## Atmospheric and Solar Radiation

**11-50C** The solar constant represents the rate at which solar energy is incident on a surface normal to sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun. It value is $G_s = 1353 \text{ W/m}^2$. The solar constant is used to estimate the effective surface temperature of the sun from the requirement that

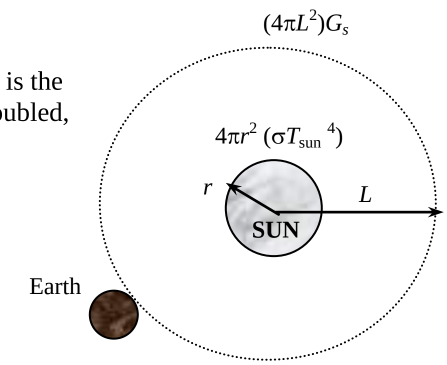
$$(4\pi L^2)G_{s1} = (4\pi r^2)\sigma T_{sun}{}^4$$

where L is the mean distance between the sun and the earth and r is the radius of the sun. If the distance between the earth and the sun doubled, the value of $G_s$ drops to one-fourth since

$$4\pi(2L)^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = 4\pi L^2 G_{s1} \longrightarrow \mathbf{G_{s2} = \frac{G_{s1}}{4}}$$




**11-51C** The amount of solar radiation incident on earth will decrease by a factor of

$$\text{Reduction factor} = \frac{\sigma T_{sun}{}^4}{\sigma T_{sun,new}{}^4} = \frac{5762^4}{2000^4} = 68.9$$

(or to 1.5% of what it was). Also, the fraction of radiation in the visible range would be much smaller.

**11-52C** Air molecules scatter blue light much more than they do red light. This molecular scattering in all directions is what gives the sky its bluish color. At sunset, the light travels through a thicker layer of atmosphere, which removes much of the blue from the natural light, letting the red dominate.

**11-53C** The reason for different seasons is the tilt of the earth which causes the solar radiation to travel through a longer path in the atmosphere in winter, and a shorter path in summer. Therefore, the solar radiation is attenuated much more strongly in winter.

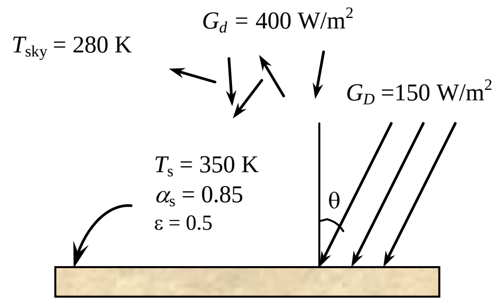
**11-54C** The gas molecules and the suspended particles in the atmosphere emit radiation as well as absorbing it. Although this emission is far from resembling the distribution of radiation from a blackbody, it is found convenient in radiation calculations to treat the atmosphere as a blackbody at some lower fictitious temperature that emits an equivalent amount of radiation energy. This fictitious temperature is called the effective sky temperature $T_{sky}$.

**11-55C** There is heat loss from both sides of the bridge (top and bottom surfaces of the bridge) which reduces temperature of the bridge surface to very low values. The relatively warm earth under a highway supply heat to the surface continuously, making the water on it less likely to freeze.

**11-56C** Due to its nearly horizontal orientation, windshield exchanges heat with the sky that is at very low temperature. Side windows on the other hand exchange heat with surrounding surfaces that are at relatively high temperature.

**11-57C** Because of different wavelengths of solar radiation and radiation originating from surrounding bodies, the surfaces usually have quite different absorptivities. Solar radiation is concentrated in the short wavelength region and the surfaces in the infrared region.

**11-58** A surface is exposed to solar and sky radiation. The net rate of radiation heat transfer is to be determined.




***Properties*** The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.85$ and $\varepsilon = 0.5$.

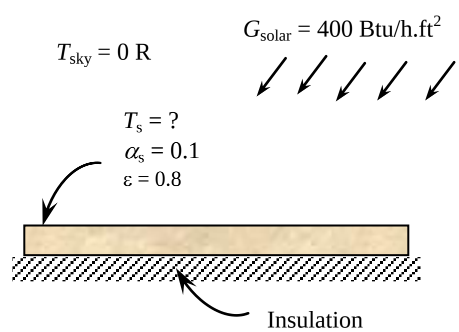
***Analysis*** The total solar energy incident on the surface is

$$\begin{aligned} G_{solar} &= G_D \cos\theta + G_d \\ &= (350\,\text{W/m}^2)\cos 30° + (400\,\text{W/m}^2) \\ &= 703.1\,\text{W/m}^2 \end{aligned}$$

Then the net rate of radiation heat transfer in this case becomes

$$\begin{aligned} \dot{q}_{net,rad} &= \alpha_s G_{solar} - \varepsilon\sigma(T_s^4 - T_{sky}^4) \\ &= 0.85(703.1\,\text{W/m}^2) - 0.5(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(350\,\text{K})^4 - (280\,\text{K})^4\right] \\ &= \mathbf{347\ W/m^2}\ \ \text{(to the surface)} \end{aligned}$$

**11-59E** A surface is exposed to solar and sky radiation. The equilibrium temperature of the surface is to be determined.

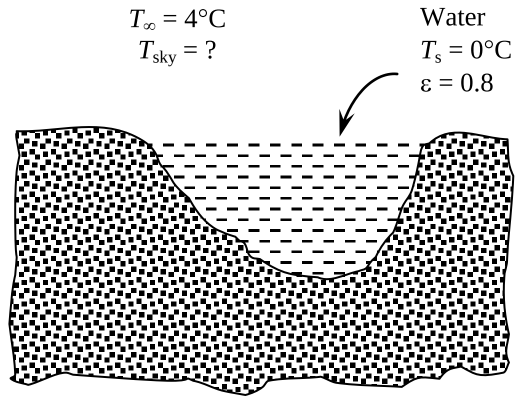



***Properties*** The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.10$ and $\varepsilon = 0.8$.

***Analysis*** The equilibrium temperature of the surface in this case is

$$\begin{aligned} \dot{q}_{net,rad} &= \alpha_s G_{solar} - \varepsilon\sigma(T_s^4 - T_{sky}^4) = 0 \\ \alpha_s G_{solar} &= \varepsilon\sigma(T_s^4 - T_{sky}^4) \\ 0.1(400\,\text{Btu/h.ft}^2) &= 0.8(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\left[T_s^4 - (0\,\text{R})^4\right] \\ T_s &= \mathbf{413.3\ R} \end{aligned}$$

**11-60** Water is observed to have frozen one night while the air temperature is above freezing temperature. The effective sky temperature is to be determined.




***Properties*** The emissivity of water is ε = 0.95 (Table A-21).

***Analysis*** Assuming the water temperature to be 0°C, the value of the effective sky temperature is determined from an energy balance on water to be

$$h(T_{air} - T_{surface}) = \varepsilon\sigma(T_s^4 - T_{sky}^4)$$

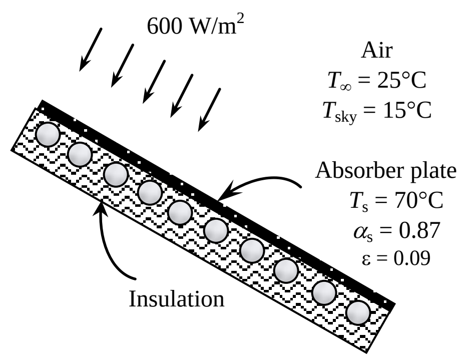
and

$$(18\text{ W/m}^2\cdot°\text{C})(4°\text{C} - 0°\text{C}) = 0.95(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(273\text{K})^4 - T_{sky}^4\right]$$

$$\longrightarrow T_{sky} = \mathbf{254.8\ K}$$

Therefore, the effective sky temperature must have been below 255 K.

**11-61** The absorber plate of a solar collector is exposed to solar and sky radiation. The net rate of solar energy absorbed by the absorber plate is to be determined.




***Properties*** The solar absorptivity and emissivity of the surface are given to $\alpha_s$ = 0.87 and ε = 0.09.

***Analysis*** The net rate of solar energy delivered by the absorber plate to the water circulating behind it can be determined from an energy balance to be

$$\dot{q}_{net} = \dot{q}_{gain} - \dot{q}_{loss}$$

$$\dot{q}_{net} = \alpha_s G_{solar} - \ \varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})$$

Then,

$$\dot{q}_{net} = 0.87(600\text{ W/m}^2) - 0.09(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(70+273\text{K})^4 - (15+273\text{K})^4\right]$$
$$-(10\text{ W/m}^2\cdot\text{K})(70°\text{C} - 25°\text{C})$$
$$= \mathbf{36.5\ W/m^2}$$

Therefore, heat is gained by the plate and transferred to water at a rate of 36.5 W per m$^2$ surface area.

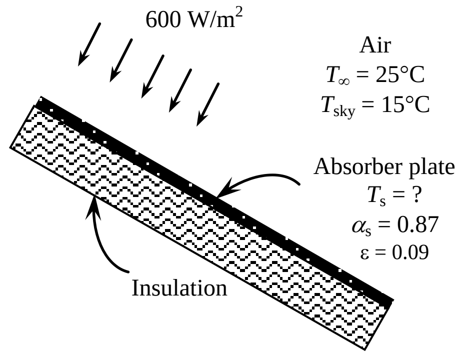
**11-62** "!PROBLEM 11-62"

```
"GIVEN"
"alpha_s=0.87 parameter to be varied"
epsilon=0.09
G_solar=600 "[W/m^2]"
T_air=25+273 "[K]"
T_sky=15+273 "[K]"
T_s=70+273 "[K]"
h=10 "[W/m^2-C]"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
q_dot_net=q_dot_gain-q_dot_loss "energy balance"
q_dot_gain=alpha_s*G_solar
q_dot_loss=epsilon*sigma*(T_s^4-T_sky^4)+h*(T_s-T_air)
```

| $\alpha_s$ | $q_{net}$ [W/m$^2$] |
|---|---|
| 0.5 | -185.5 |
| 0.525 | -170.5 |
| 0.55 | -155.5 |
| 0.575 | -140.5 |
| 0.6 | -125.5 |
| 0.625 | -110.5 |
| 0.65 | -95.52 |
| 0.675 | -80.52 |
| 0.7 | -65.52 |
| 0.725 | -50.52 |
| 0.75 | -35.52 |
| 0.775 | -20.52 |
| 0.8 | -5.525 |
| 0.825 | 9.475 |
| 0.85 | 24.48 |
| 0.875 | 39.48 |
| 0.9 | 54.48 |
| 0.925 | 69.48 |
| 0.95 | 84.48 |
| 0.975 | 99.48 |
| 1 | 114.5 |

**11-63** The absorber surface of a solar collector is exposed to solar and sky radiation. The equilibrium temperature of the absorber surface is to be determined if the backside of the plate is insulated.




***Properties*** The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.87$ and $\varepsilon = 0.09$.

***Analysis*** The backside of the absorbing plate is insulated (instead of being attached to water tubes), and thus

$$\dot{q}_{net} = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})$$

$$(0.87)(600\,\text{W/m}^2) = (0.09)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(T_s)^4 - (288\,\text{K})^4\right] + (10\,\text{W/m}^2\cdot\text{K})(T_s - 298\,\text{K})$$

$$T_s = \mathbf{346\ K}$$

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## Special Topic: Solar Heat Gain Through Windows

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**11-64C** (*a*) The spectral distribution of solar radiation beyond the earth's atmosphere resembles the energy emitted by a black body at 5982°C, with about 39 percent in the visible region (0.4 to 0.7 μm), and the 52 percent in the near infrared region (0.7 to 3.5 μm). (*b*) At a solar altitude of 41.8°, the total energy of direct solar radiation incident at sea level on a clear day consists of about 3 percent ultraviolet, 38 percent visible, and 59 percent infrared radiation.

**11-65C** A window that transmits visible part of the spectrum while absorbing the infrared portion is ideally suited for minimizing the air-conditioning load since such windows provide maximum daylighting and minimum solar heat gain. The ordinary window glass approximates this behavior remarkably well.

**11-66C** The **solar heat gain coefficient (**SHGC) is defined as the fraction of incident solar radiation that enters through the glazing. The solar heat gain of a glazing relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87, is called the **shading coefficient**. They are related to each other by

$$\text{SC} = \frac{\text{Solar heat gain of product}}{\text{Solar heat gain of reference glazing}} = \frac{\text{SHGC}}{\text{SHGC}_{\text{ref}}} = \frac{\text{SHGC}}{0.87} = 1.15\times\text{SHGC}$$

For single pane clear glass window, SHGC = 0.87 and SC = 1.0.

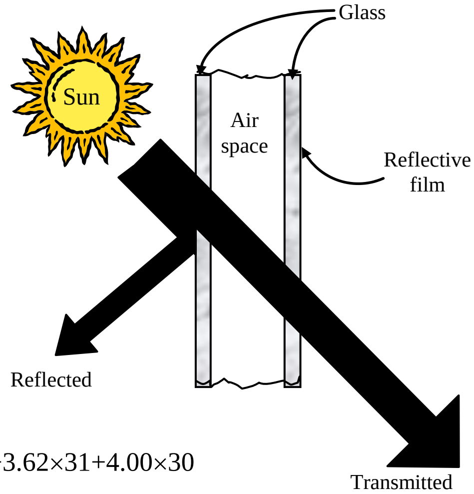
**11-67C** The SC (shading coefficient) of a device represents the solar heat gain relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87. The shading coefficient of a 3-mm thick *clear glass* is SC = 1.0 whereas SC = 0.88 for 3-mm thick *heat absorbing glass*.

**11-68C** A device that blocks solar radiation and thus reduces the solar heat gain is called a shading device. External shading devices are more effective in reducing the solar heat gain since they intercept sun's rays before they reach the glazing. The solar heat gain through a window can be reduced by as much as 80 percent by exterior shading. *Light colored* shading devices maximize the back reflection and thus minimize the solar gain. *Dark colored* shades, on the other hand, minimize the back refection and thus maximize the solar heat gain.

**11-69C** A low-e coating on the inner surface of a window glass reduces both the (*a*) heat loss in winter and (*b*) heat gain in summer. This is because the radiation heat transfer to or from the window is proportional to the emissivity of the inner surface of the window. In winter, the window is colder and thus radiation heat loss from the room to the window is low. In summer, the window is hotter and the radiation transfer from the window to the room is low.

**11-70C** Glasses coated with reflective films on the outer surface of a window glass reduces solar heat both in summer and in winter.

**11-71** The net annual cost savings due to installing reflective coating on the West windows of a building and the simple payback period are to be determined.




***Assumptions*** **1** The calculations given below are for an average year. **2** The unit costs of electricity and natural gas remain constant.

***Analysis*** Using the daily averages for each month and noting the number of days of each month, the total solar heat flux incident on the glazing during summer and winter months are determined to be

$$Q_{\text{solar, summer}} = 4.24\times30 + 4.16\times31 + 3.93\times31 + 3.48\times30$$
$$= 482 \text{ kWh/year}$$

$$Q_{\text{solar, winter}} = 2.94\times31 + 2.33\times30 + 2.07\times31 + 2.35\times31 + 3.03\times28 + 3.62\times31 + 4.00\times30$$
$$= 615 \text{ kWh/year}$$

Then the decrease in the annual cooling load and the increase in the annual heating load due to reflective film become

$$\text{Cooling load decrease} = Q_{\text{solar, summer}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (482 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.261)$$
$$= 14{,}605 \text{ kWh/year}$$

$$\text{Heating load increase} = Q_{\text{solar, winter}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (615 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.261)$$
$$= 18{,}635 \text{ kWh/year} = 635.8 \text{ therms/year}$$

since 1 therm = 29.31 kWh. The corresponding decrease in cooling costs and increase in heating costs are

$$\text{Decrease in cooling costs} = (\text{Cooling load decrease})(\text{Unit cost of electricity})/\text{COP}$$
$$= (14{,}605 \text{ kWh/year})(\$0.09/\text{kWh})/3.2 = \$411/\text{year}$$

$$\text{Increase in heating costs} = (\text{Heating load increase})(\text{Unit cost of fuel})/\text{Efficiency}$$
$$= (635.8 \text{ therms/year})(\$0.45/\text{therm})/0.80 = \$358/\text{year}$$

Then the net annual cost savings due to reflective films become

$$\text{Cost Savings} = \text{Decrease in cooling costs} - \text{Increase in heating costs} = \$411 - 358 = \mathbf{\$53/year}$$

The implementation cost of installing films is

$$\text{Implementation Cost} = (\$20/\text{m}^2)(60 \text{ m}^2) = \$1200$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$1200}{\$53/\text{year}} = \mathbf{23\ years}$$

***Discussion*** The reflective films will pay for themselves in this case in about 23 years, which is unacceptable to most manufacturers since they are not usually interested in any energy conservation measure which does not pay for itself within 3 years.

**11-72** A house located at 40º N latitude has ordinary double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

***Assumptions*** The calculations are performed for an average day in a given month.

***Properties*** The shading coefficient of a double pane window with 6-mm thick glasses is SC = 0.82 (Table 11-5). The incident radiation at different windows at different times are given as (Table 11-4)

| Month | Time | Solar radiation incident on the surface, W/m$^2$ | | | |
|---|---|---|---|---|---|
| | | North | East | South | West |
| July | 9:00 | 117 | 701 | 190 | 114 |
| July | 12:00 | 138 | 149 | 395 | 149 |
| July | 15:00 | 117 | 114 | 190 | 701 |
| January | Daily total | 446 | 1863 | 5897 | 1863 |

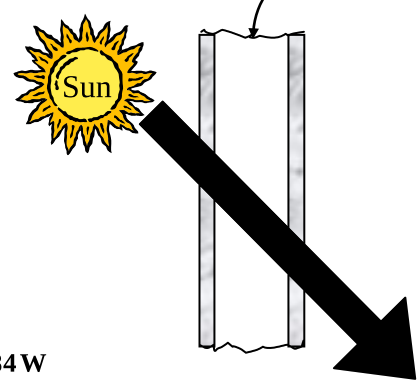
***Analysis*** The solar heat gain coefficient (SHGC) of the windows is determined from Eq.11-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times 0.82 = 0.7134$$

The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar,incident}} = 0.7134\times A_{\text{glazing}}\times\dot{q}_{\text{solar,incident}}$$

Then the rates of heat gain at the 4 walls at 3 different times in July become




*North wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{334\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(4\text{ m}^2)\times(138\text{ W/m}^2) = \mathbf{394\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{334\ W}$$

*East wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{3001\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{638\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{488\ W}$$

*South wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{1084\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(8\text{ m}^2)\times(395\text{ W/m}^2) = \mathbf{2254\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{1084\ W}$$

*West wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{488\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{638\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{3001\ W}$$

Similarly, the solar heat gain of the house through all of the windows in January is determined to be

*January*:

$$\dot{Q}_{\text{solar gain, North}} = 0.7134\times(4\text{ m}^2)\times(446\text{ Wh/m}^2\cdot\text{day}) = 1273\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain,East}} = 0.7134\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 7974\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain, South}} = 0.7134\times(8\text{ m}^2)\times(5897\text{ Wh/m}^2\cdot\text{day}) = 33{,}655\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain,West}} = 0.7134\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 7974\text{ Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 446+1863+5897+1863 = 58{,}876\text{ Wh/day} \cong \mathbf{58.9\ kWh/day}$$

**11-73** A house located at 40º N latitude has gray-tinted double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

***Assumptions*** The calculations are performed for an average day in a given month.

***Properties*** The shading coefficient of a gray-tinted double pane window with 6-mm thick glasses is SC = 0.58 (Table 11-5). The incident radiation at different windows at different times are given as (Table 11-4)

| Month | Time | Solar radiation incident on the surface, W/m$^2$ | | | |
|---|---|---|---|---|---|
| | | North | East | South | West |
| July | 9:00 | 117 | 701 | 190 | 114 |
| July | 12:00 | 138 | 149 | 395 | 149 |
| July | 15:00 | 117 | 114 | 190 | 701 |
| January | Daily total | 446 | 1863 | 5897 | 1863 |

***Analysis*** The solar heat gain coefficient (SHGC) of the windows is determined from Eq.11-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times 0.58 = 0.5046$$

The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC\times A_{\text{glazing}}\times \dot{q}_{\text{solar,incident}} = 0.5046\times A_{\text{glazing}}\times \dot{q}_{\text{solar,incident}}$$

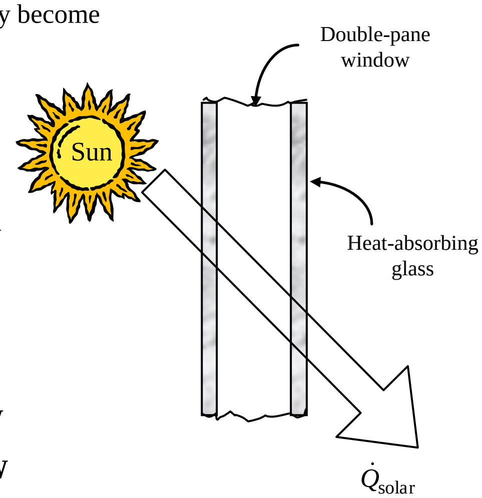
Then the rates of heat gain at the 4 walls at 3 different times in July become

*North wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{236\text{ W}}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046\times(4\text{ m}^2)\times(138\text{ W/m}^2) = \mathbf{279\text{ W}}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{236\text{ W}}$$




*East wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{2122\text{ W}}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{461\text{ W}}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{345\text{ W}}$$

*South wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{7674\text{ W}}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046\times(8\text{ m}^2)\times(395\text{ W/m}^2) = \mathbf{1595\text{ W}}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{767\text{ W}}$$

*West wall*:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{345\text{ W}}$$

$$\dot{Q}_{\text{solar gain,12:00}} = 0.5046\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{451\text{ W}}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{2122\text{ W}}$$

Similarly, the solar heat gain of the house through all of the windows in January is determined to be

*January*:

$$\dot{Q}_{\text{solar gain, North}} = 0.5046\times(4\text{ m}^2)\times(446\text{ Wh/m}^2\cdot\text{day}) = 900\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain,East}} = 0.5046\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 5640\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain, South}} = 0.5046\times(8\text{ m}^2)\times(5897\text{ Wh/m}^2\cdot\text{day}) = 23{,}805\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain,West}} = 0.5046\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 5640\text{ Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 900+5640+23{,}805+5640 = 35{,}985\text{ Wh/day} = \mathbf{35.895\text{ kWh/day}}$$

**11-74** A building at 40º N latitude has double pane heat absorbing type windows that are equipped with light colored venetian blinds. The total solar heat gains of the building through the south windows at solar noon in April for the cases of with and without the blinds are to be determined.

***Assumptions*** The calculations are performed for an "average" day in April, and may vary from location to location.

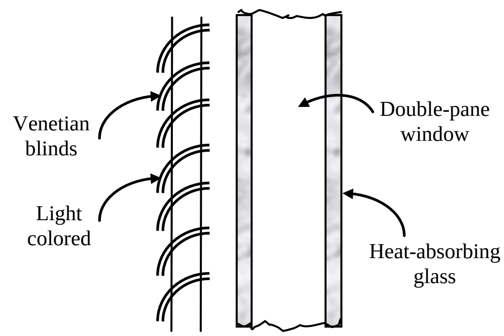
***Properties*** The shading coefficient of a double pane heat absorbing type windows is SC = 0.58 (Table 11-5). It is given to be SC = 0.30 in the case of blinds. The solar radiation incident at a South-facing surface at 12:00 noon in April is 559 W/m$^2$ (Table 11-4).

***Analysis*** The solar heat gain coefficient (SHGC) of the windows without the blinds is determined from Eq.11-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times 0.58 = 0.5046$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, no blinds}} &= SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar,incident}}\\ &= 0.5046(200\,\text{m}^2)(559\,\text{W/m}^2)\\ &= \mathbf{56{,}414\,W}\end{aligned}$$




In the case of windows equipped with venetian blinds, the SHGC and the rate of solar heat gain become

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times 0.30 = 0.261$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, no blinds}} &= SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar,incident}}\\ &= 0.261(200\,\text{m}^2)(559\,\text{W/m}^2)\\ &= \mathbf{29{,}180\,W}\end{aligned}$$

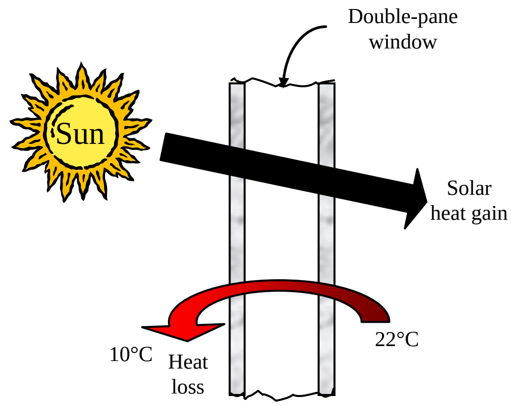
***Discussion*** Note that light colored venetian blinds significantly reduce the solar heat, and thus air-conditioning load in summers.

**11-75** A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through an east window in a typical day in January.

***Assumptions*** **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

***Properties*** The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 11-5). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 $W/m^2.°C$. (Table 9-6). The total solar radiation incident at an East-facing surface in January during a typical day is 1863 $Wh/m^2$ (Table 11-4).

***Analysis*** The solar heat gain coefficient (SHGC) of the windows is determined from Eq.11-57 to be




$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.88 = 0.7656$$

Then the solar heat gain through the window per unit area becomes

$$\begin{aligned} Q_{\text{solar gain}} &= SHGC\times A_{\text{glazing}}\times q_{\text{solar, daily total}} \\ &= 0.7656(1\,\text{m}^2)(1863\,\text{Wh/m}^2) \\ &= \mathbf{1426\,Wh = 1.426\,kWh} \end{aligned}$$

The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned} Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\Delta t = U_{\text{window}}A_{\text{window}}(T_i - T_{0,\text{ave}})(1\,\text{day}) \\ &= (4.55\,\text{W/m}^2\cdot°\text{C})(1\,\text{m}^2)(22-10)°\text{C}(24\,\text{h}) \\ &= \mathbf{1310\,Wh = 1.31\,kWh} \end{aligned}$$

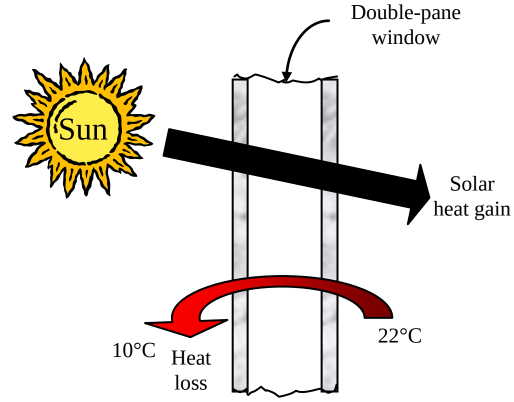
Therefore, the house is **gaining** more heat than it is loosing through the East windows during a typical day in January.

**11-76** A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through a South window in a typical day in January.

***Assumptions*** **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

***Properties*** The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 11-5). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 W/m$^2$.°C (Table 9-6). The total solar radiation incident at a South-facing surface in January during a typical day is 5897 Wh/m$^2$ (Table 11-5).

***Analysis*** The solar heat gain coefficient (SHGC) of the windows is determined from Eq.11-57 to be




$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.88 = 0.7656$$

Then the solar heat gain through the window per unit area becomes

$$\begin{aligned} Q_{\text{solar gain}} &= SHGC\times A_{\text{glazing}}\times q_{\text{solar,daily total}} \\ &= 0.7656(1\,\text{m}^2)(5897\,\text{Wh/m}^2) \\ &= \mathbf{4515\,Wh = 4.515\ kWh} \end{aligned}$$

The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned} Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\Delta t = U_{\text{window}}A_{\text{window}}(T_i - T_{0,\text{ave}})(1\,\text{day}) \\ &= (4.55\,\text{W/m}^2\cdot{}^\circ\text{C})(1\,\text{m}^2)(22-10)^\circ\text{C}(24\,\text{h}) \\ &= \mathbf{1310\,Wh = 1.31\,kWh} \end{aligned}$$

Therefore, the house is **gaining** much more heat than it is loosing through the South windows during a typical day in January.

**11-77E** A house has 1/8-in thick single pane windows with aluminum frames on a West wall. The rate of net heat gain (or loss) through the window at 3 PM during a typical day in January is to be determined.

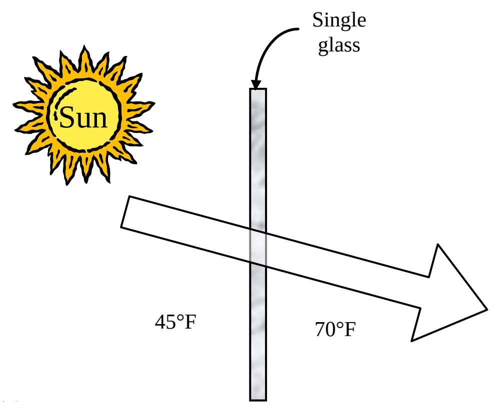
***Assumptions*** **1** The calculations are performed for an "average" day in January. **2** The frame area relative to glazing area is small so that the glazing area can be taken to be the same as the window area.

***Properties*** The shading coefficient of a 1/8-in thick single pane window is SC = 1.0 (Table 11-5). The overall heat transfer coefficient for 1/8-in thick single pane windows with aluminum frames is 6.63 $W/m^2.°C$ = 1.17 $Btu/h.ft^2.°F$ (Table 9-6). The total solar radiation incident at a West-facing surface at 3 PM in January during a typical day is 557 $W/m^2$ = 177 $Btu/h.ft^2$ (Table 11-4).

***Analysis*** The solar heat gain coefficient (SHGC) of the windows is determined from Eq.11-57 to be

$$SHGC = 0.87 \times SC = 0.87 \times 1.0 = 0.87$$

The window area is: $A_{window} = (9\text{ ft})(15\text{ft}) = 135\text{ ft}^2$




Then the rate of solar heat gain through the window at 3 PM becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, 3 PM}} &= SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, 3 PM}} \\ &= 0.87(135\text{ ft}^2)(177\text{ Btu/h.ft}^2) \\ &= 20{,}789\text{ Btu/h}\end{aligned}$$

The rate of heat loss through the window at 3 PM is

$$\begin{aligned}\dot{Q}_{\text{loss, window}} &= U_{\text{window}} A_{\text{window}} (T_i - T_0) \\ &= (1.17\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(135\text{ ft}^2)(70-45)°\text{F} \\ &= 3949\text{ Btu/h}\end{aligned}$$

The house will be gaining heat at 3 PM since the solar heat gain is larger than the heat loss. The rate of net heat gain through the window is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{solar gain, 3 PM}} - \dot{Q}_{\text{loss, window}} = 20{,}789 - 394 = \mathbf{16{,}840\ Btu/h}$$

***Discussion*** The actual heat gain will be less because of the area occupied by the window frame.

**11-78** A building located near 40º N latitude has equal window areas on all four sides. The side of the building with the highest solar heat gain in summer is to be determined.

***Assumptions*** The shading coefficients of windows on all sides of the building are identical.

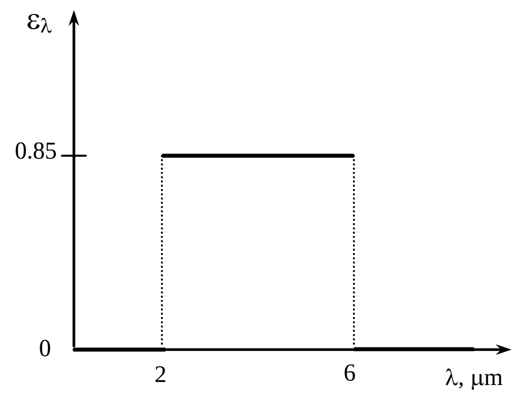
***Analysis*** The reflective films should be installed on the side that receives the most incident solar radiation in summer since the window areas and the shading coefficients on all four sides are identical. The incident solar radiation at different windows in July are given to be (Table 11-5)

| Month | Time | The daily total solar radiation incident on the surface, $Wh/m^2$ | | | |
|---|---|---|---|---|---|
| | | North | East | South | West |
| July | Daily total | 1621 | 4313 | 2552 | 4313 |

Therefore, the reflective film should be installed on the **East** or **West** windows (instead of the South windows) in order to minimize the solar heat gain and thus the cooling load of the building.

## Review Problems

**11-79** The variation of emissivity of an opaque surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.




***Analysis*** The average emissivity of the surface can be determined from

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where $f_{\lambda_1}$ and $f_{\lambda_2}$ are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 11-1 to be

$$\lambda_1 T = (2\ \mu\text{m})(1200\ \text{K}) = 2400\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.140256$$

$$\lambda_2 T = (6\ \mu\text{m})(1200\ \text{K}) = 7200\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.819217$$

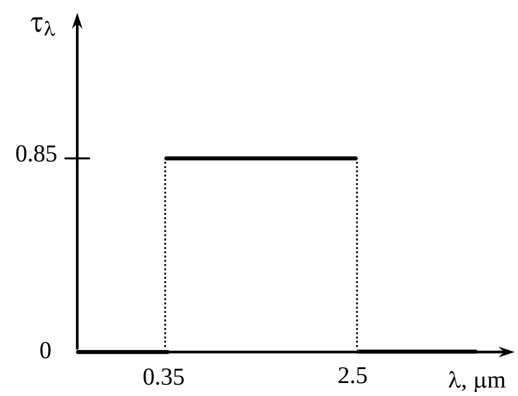
and

$$\varepsilon = (0.0)(0.140256) + (0.85)(0.819217 - 0.140256) + (0.0)(1 - 0.819217) = \mathbf{0.577}$$

Then the emissive flux of the surface becomes

$$E = \varepsilon\sigma T^4 = (0.577)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\,\text{K})^4 = \mathbf{67{,}853\ \ W/m^2}$$

**11-80** The variation of transmissivity of glass with wavelength is given. The transmissivity of the glass for solar radiation and for light are to be determined.




***Analysis*** For solar radiation, $T$ = 5800 K. The average transmissivity of the surface can be determined from

$$\tau(T) = \tau_1 f_{\lambda_1} + \tau_2 (f_{\lambda_2} - f_{\lambda_1}) + \tau_3 (1 - f_{\lambda_2})$$

where $f_{\lambda_1}$ and $f_{\lambda_2}$ are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 11-1 to be

$$\lambda_1 T = (0.35\ \mu\text{m})(5800\text{K}) = 2030\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.071852$$

$$\lambda_2 T = (2.5\ \mu\text{m})(5800\text{K}) = 14{,}500\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.966440$$

and

$$\tau = (0.0)(0.071852) + (0.85)(0.966440 - 0.071852) + (0.0)(1 - 0.966440) = \mathbf{0.760}$$

For light, we take $T$ = 300 K. Repeating the calculations at this temperature we obtain

$$\lambda_1 T = (0.35\ \mu\text{m})(300\text{K}) = 105\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.00$$

$$\lambda_2 T = (2.5\ \mu\text{m})(300\text{K}) = 750\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000012$$

$$\tau = (0.0)(0.00) + (0.85)(0.000012 - 0.00) + (0.0)(1 - 0.000012) = \mathbf{0.00001}$$

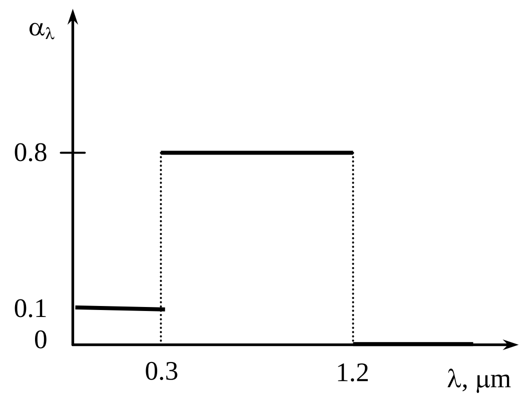
**11-81** A hole is drilled in a spherical cavity. The maximum rate of radiation energy streaming through the hole is to be determined.

***Analysis*** The maximum rate of radiation energy streaming through the hole is the blackbody radiation, and it can be determined from

$$E = A\sigma T^4 = \pi(0.0025\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(600\,\text{K})^4 = \mathbf{0.144\ W}$$

The result would not change for a different diameter of the cavity.

**11-82** The variation of absorptivity of a surface with wavelength is given. The average absorptivity of the surface is to be determined for two source temperatures.




***Analysis*** (*a*) <u>*T* = 1000 K</u>. The average absorptivity of the surface can be determined from

$$\alpha(T) = \alpha_1 f_{0\text{-}\lambda_1} + \alpha_2 f_{\lambda_1\text{-}\lambda_2} + \alpha_3 f_{\lambda_2\text{-}\infty}$$
$$= \alpha_1 f_{\lambda_1} + \alpha_2(f_{\lambda_2} - f_{\lambda_1}) + \alpha_3(1 - f_{\lambda_2})$$

where $f_{\lambda_1}$ and $f_{\lambda_2}$ are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (0.3\ \mu\text{m})(1000\text{K}) = 300\ \mu\text{mK} \longrightarrow \text{f}_{\lambda_1} = 0.0$$
$$\lambda_2 T = (1.2\ \mu\text{m})(1000\text{K}) = 1200\ \mu\text{mK} \longrightarrow \text{f}_{\lambda_2} = 0.002134$$

$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1}$ since $f_0 = 0$ and $\text{f}_{\lambda_2-\infty} = f_\infty - f_{\lambda_2}$ since $f_\infty = 1$.

and,

$$\alpha = (0.1)0.0 + (0.8)(0.002134 - 0.0) + (0.0)(1 - 0.002134) = \mathbf{0.0017}$$

(*a*) <u>*T* = 3000 K</u>.

$$\lambda_1 T = (0.3\ \mu\text{m})(3000\text{K}) = 900\ \mu\text{mK} \longrightarrow \text{f}_{\lambda_1} = 0.000169$$
$$\lambda_2 T = (1.2\ \mu\text{m})(3000\text{K}) = 3600\ \mu\text{mK} \longrightarrow \text{f}_{\lambda_2} = 0.403607$$

$$\alpha = (0.1)0.000169 + (0.8)(0.403607 - 0.000169) + (0.0)(1 - 0.403607) = \mathbf{0.323}$$

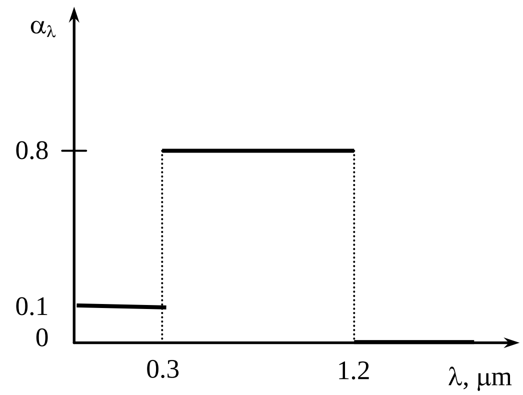
**11-83** The variation of absorptivity of a surface with wavelength is given. The surface receives solar radiation at a specified rate. The solar absorptivity of the surface and the rate of absorption of solar radiation are to be determined.

***Analysis*** For solar radiation, $T$ = 5800 K. The solar absorptivity of the surface is

$$\lambda_1 T = (0.3\ \mu\text{m})(5800\text{K}) = 1740\ \mu\text{mK} \rightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T = (1.2\ \mu\text{m})(5800\text{K}) = 6960\ \mu\text{mK} \rightarrow f_{\lambda_2} = 0.805713$$

$$\begin{aligned}\alpha &= (0.1)0.033454 + (0.8)(0.805713 - 0.033454) \\ &\quad + (0.0)(1 - 0.805713) \\ &= \mathbf{0.621}\end{aligned}$$




The rate of absorption of solar radiation is determined from

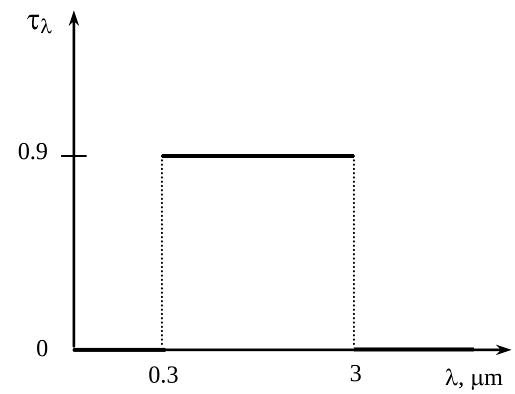
$$E_{\text{absorbed}} = \alpha I = 0.621(820\ \text{W/m}^2) = \mathbf{509\ W/m^2}$$

**11-84** The spectral transmissivity of a glass cover used in a solar collector is given. Solar radiation is incident on the collector. The solar flux incident on the absorber plate, the transmissivity of the glass cover for radiation emitted by the absorber plate, and the rate of heat transfer to the cooling water are to be determined.

***Analysis*** (*a*) For solar radiation, $T$ = 5800 K. The average transmissivity of the surface can be determined from

$$\tau(T) = \tau_1 f_{\lambda_1} + \tau_2 (f_{\lambda_2} - f_{\lambda_1}) + \tau_3 (1 - f_{\lambda_2})$$

where $f_{\lambda_1}$ and $f_{\lambda_2}$ are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 11-1 to be




$$\lambda_1 T = (0.3\ \mu\text{m})(5800\text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T = (3\ \mu\text{m})(5800\text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.978746$$

and

$$\tau = (0.0)(0.033454) + (0.9)(0.978746 - 0.033454) + (0.0)(1 - 0.978746) = 0.851$$

Since the absorber plate is black, all of the radiation transmitted through the glass cover will be absorbed by the absorber plate and therefore, the solar flux incident on the absorber plate is same as the radiation absorbed by the absorber plate:

$$E_{\text{abs. plate}} = \tau I = 0.851(950\ \text{W/m}^2) = \mathbf{808.5\ W/m^2}$$

(*b*) For radiation emitted by the absorber plate, we take $T$ = 300 K, and calculate the transmissivity as follows:

$$\lambda_1 T = (0.3\ \mu\text{m})(300\text{K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

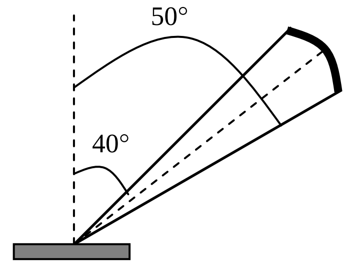
$$\lambda_2 T = (3\ \mu\text{m})(300\text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000169$$

$$\tau = (0.0)(0.0) + (0.9)(0.000169 - 0.0) + (0.0)(1 - 0.000169) = \mathbf{0.00015}$$

(*c*) The rate of heat transfer to the cooling water is the difference between the radiation absorbed by the absorber plate and the radiation emitted by the absorber plate, and it is determined from

$$\dot{Q}_{\text{water}} = (\tau_{\text{solar}} - \tau_{\text{room}})I = (0.851 - 0.00015)(950\ \text{W/m}^2) = \mathbf{808.3\ W/m^2}$$

**11-85** A small surface emits radiation. The rate of radiation energy emitted through a band is to be determined.




***Assumptions*** Surface *A* emits diffusely as a blackbody.

***Analysis*** The rate of radiation emission from a surface per unit surface area in the direction ($\theta,\phi$) is given as

$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

The total rate of radiation emission through the band between 40° and 50° can be expressed as

$$E = \int_{\phi=0}^{2\pi}\int_{\theta=40}^{50} I_e(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_b(0.1736\pi) = \frac{\sigma T^4}{\pi}(0.1736\pi) = 0.1736\sigma T^4$$

since the blackbody radiation intensity is constant ($I_b$ = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=40}^{50}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=40}^{50}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 50 - \sin^2 40) = 0.1736\pi$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 1 cm² in the specified band becomes

$$\dot{Q}_e = EdA = 0.1736\sigma T^4 dA = 0.1736\times(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(600\,\text{K})^4(1\times10^{-4}\ \text{m}^2) = \mathbf{0.128\ W}$$

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**11-86 ….. 11-87 Design and Essay Problems**

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