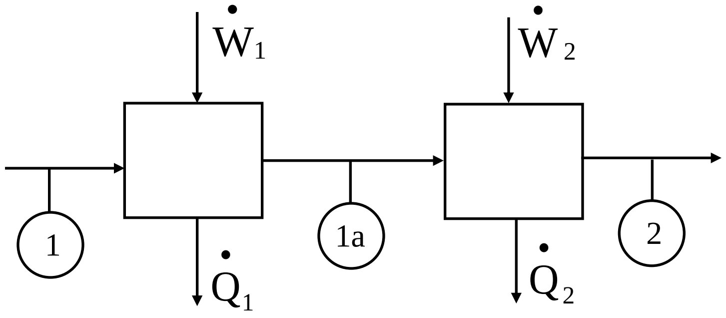
Let $P_2/P_1 = 50$ & $k=1.4$

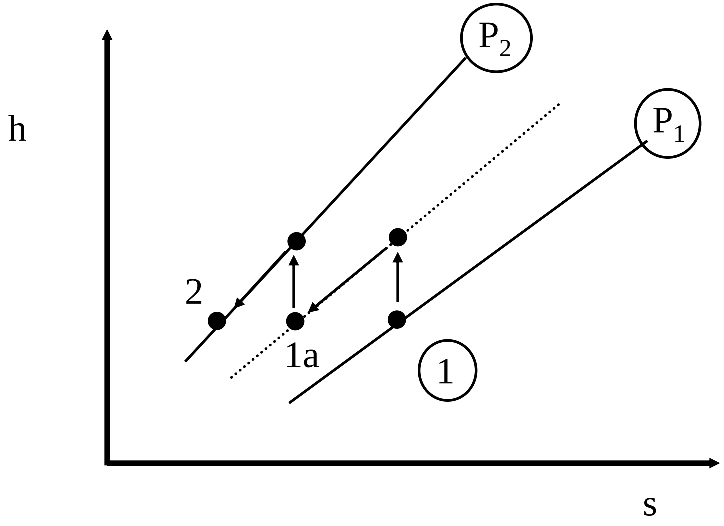
$$\frac{\dot{W}_{comp,\, revS=cont}}{\dot{W}_{comp,\, revT=cont}} = \frac{\left[\left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)} - 1\right]}{\ln\left[\left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)}\right]} = \frac{2.057}{1.118} = 1.84$$

Compression at lower temperature requires less work because density is higher!

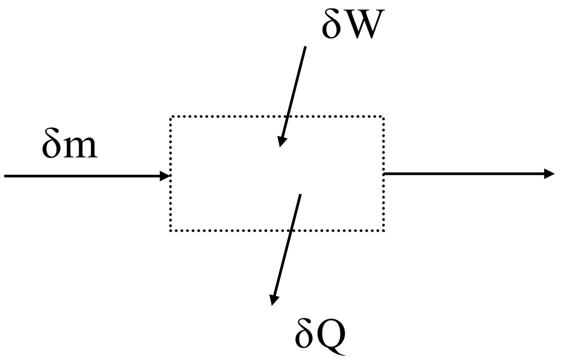
How do we accomplish this?

Intercooling stages:





Why is compression at lower temperature good?



$$-\delta Q + \delta W = \delta m\,(h + dh) - \delta m\,h$$

$$\text{or:} \qquad -\delta Q + \delta W = \delta m\,dh$$

$$\text{also:} \qquad dS_{gen} = \frac{\delta Q}{T} + \delta m\,ds = 0$$

$$\text{thus:} \qquad \frac{\delta Q}{\delta m} = -T\,ds \qquad \text{for reversible case}$$

$$\frac{\delta W}{\delta m} = -T\,ds + dh$$

But from Gibbs equation:

$$T\,ds = dh - v\,dP$$

Thus:

$$\frac{\delta W}{\delta m} = v\,dP$$

For a given dP, δW is less for lower specific volume, v

“It take less work to compress a denser fluid”

Example, compare water (pump) and gas (compressor) work.