Chapter 11
FUNDAMENTALS OF THERMAL RADIATION

Electromagnetic and Thermal Radiation

11-1C Electromagnetic waves are caused by accelerated charges or changing electric currents giving rise to electric and magnetic fields. Sound waves are caused by disturbances. Electromagnetic waves can travel in vacuum, sound waves cannot.

11-2C Electromagnetic waves are characterized by their frequency v and wavelength λ. These two properties in a medium are related by $\lambda = c / v$ where c is the speed of light in that medium.

11-3C Visible light is a kind of electromagnetic wave whose wavelength is between 0.40 and 0.76 μm. It differs from the other forms of electromagnetic radiation in that it triggers the sensation of seeing in the human eye.

11-4C Infrared radiation lies between 0.76 and 100 μm whereas ultraviolet radiation lies between the wavelengths 0.01 and 0.40 μm. The human body does not emit any radiation in the ultraviolet region since bodies at room temperature emit radiation in the infrared region only.

11-5C Thermal radiation is the radiation emitted as a result of vibrational and rotational motions of molecules, atoms and electrons of a substance, and it extends from about 0.1 to 100 μm in wavelength. Unlike the other forms of electromagnetic radiation, thermal radiation is emitted by bodies because of their temperature.

11-6C Light (or visible) radiation consists of narrow bands of colors from violet to red. The color of a surface depends on its ability to reflect certain wavelength. For example, a surface that reflects radiation in the wavelength range 0.63-0.76 μm while absorbing the rest appears red to the eye. A surface that reflects all the light appears white while a surface that absorbs the entire light incident on it appears black. The color of a surface at room temperature is not related to the radiation it emits.

11-7C Radiation in opaque solids is considered surface phenomena since only radiation emitted by the molecules in a very thin layer of a body at the surface can escape the solid.

11-8C Because the snow reflects almost all of the visible and ultraviolet radiation, and the skin is exposed to radiation both from the sun and from the snow.

11-9C Microwaves in the range of 10^2 to 10^5 μm are very suitable for use in cooking as they are reflected by metals, transmitted by glass and plastics and absorbed by food (especially water) molecules. Thus the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food with no conduction and convection thermal resistances involved. In conventional cooking, on the other hand, conduction and convection thermal resistances slow down the heat transfer, and thus the heating process.

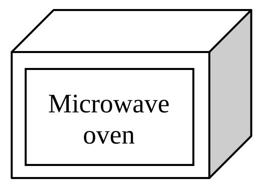
11-10 Electricity is generated and transmitted in power lines at a frequency of 60 Hz. The wavelength of the electromagnetic waves is to be determined.

Analysis The wavelength of the electromagnetic waves is

$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{60 \text{ Hz}(1/\text{s})} = \mathbf{4.997\times10^6 \text{ m}}$$

11-11 A microwave oven operates at a frequency of 2.8×10^9 Hz. The wavelength of these microwaves and the energy of each microwave are to be determined.

Analysis The wavelength of these microwaves is



$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{2.8\times10^9 \text{ Hz}(1/\text{s})} = 0.107 \text{ m} = \mathbf{107 \text{ mm}}$$

Then the energy of each microwave becomes

$$e = hv = \frac{hc}{\lambda} = \frac{(6.625\times10^{-34} \text{ Js})(2.998\times10^8 \text{ m/s})}{0.107 \text{ m}} = \mathbf{1.86\times10^{-24} \text{ J}}$$

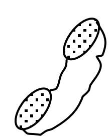
11-12 A radio station is broadcasting radiowaves at a wavelength of 200 m. The frequency of these waves is to be determined.

Analysis The frequency of the waves is determined from

$$\lambda = \frac{c}{v} \longrightarrow v = \frac{c}{\lambda} = \frac{2.998\times10^8 \text{ m/s}}{200 \text{ m}} = \mathbf{1.5\times10^6 \text{ Hz}}$$

11-13 A cordless telephone operates at a frequency of 8.5×10^8 Hz. The wavelength of these telephone waves is to be determined.

Analysis The wavelength of the telephone waves is



$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{8.5\times10^8 \text{ Hz}(1/\text{s})} = 0.35 \text{ m} = \mathbf{350 \text{ mm}}$$

Blackbody Radiation

11-14C A blackbody is a perfect emitter and absorber of radiation. A blackbody does not actually exist. It is an idealized body that emits the maximum amount of radiation that can be emitted by a surface at a given temperature.

11-15C *Spectral blackbody emissive power* is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area and per unit wavelength about wavelength λ. The integration of the spectral blackbody emissive power over the entire wavelength spectrum gives the *total blackbody emissive power*,

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)d\lambda = \sigma T^4$$

The spectral blackbody emissive power varies with wavelength, the total blackbody emissive power does not.

11-16C We defined the blackbody radiation function f_λ because the integration $\int_0^\infty E_{b\lambda}(T)d\lambda$ cannot be performed. The blackbody radiation function f_λ represents the fraction of radiation emitted from a blackbody at temperature T in the wavelength range from $\lambda = 0$ to λ. This function is used to determine the fraction of radiation in a wavelength range between λ_1 and λ_2.

11-17C The larger the temperature of a body , the larger the fraction of the radiation emitted in shorter wavelengths. Therefore, the body at 1500 K will emit more radiation in the shorter wavelength region. The body at 1000 K emits more radiation at $20\ \mu m$ than the body at 1500 K since $\lambda T = \text{constant}$.

11-18 An isothermal cubical body is suspended in the air. The rate at which the cube emits radiation energy and the spectral blackbody emissive power are to be determined.

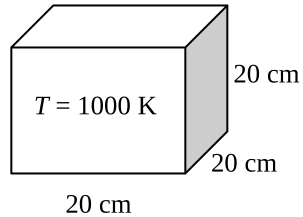
Assumptions The body behaves as a black body.

Analysis (*a*) The total blackbody emissive power is determined from Stefan-Boltzman Law to be

$$A_s = 6a^2 = 6(0.2^2) = 0.24\,\text{m}^2$$

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\,\text{K})^4(0.24\,\text{m}^2) = \mathbf{1.36\times10^4\ W}$$

(*b*) The spectral blackbody emissive power at a wavelength of 4 μm is determined from Plank's distribution law,



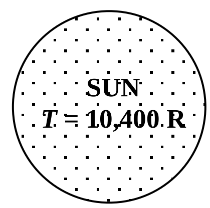
$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]} = \frac{3.743\times10^8\ \text{W}\cdot\mu\text{m}^4/\text{m}^2}{(4\,\mu\text{m})^5\left[\exp\left(\frac{1.4387\times10^4\ \mu\text{m}\cdot\text{K}}{(4\,\mu\text{m})(1000\text{K})}\right)-1\right]}$$

$$= \mathbf{10.3\,kW/m^2\cdot\mu m}$$

11-19E The sun is at an effective surface temperature of 10,372 R. The rate of infrared radiation energy emitted by the sun is to be determined.

Assumptions The sun behaves as a black body.

Analysis Noting that T = 10,400 R = 5778 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11-2 to be



$$\lambda_1 T = (0.76\,\mu\text{m})(5778\text{K}) = 4391.3\mu\text{mK} \longrightarrow f_{\lambda_1} = 0.547370$$

$$\lambda_2 T = (100\,\mu\text{m})(5778\text{K}) = 577{,}800\mu\text{mK} \longrightarrow f_{\lambda_2} = 1.0$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 1.0 - 0.547 = 0.453 \qquad \text{(or 45.3\%)}$$

The total blackbody emissive power of the sun is determined from Stefan-Boltzman Law to be

$$E_b = \sigma T^4 = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(10{,}400\,\text{R})^4 = 2.005\times10^7\ \text{Btu/h.ft}^2$$

Then,

$$E_{\text{infrared}} = (0.451)E_b = (0.453)(2.005\times10^7\ \text{Btu/h.ft}^2) = \mathbf{9.08\times10^6\ Btu/h.ft^2}$$

11-20E "!PROBLEM 11-20"

"GIVEN"
T=5780 "[K]"
"lambda=0.01[micrometer], parameter to be varied"

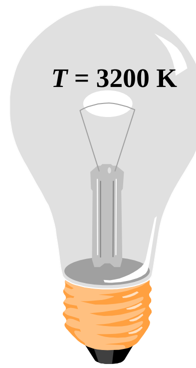
"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 "[W-micrometer^4/m^2]"
C_2=1.439E4 "[micrometer-K]"

λ [micrometer]	$E_{b,\lambda}$ [W/m^2-micrometer]
0.01	2.820E-90
10.11	12684
20.21	846.3
30.31	170.8
40.41	54.63
50.51	22.52
60.62	10.91
70.72	5.905
80.82	3.469
90.92	2.17
…	…
…	…
909.1	0.0002198
919.2	0.0002103
929.3	0.0002013
939.4	0.0001928
949.5	0.0001847
959.6	0.000177
969.7	0.0001698
979.8	0.0001629
989.9	0.0001563
1000	0.0001501

11-21 The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a black body.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T = 3200 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11-2 to be



$$\lambda_1 T = (0.40\ \mu\text{m})(3200\ \text{K}) = 1280\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0043964$$

$$\lambda_2 T = (0.76\ \mu\text{m})(3200\ \text{K}) = 2432\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.147114$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.14711424 - 0.0043964 = \mathbf{0.142718} \quad \text{(or 14.3\%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{maxpower}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow \lambda_{\text{maxpower}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{3200\,\text{K}} = \mathbf{0.905mm}$$

11-22 "!PROBLEM 11-22"

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"GIVEN"
"T=3200 [K], parameter to be varied"
lambda_1=0.40 "[micrometer]"
lambda_2=0.76 "[micrometer]"

"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 "[W-micrometer^4/m^2]"
C_2=1.439E4 "[micrometer-K]"

f_lambda=integral(E_b_lambda, lambda, lambda_1, lambda_2)/E_b
E_b=sigma*T^4
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"
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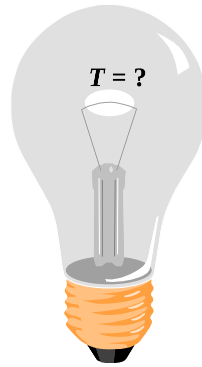
T [K]	f_λ
1000	0.000007353
1200	0.0001032
1400	0.0006403
1600	0.002405
1800	0.006505
2000	0.01404
2200	0.02576
2400	0.04198
2600	0.06248
2800	0.08671
3000	0.1139
3200	0.143
3400	0.1732
3600	0.2036
3800	0.2336
4000	0.2623

11-23 An incandescent light bulb emits 15% of its energy at wavelengths shorter than 1 μm. The temperature of the filament is to be determined.



Assumptions The filament behaves as a black body.

Analysis From the Table 11-2 for the fraction of the radiation, we read

$$f_\lambda = 0.15 \longrightarrow \lambda T = 2445\,\mu\text{mK}$$

For the wavelength range of $\lambda_1 = 0.0$ μm to $\lambda_2 = 1.0$ μm

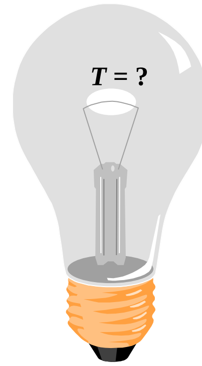
$$\lambda = 1\,\mu\text{m} \longrightarrow \lambda T = 2445\,\mu\text{mK} \longrightarrow T = \mathbf{2445\,K}$$

11-24 Radiation emitted by a light source is maximum in the blue range. The temperature of this light source and the fraction of radiation it emits in the visible range are to be determined.

Assumptions The light source behaves as a black body.

Analysis The temperature of this light source is

$$(\lambda T)_{\text{maxpower}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow T = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{0.47\ \mu\text{m}} = \mathbf{6166\ K}$$



The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T = 6166 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(6166\text{K}) = 2466\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.15444$$

$$\lambda_2 T = (0.76\ \mu\text{m})(6166\text{K}) = 4686\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.59141$$

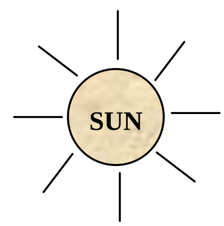
Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.59141 - 0.15444 \cong \mathbf{0.437} \quad \text{(or 43.7\%)}$$

11-25 A glass window transmits 90% of the radiation in a specified wavelength range and is opaque for radiation at other wavelengths. The rate of radiation transmitted through this window is to be determined for two cases.

Assumptions The sources behave as a black body.

Analysis The surface area of the glass window is

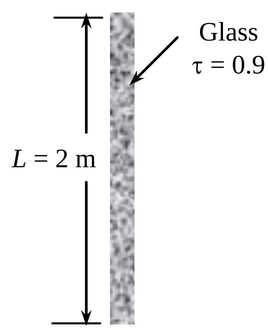


$$A_s = 4\ \text{m}^2$$

(*a*) For a blackbody source at 5800 K, the total blackbody radiation emission is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{kW/m}^2.\text{K})^4(5800\text{K})^4(4\ \text{m}^2) = 2.567\times10^5\ \text{kW}$$

The fraction of radiation in the range of 0.3 to 3.0 μm is



$$\lambda_1 T = (0.30\ \mu\text{m})(5800\text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.03345$$

$$\lambda_2 T = (3.0\ \mu\text{m})(5800\text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.97875$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.97875 - 0.03345 = 0.9453$$

Noting that 90% of the total radiation is transmitted through the window,

$$E_{\text{transmit}} = 0.90\Delta f E_b(T)$$
$$= (0.90)(0.9453)(2.567\times10^5\ \text{kW}) = \mathbf{2.184\times10^5\ kW}$$

(*b*) For a blackbody source at 1000 K, the total blackbody emissive power is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\text{K})^4(4\ \text{m}^2) = 226.8\ \text{kW}$$

The fraction of radiation in the visible range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(1000\text{K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0000$$

$$\lambda_2 T = (3.0\ \mu\text{m})(1000\text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.273232$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.273232 - 0$$

and

$$E_{\text{transmit}} = 0.90\Delta f E_b(T) = (0.90)(0.273232)(226.8\ \text{kW}) = \mathbf{55.8\ kW}$$

Radiation Intensity

11-26C A solid angle represents an opening in space, whereas a plain angle represents an opening in a plane. For a sphere of unit radius, the solid angle about the origin subtended by a given surface on the sphere is equal to the area of the surface. For a cicle of unit radius, the plain angle about the origin subtended by a given arc is equal to the length of the arc. The value of a solid angle associated with a sphere is 4π.

11-27C The intensity of emitted radiation $I_e(\theta, \phi)$ is defined as the rate at which radiation energy $d\dot{Q}_e$ is emitted in the (θ, ϕ) direction per unit area normal to this direction and per unit solid angle about this direction. For a diffusely emitting surface, the emissive power is related to the intensity of emitted radiation by $E = \pi I_e$ (or $E_\lambda = \pi I_{\lambda,e}$ for spectral quantities).

11-28C Irradiation *G* is the radiation flux incident on a surface from all directions. For diffusely incident radiation, irradiation on a surface is related to the intensity of incident radiation by $G = \pi I_i$ (or $G_\lambda = \pi I_{\lambda,i}$ for spectral quantities).

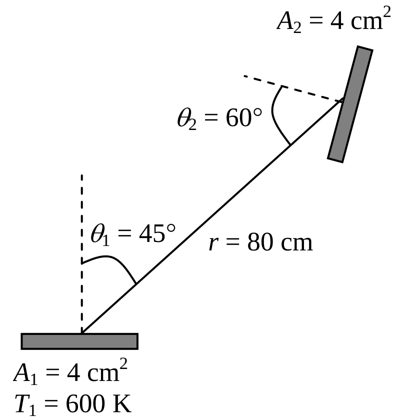
11-29C Radiosity *J* is the rate at which radiation energy leaves a unit area of a surface by emission and reflection in all directions.. For a diffusely emitting and reflecting surface, radiosity is related to the intensity of emitted and reflected radiation by $J = \pi I_{e+r}$ (or $J_\lambda = \pi I_{\lambda,e+r}$ for spectral quantities).

11-30C When the variation of a spectral radiation quantity with wavelength is known, the correcponding total quantity is determined by integrating that quantity with respect to wavelength from λ = 0 to λ = ∞.

11-31 A surface is subjected to radiation emitted by another surface. The solid angle subtended and the rate at which emitted radiation is received are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from Eq. 11-12 to be



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(4\,\text{cm}^2)\cos 60°}{(80\,\text{cm})^2} = \mathbf{3.125 \times 10^{-4}\ sr}$$

since the normal of A_2 makes 60° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle ω_{2-1}. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(800\,\text{K})^4}{\pi} = 7393\,\text{W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1} \\ &= (7393\,\text{W/m}^2\cdot\text{sr})(4\times10^{-4}\cos 45°\ \text{m}^2)(3.125\times10^{-4}\ \text{sr}) \\ &= \mathbf{6.534\times10^{-4}\ W}\end{aligned}$$

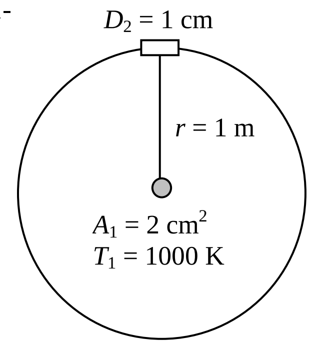
Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 6.534×10^{-4} W.

If A_2 were directly above A_1 at a distance 80 cm, θ_1 = 0° and the rate of radiation energy emitted by A_1 becomes zero.

11-32 Radiation is emitted from a small circular surface located at the center of a sphere. Radiation energy streaming through a hole located on top of the sphere and the side of sphere are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis (*a*) Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from Eq. 11-12 to be



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2}{r^2} = \frac{\pi(0.005\,\text{m})^2}{(1\,\text{m})^2} = 7.854\times10^{-5}\ \text{sr}$$

since A_2 were positioned normal to the direction of viewing.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

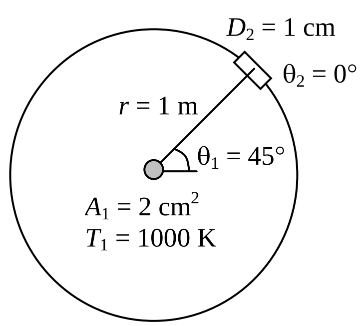
$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1000\,\text{K})^4}{\pi} = 18{,}048\ \text{W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\ \text{W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 0°\ \text{m}^2)(7.854\times10^{-5}\ \text{sr})\\ &= \mathbf{2.835\times10^{-4}\ W}\end{aligned}$$

where θ_1 = 0°. Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.835×10^{-4} W.

(*b*) In this orientation, θ_1 = 45° and θ_2 = 0°. Repeating the calculation we obtain the rate of radiation to be

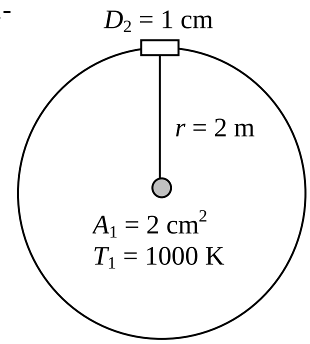


$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\ \text{W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 45°\ \text{m}^2)(7.854\times10^{-5}\ \text{sr})\\ &= \mathbf{2.005\times10^{-4}\ W}\end{aligned}$$

11-33 Radiation is emitted from a small circular surface located at the center of a sphere. Radiation energy streaming through a hole located on top of the sphere and the side of sphere are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis (*a*) Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from Eq. 11-12 to be



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2}{r^2} = \frac{\pi(0.005\,\text{m})^2}{(2\,\text{m})^2} = 1.963\times10^{-5}\ \text{sr}$$

since A_2 were positioned normal to the direction of viewing.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

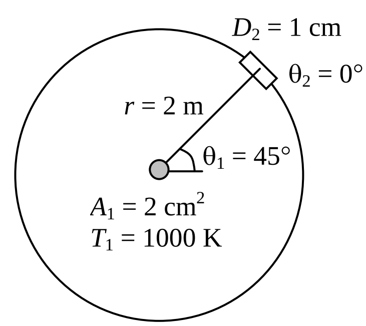
$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1000\,\text{K})^4}{\pi} = 18{,}048\,\text{W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\,\text{W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 0^\circ\ \text{m}^2)(1.963\times10^{-5}\ \text{sr})\\ &= \mathbf{7.087\times10^{-5}\ W}\end{aligned}$$

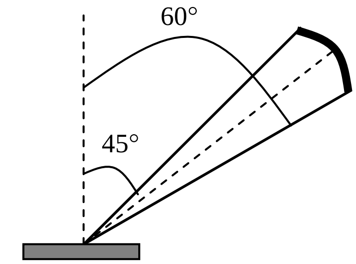
where $\theta_1 = 0°$. Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.835×10^{-4} W.

(*b*) In this orientation, $\theta_1 = 45°$ and $\theta_2 = 0°$. Repeating the calculation we obtain the rate of radiation as



$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\,\text{W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 45^\circ\ \text{m}^2)(1.963\times10^{-5}\ \text{sr})\\ &= \mathbf{5.010\times10^{-5}\ W}\end{aligned}$$

11-34 A small surface emits radiation. The rate of radiation energy emitted through a band is to be determined.



Assumptions Surface *A* emits diffusely as a blackbody.

Analysis The rate of radiation emission from a surface per unit surface area in the direction (θ,ϕ) is given as

$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

The total rate of radiation emission through the band between 60° and 45° can be expressed as

$$E = \int_{\phi=0}^{2\pi}\int_{\theta=45}^{60} I_e(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_b\frac{\pi}{4} = \frac{\sigma T^4}{\pi}\frac{\pi}{4} = \frac{\sigma T^4}{4}$$

since the blackbody radiation intensity is constant (I_b = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=45}^{60}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=45}^{60}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 60 - \sin^2 45) = \pi/4$$

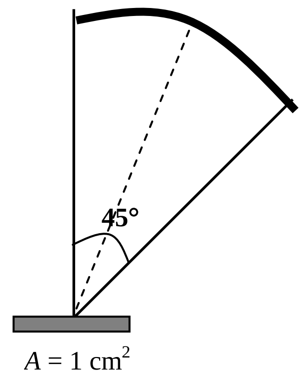
Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 1 cm^2 in the specified band becomes

$$\dot{Q}_e = EdA = \frac{\sigma T^4}{4}dA = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1500\text{ K})^4}{4}(1\times10^{-4}\text{ m}^2) = \mathbf{7.18\ W}$$

11-35 A small surface is subjected to uniform incident radiation. The rates of radiation emission through two specified bands are to be determined.

Assumptions The intensity of incident radiation is constant.

Analysis (*a*) The rate at which radiation is incident on a surface per unit surface area in the direction (θ,ϕ) is given as



$$dG = \frac{d\dot{Q}_i}{dA} = I_i(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

The total rate of radiation emission through the band between 0° and 45° can be expressed as

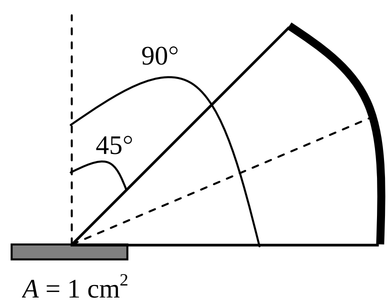
$$G_1 = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{45} I_i(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_i\ \frac{\pi}{2}$$

since the incident radiation is constant (I_i = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{45}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=0}^{45}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 45 - \sin^2 0) = \pi/2$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 1 cm^2 in the specified band becomes

$$\dot{Q}_{i,1} = G_1 dA = 0.5\pi I_i dA = 0.5\pi(2.2\times10^4\ \text{W/m}^2\cdot\text{sr})(1\times10^{-4}\ \text{m}^2) = \mathbf{3.46\ W}$$



(*b*) Similarly, the total rate of radiation emission through the band between 45° and 90° can be expressed as

$$G_1 = \int_{\phi=0}^{2\pi}\int_{\theta=45}^{90} I_i(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_i\ \frac{\pi}{2}$$

since

$$\int_{\phi=0}^{2\pi}\int_{\theta=45}^{90}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=45}^{90}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 90 - \sin^2 45) = \pi/2$$

and

$$\dot{Q}_{i,2} = G_2 dA = 0.5\pi I_i dA = 0.5\pi(2.2\times10^4\ \text{W/m}^2\cdot\text{sr})(1\times10^{-4}\ \text{m}^2) = \mathbf{3.46\ W}$$

Discussion Note that the viewing area for the band 0° - 45° is much smaller, but the radiation energy incident through it is equal to the energy streaming through the remaining area.

Radiation Properties

11-36C The emissivity ε is the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature. The fraction of radiation absorbed by the surface is called the absorptivity α,

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \quad \text{and} \quad \alpha = \frac{\text{absorbed radiation}}{\text{incident radiation}} = \frac{G_{abs}}{G}$$

When the surface temperature is equal to the temperature of the source of radiation, the total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature $\varepsilon_\lambda(T) = \alpha_\lambda(T)$.

11-37C The fraction of irradiation reflected by the surface is called reflectivity ρ and the fraction transmitted is called the transmissivity τ

$$\rho = \frac{G_{ref}}{G} \quad \text{and} \quad \tau = \frac{G_{tr}}{G}$$

Surfaces are assumed to reflect in a perfectly spectral or diffuse manner for simplicity. In spectral (or mirror like) reflection, the angle of reflection equals the angle of incidence of the radiation beam. In diffuse reflection, radiation is reflected equally in all directions.

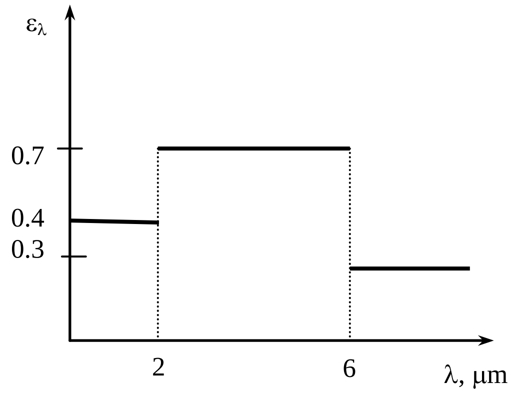
11-38C A body whose surface properties are independent of wavelength is said to be a graybody. The emissivity of a blackbody is one for all wavelengths, the emissivity of a graybody is between zero and one.

11-39C The heating effect which is due to the non-gray characteristic of glass, clear plastic, or atmospheric gases is known as the greenhouse effect since this effect is utilized primarily in greenhouses. The combustion gases such as CO_2 and water vapor in the atmosphere transmit the bulk of the solar radiation but absorb the infrared radiation emitted by the surface of the earth, acting like a heat trap. There is a concern that the energy trapped on earth will eventually cause global warming and thus drastic changes in weather patterns.

11-40C Glass has a transparent window in the wavelength range 0.3 to 3 μm and it is not transparent to the radiation which has wavelength range greater than 3 μm. Therefore, because the microwaves are in the range of 10^2 to 10^5 μm, the harmful microwave radiation cannot escape from the glass door.

11-41 The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from



$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b_\lambda}(T)d\lambda}{\sigma T^4}$$

$$= \varepsilon_1 f_{0\text{-}\lambda_1} + \varepsilon_2 f_{\lambda_1\text{-}\lambda_2} + \varepsilon_3 f_{\lambda_2\text{-}\infty}$$

$$= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (2\ \mu\text{m})(1000\ \text{K}) = 2000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.066728$$

$$\lambda_2 T = (6\ \mu\text{m})(1000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.737818$$

$$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1} \text{ since } f_0 = 0 \text{ and } f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} \text{ since } f_\infty = 1.$$

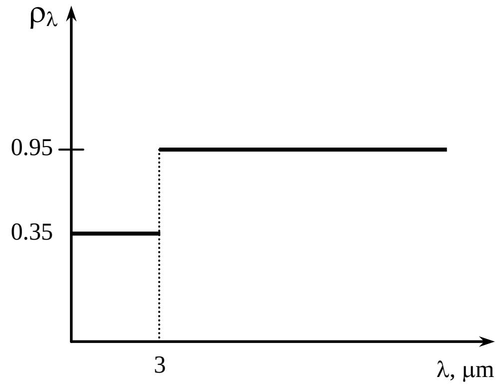
and,

$$\varepsilon = (0.4)0.066728 + (0.7)(0.737818 - 0.066728) + (0.3)(1 - 0.737818) = \mathbf{0.575}$$

Then the emissive power of the surface becomes

$$E = \varepsilon\sigma T^4 = 0.575(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\,\text{K})^4 = \mathbf{32.6\ kW/m^2}$$

11-42 The variation of reflectivity of a surface with wavelength is given. The average reflectivity, emissivity, and absorptivity of the surface are to be determined for two source temperatures.



Analysis The average reflectivity of this surface for solar radiation (T = 5800 K) is determined to be

$$\lambda T = (3\ \mu\text{m})(5800\ \text{K}) = 17400\ \mu\text{mK} \rightarrow f_\lambda = 0.978746$$

$$\begin{aligned}\rho(T) &= \rho_1 f_{0-\lambda_1}(T) + \rho_2 f_{\lambda_1-\infty}(T)\\ &= \rho_1 f_{\lambda_1} + \rho_2(1-f_{\lambda_1})\\ &= (0.35)(0.978746) + (0.95)(1-0.978746)\\ &= \mathbf{0.362}\end{aligned}$$

Noting that this is an opaque surface, $\tau = 0$

At T = 5800 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1-\rho = 1-0.362 = \mathbf{0.638}$

Repeating calculations for radiation coming from surfaces at T = 300 K,

$$\lambda T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0001685$$

$$\rho(T) = (0.35)(0.0001685) + (0.95)(1-0.0001685) = \mathbf{0.95}$$

At T = 300 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1-\rho = 1-0.95 = \mathbf{0.05}$

and $\varepsilon = \alpha = \mathbf{0.05}$

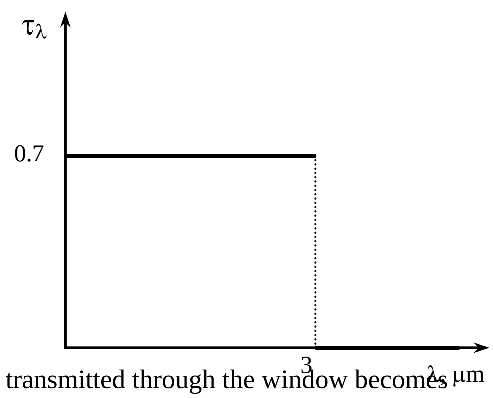
The temperature of the aluminum plate is close to room temperature, and thus emissivity of the plate will be equal to its absorptivity at room temperature. That is,

$$\varepsilon = \varepsilon_{room} = 0.05$$
$$\alpha = \alpha_s = 0.638$$

which makes it suitable as a solar collector. ($\alpha_s = 1$ and $\varepsilon_{\text{room}} = 0$ for an ideal solar collector)

11-43 The variation of transmissivity of the glass window of a furnace at a specified temperature with wavelength is given. The fraction and the rate of radiation coming from the furnace and transmitted through the window are to be determined.

Assumptions The window glass behaves as a black body.



Analysis The fraction of radiation at wavelengths smaller than 3 μm is

$$\lambda T = (3\ \mu\text{m})(1200\ \text{K}) = 3600\ \mu\text{mK} \longrightarrow f_\lambda = 0.403607$$

The fraction of radiation coming from the furnace and transmitted through the window is

$$\begin{aligned}\tau(T) &= \tau_1 f_\lambda + \tau_2(1-f_\lambda)\\ &= (0.7)(0.403607) + (0)(1-0.403607)\\ &= \mathbf{0.283}\end{aligned}$$

Then the rate of radiation coming from the furnace and transmitted through the window becomes

$$G_{tr} = \tau A\sigma T^4 = 0.283(0.25\times 0.25\ \text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\ \text{K})^4 = \mathbf{2076\ W}$$

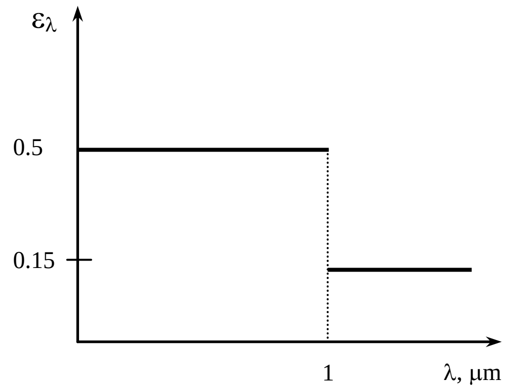
11-44 The variation of emissivity of a tungsten filament with wavelength is given. The average emissivity, absorptivity, and reflectivity of the filament are to be determined for two temperatures.

Analysis (*a*) T = 2000 K

$$\lambda_1 T = (1\,\mu\text{m})(2000\text{K}) = 2000\,\mu\text{mK} \longrightarrow f_{\lambda_1} = 0.066728$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1}) \\ &= (0.5)(0.066728) + (0.15)(1 - 0.066728) \\ &= \mathbf{0.173}\end{aligned}$$



From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.173} \quad \text{(at 2000 K)}$$

and

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.173 = \mathbf{0.827}$$

(*b*) T = 3000 K

$$\lambda_1 T = (1\,\mu\text{m})(3000\text{ K}) = 3000\,\mu\text{mK} \longrightarrow f_{\lambda_1} = 0.273232$$

Then

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1}) \\ &= (0.5)(0.273232) + (0.15)(1 - 0.273232) \\ &= \mathbf{0.246}\end{aligned}$$

From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.246} \quad \text{(at 3000 K)}$$

and

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.246 = \mathbf{0.754}$$

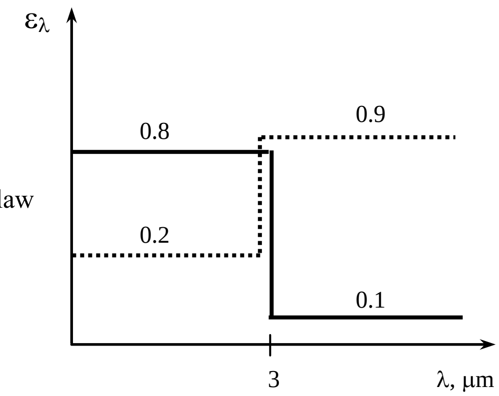
11-45 The variations of emissivity of two surfaces are given. The average emissivity, absorptivity, and reflectivity of each surface are to be determined at the given temperature.

Analysis For the first surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\ \text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.890029) + (0.9)(1 - 0.890029) \\ &= \mathbf{0.28}\end{aligned}$$



The absorptivity and reflectivity are determined from Kirchhoff's law

$$\varepsilon = \alpha = \mathbf{0.28} \quad (\text{at } 3000\ \text{K})$$

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.28 = \mathbf{0.72}$$

For the second surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\ \text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.8)(0.890029) + (0.1)(1 - 0.890029) \\ &= \mathbf{0.72}\end{aligned}$$

Then,

$$\varepsilon = \alpha = \mathbf{0.72} \quad (\text{at } 3000\ \text{K})$$

$$\alpha + \rho = 1 \rightarrow \rho = 1 - \alpha = 1 - 0.72 = \mathbf{0.28}$$

Discussion The second surface is more suitable to serve as a solar absorber since its absorptivity for short wavelength radiation (typical of radiation emitted by a high-temperature source such as the sun) is high, and its emissivity for long wavelength radiation (typical of emitted radiation from the absorber plate) is low.

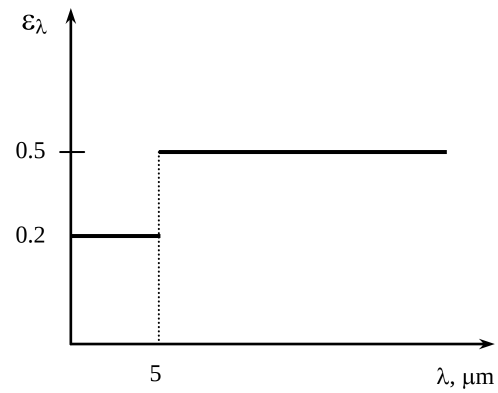
11-46 The variation of emissivity of a surface with wavelength is given. The average emissivity and absorptivity of the surface are to be determined for two temperatures.

Analysis (*a*) For T = 5800 K:

$$\lambda_1 T = (5\ \mu\text{m})(5800\ \text{K}) = 29{,}000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.99534$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})\\ &= (0.2)(0.99534) + (0.9)(1 - 0.99534)\\ &= \mathbf{0.203}\end{aligned}$$



(*b*) For T = 300 K:

$$\lambda_1 T = (5\mu\text{m})(300\ \text{K}) = 1500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.013754$$

and

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})\\ &= (0.2)(0.013754) + (0.9)(1 - 0.013754)\\ &= \mathbf{0.89}\end{aligned}$$

The absorptivities of this surface for radiation coming from sources at 5800 K and 300 K are, from Kirchhoff's law,

$$\alpha = \varepsilon = \mathbf{0.203} \quad \text{(at 5800 K)}$$

$$\alpha = \varepsilon = \mathbf{0.89} \quad \text{(at 300 K)}$$

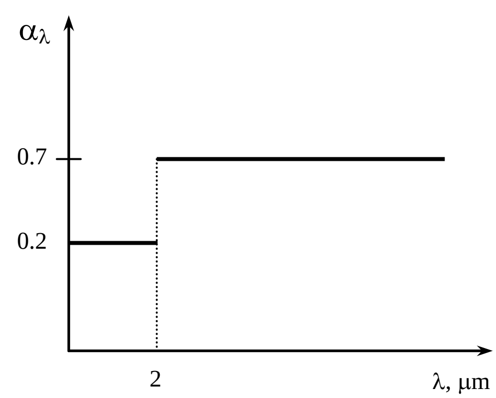
11-47 The variation of absorptivity of a surface with wavelength is given. The average absorptivity, reflectivity, and emissivity of the surface are to be determined at given temperatures.

Analysis For T = 2500 K:

$$\lambda_1 T = (2\ \mu\text{m})(2500\ \text{K}) = 5000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.633747$$

The average absorptivity of this surface is

$$\begin{aligned}\alpha(T) &= \alpha_1 f_{\lambda_1} + \alpha_2(1 - f_{\lambda_1})\\ &= (0.2)(0.633747) + (0.7)(1 - 0.633747)\\ &= \mathbf{0.38}\end{aligned}$$



Then the reflectivity of this surface becomes

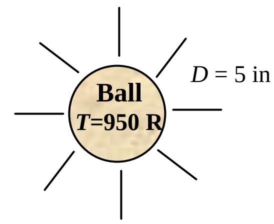
$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.38 = \mathbf{0.62}$$

Using Kirchhoff's law, $\alpha = \varepsilon$, the average emissivity of this surface at T = 3000 K is determined to be

$$\lambda T = (2\mu\text{m})(3000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda} = 0.737818$$

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})\\ &= (0.2)(0.737818) + (0.7)(1 - 0.737818)\\ &= \mathbf{0.33}\end{aligned}$$

11-48E A spherical ball emits radiation at a certain rate. The average emissivity of the ball is to be determined at the given temperature.



Analysis The surface area of the ball is

$$A = \pi D^2 = \pi(5/12\text{ ft})^2 = 0.5454\text{ ft}^2$$

Then the average emissivity of the ball at this temperature is determined to be

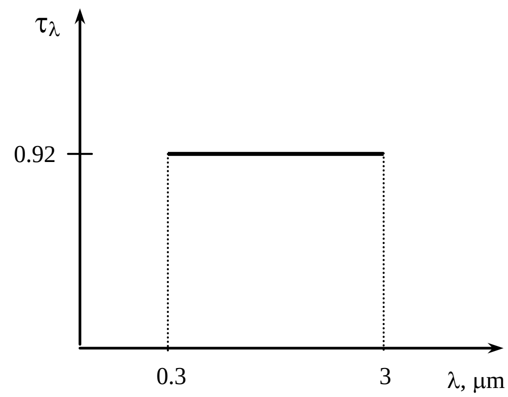
$$E = \varepsilon A\sigma T^4 \longrightarrow \varepsilon = \frac{E}{A\sigma T^4} = \frac{120\text{ Btu/h}}{(0.5454\text{ft}^2)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\cdot\text{R}^4)(950\text{ R})^4} = \mathbf{0.158}$$

11-49 The variation of transmissivity of a glass is given. The average transmissivity of the pane at two temperatures and the amount of solar radiation transmitted through the pane are to be determined.

Analysis For T=5800 K:

$$\lambda_1 T_1 = (0.3\ \mu\text{m})(5800\text{ K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.035$$

$$\lambda_2 T_1 = (3\ \mu\text{m})(5800\text{ K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.977$$



The average transmissivity of this surface is

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.9)(0.977 - 0.035) = \mathbf{0.848}$$

For T=300 K:

$$\lambda_1 T_2 = (0.3\ \mu\text{m})(300\text{ K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T_2 = (3\ \mu\text{m})(300\text{ K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.0001685$$

Then,

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.9)(0.0001685 - 0.0) = \mathbf{0.00015} \approx \mathbf{0}$$

The amount of solar radiation transmitted through this glass is

$$G_{\text{tr}} = \tau G_{\text{incident}} = 0.848(650\text{ W/m}^2) = \mathbf{551\ W/m^2}$$