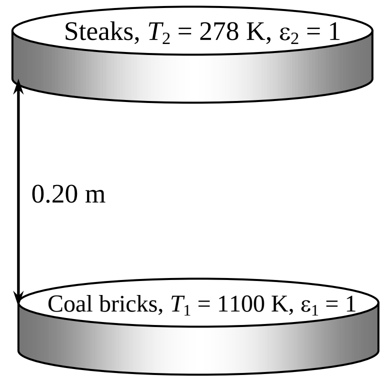
12-41 A circular grill is considered. The bottom of the grill is covered with hot coal bricks, while the wire mesh on top of the grill is covered with steaks. The initial rate of radiation heat transfer from coal bricks to the steaks is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are $\varepsilon = 1$ for all surfaces since they are black or reradiating.



Analysis We consider the coal bricks to be surface 1, the steaks to be surface 2 and the side surfaces to be surface 3. First we determine the view factor between the bricks and the steaks (Table 12-1),

$$R_i = R_j = \frac{r_i}{L} = \frac{0.15\,\text{m}}{0.20\,\text{m}} = 0.75$$

$$S = 1 + \frac{1 + R_j^{\,2}}{R_i^{\,2}} = \frac{1 + 0.75^2}{0.75^2} = 3.7778$$

$$F_{12} = F_{ij} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{R_j}{R_i}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{3.7778 - \left[3.7778^2 - 4\left(\frac{0.75}{0.75}\right)^2\right]^{1/2}\right\} = 0.2864$$

(It can also be determined from Fig. 12-7).

Then the initial rate of radiation heat transfer from the coal bricks to the stakes becomes

$$\begin{aligned}\dot{Q}_{12} &= F_{12} A_1 \sigma (T_1^{\,4} - T_2^{\,4}) \\ &= (0.2864)[\pi(0.3\,\text{m})^2/4](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1100\,\text{K})^4 - (278\,\text{K})^4] \\ &= \mathbf{1674\ W}\end{aligned}$$

When the side opening is closed with aluminum foil, the entire heat lost by the coal bricks must be gained by the stakes since there will be no heat transfer through a reradiating surface. The grill can be considered to be three-surface enclosure. Then the rate of heat loss from the room can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}}$$

where

$$E_{b1} = \sigma T_1^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$
$$E_{b2} = \sigma T_2^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(18+273\ \text{K})^4 = 407\ \text{W/m}^2$$

and

$$A_1 = A_2 = \frac{\pi(0.3\ \text{m})^2}{4} = 0.07069\ \text{m}^2$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(0.07069\,\text{m}^2)(0.2864)} = 49.39\,\text{m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(0.07069\,\text{m}^2)(1 - 0.2864)} = 19.82\,\text{m}^{-2}$$

Substituting,

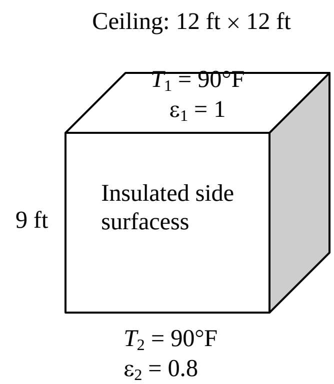
$$\dot{Q}_{12} = \frac{(83{,}015 - 407)\ \text{W/m}^2}{\left(\dfrac{1}{49.39\,\text{m}^{-2}} + \dfrac{1}{2(19.82\,\text{m}^{-2})}\right)^{-1}} = \mathbf{3757\ \ W}$$

12-42E A room is heated by lectric resistance heaters placed on the ceiling which is maintained at a uniform temperature. The rate of heat loss from the room through the floor is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** There is no heat loss through the side surfaces.

Properties The emissivities are ε = 1 for the ceiling and ε = 0.8 for the floor. The emissivity of insulated (or reradiating) surfaces is also 1.

Analysis The room can be considered to be three-surface enclosure with the ceiling surface 1, the floor surface 2 and the side surfaces surface 3. We assume steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. Then the rate of heat loss from the room through its floor can be determined from



$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1} + R_2}$$

where

$$E_{b1} = \sigma T_1^{\,4} = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(90+460\,\text{R})^4 = 157\,\text{Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^{\,4} = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(65+460\,\text{R})^4 = 130\,\text{Btu/h.ft}^2$$

and

$$A_1 = A_2 = (12\text{ ft})^2 = 144\text{ ft}^2$$

The view factor from the floor to the ceiling of the room is $F_{12} = 0.27$ (From Figure 12-42). The view factor from the ceiling or the floor to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.27 = 0.73$$

since the ceiling is flat and thus $F_{11} = 0$. Then the radiation resistances which appear in the equation above become

$$R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2} = \frac{1-0.8}{(144\,\text{ft}^2)(0.8)} = 0.00174\text{ft}^{-2}$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(144\,\text{ft}^2)(0.27)} = 0.02572\text{ft}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(144\,\text{ft}^2)(0.73)} = 0.009513\text{ft}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(157-130)\,\text{Btu/h.ft}^2}{\left(\dfrac{1}{0.02572\text{ft}^{-2}} + \dfrac{1}{2(0.009513\text{ft}^{-2})}\right)^{-1} + 0.00174\text{ft}^{-2}} = \mathbf{2130\ Btu/h}$$

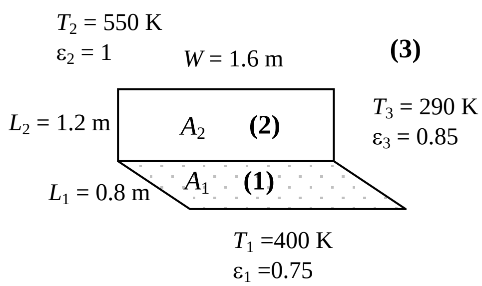
12-43 Two perpendicular rectangular surfaces with a common edge are maintained at specified temperatures. The net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the horizontal rectangle and the surroundings are ε = 0.75 and ε = 0.85, respectively.

Analysis We consider the horizontal rectangle to be surface 1, the vertical rectangle to be surface 2 and the surroundings to be surface 3. This system can be considered to be a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from

$$\left.\begin{aligned}\frac{L_1}{W}=\frac{0.8}{1.6}=0.5\\ \frac{L_2}{W}=\frac{1.2}{1.6}=0.75\end{aligned}\right\}F_{12}=0.27 \qquad \text{(Fig. 12-6)}$$



The surface areas are

$$A_1=(0.8\,\text{m})(1.6\,\text{m})=1.28\,\text{m}^2$$

$$A_2=(1.2\,\text{m})(1.6\,\text{m})=1.92\,\text{m}^2$$

$$A_3=2\times\frac{1.2\times0.8}{2}+\sqrt{0.8^2+1.2^2}\times1.6=3.268\,\text{m}^2$$

Note that the surface area of the surroundings is determined assuming that surroundings forms flat surfaces at all openings to form an enclosure. Then other view factors are determined to be

$$A_1F_{12}=A_2F_{21}\longrightarrow(1.28)(0.27)=(1.92)F_{21}\longrightarrow F_{21}=0.18 \qquad \text{(reciprocity rule)}$$

$$F_{11}+F_{12}+F_{13}=1\longrightarrow 0+0.27+F_{13}=1\longrightarrow F_{13}=0.73 \qquad \text{(summation rule)}$$

$$F_{21}+F_{22}+F_{23}=1\longrightarrow 0.18+0+F_{23}=1\longrightarrow F_{23}=0.82 \qquad \text{(summation rule)}$$

$$A_1F_{13}=A_3F_{31}\longrightarrow(1.28)(0.73)=(3.268)F_{31}\longrightarrow F_{31}=0.29 \qquad \text{(reciprocity rule)}$$

$$A_2F_{23}=A_3F_{32}\longrightarrow(1.92)(0.82)=(3.268)F_{32}\longrightarrow F_{32}=0.48 \qquad \text{(reciprocity rule)}$$

We now apply Eq. 9-52b to each surface to determine the radiosities.

Surface 1:

$$\sigma T_1^4=J_1+\frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1-J_2)+F_{13}(J_1-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(400\,\text{K})^4=J_1+\frac{1-0.75}{0.75}\left[0.27(J_1-J_2)+0.73(J_1-J_3)\right]$$

Surface 2:

$$\sigma T_2^4=J_2\longrightarrow(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(550\,\text{K})^4=J_2$$

Surface 3:

$$\sigma T_3^4=J_3+\frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3-J_1)+F_{32}(J_3-J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(290\,\text{K})^4=J_3+\frac{1-0.85}{0.85}\left[0.29(J_1-J_2)+0.48(J_1-J_3)\right]$$

Solving the above equations, we find

$$J_1=1587\,\text{W/m}^2,\quad J_2=5188\,\text{W/m}^2,\quad J_3=811.5\,\text{W/m}^2$$

Then the net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are determined to be

$$\dot{Q}_{21}=-\dot{Q}_{12}=-A_1F_{12}(J_1-J_2)=-(1.28\,\text{m}^2)(0.27)(1587-5188)\text{W/m}^2=\mathbf{1245\,W}$$

$$\dot{Q}_{13}=A_1F_{13}(J_1-J_3)=(1.28\,\text{m}^2)(0.73)(1587-811.5)\text{W/m}^2=\mathbf{725\,W}$$

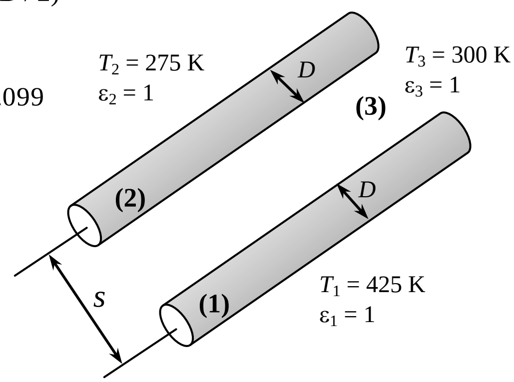
12-44 Two long parallel cylinders are maintained at specified temperatures. The rates of radiation heat transfer between the cylinders and between the hot cylinder and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis We consider the hot cylinder to be surface 1, cold cylinder to be surface 2, and the surroundings to be surface 3. Using the crossed-strings method, the view factor between two cylinders facing each other is determined to be

$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{String on surface 1}} = \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$

or $$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D} = \frac{2\left(\sqrt{0.5^2 + 0.16^2} - 0.5\right)}{\pi(0.16)} = 0.099$$



The view factor between the hot cylinder and the surroundings is

$$F_{13} = 1 - F_{12} = 1 - 0.099 = 0.901 \text{ (summation rule)}$$

The rate of radiation heat transfer between the cylinders per meter length is

$$A = \pi DL/2 = \pi(0.16\text{ m})(1\text{ m})/2 = 0.2513\text{m}^2$$

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^{\,4} - T_2^{\,4}) = (0.2513\text{m}^2)(0.099)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 275^4)\text{K}^4 = \mathbf{38.0\ W}$$

Note that half of the surface area of the cylinder is used, which is the only area that faces the other cylinder. The rate of radiation heat transfer between the hot cylinder and the surroundings per meter length of the cylinder is

$$A_1 = \pi DL = \pi(0.16\text{ m})(1\text{ m}) = 0.5027\text{m}^2$$

$$\dot{Q}_{13} = A_1F_{13}\sigma(T_1^{\,4} - T_3^{\,4}) = (0.5027\text{m}^2)(0.901)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 300^4)\text{K}^4 = \mathbf{629.8\ W}$$

12-45 A long semi-cylindrical duct with specified temperature on the side surface is considered. The temperature of the base surface for a specified heat transfer rate is to be determined.

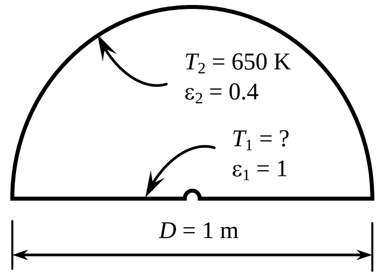
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the side surface is ε = 0.4.

Analysis We consider the base surface to be surface 1, the side surface to be surface 2. This system is a two-surface enclosure, and we consider a unit length of the duct. The surface areas and the view factor are determined as

$$A_1 = (1.0\text{ m})(1.0\text{ m}) = 1.0\text{ m}^2$$

$$A_2 = \pi DL/2 = \pi(1.0\text{ m})(1\text{ m})/2 = 1.571\text{m}^2$$



$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$

The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$1200\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot K^4)[T_1^4 - (650\text{ K})^4]}{\dfrac{1}{(1.0\text{ m}^2)(1)} + \dfrac{1-0.4}{(1.571\text{m}^2)(0.4)}} \longrightarrow T_1 = \mathbf{684.8\ K}$$

12-46 A hemisphere with specified base and dome temperatures and heat transfer rate is considered. The emissivity of the dome is to be determined.

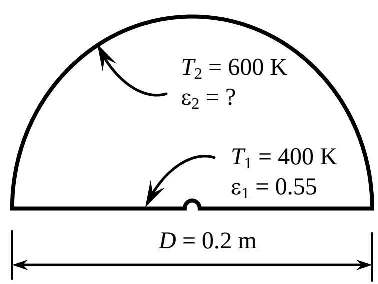
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the base surface is ε = 0.55.

Analysis We consider the base surface to be surface 1, the dome surface to be surface 2. This system is a two-surface enclosure. The surface areas and the view factor are determined as

$$A_1 = \pi D^2/4 = \pi(0.2\text{ m})^2/4 = 0.0314\text{ m}^2$$

$$A_2 = \pi D^2/2 = \pi(0.2\text{ m})^2/2 = 0.0628\text{m}^2$$



$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$

The emissivity of the dome is determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$50\text{ W} = -\frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (600\text{ K})^4]}{\dfrac{1-0.55}{(0.0314\text{m}^2)(0.55)} + \dfrac{1}{(0.0314\text{m}^2)(1)} + \dfrac{1-\varepsilon_2}{(0.0628\text{m}^2)\varepsilon_2}} \longrightarrow \varepsilon_2 = \mathbf{0.21}$$

Radiation Shields and The Radiation Effect

12-47C Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high reflectivity(low emissivity) sheet of material between the two surfaces. Such highly reflective thin plates or shells are known as radiation shields. Multilayer radiation shields constructed of about 20 shields per cm. thickness separated by evacuated space are commonly used in cryogenic and space applications to minimize heat transfer. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect.

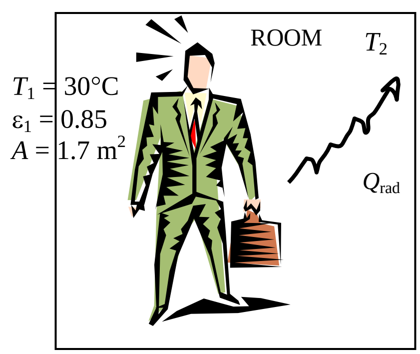
12-48C The influence of radiation on heat transfer or temperature of a surface is called the radiation effect. The radiation exchange between the sensor and the surroundings may cause the thermometer to indicate a different reading for the medium temperature. To minimize the radiation effect, the sensor should be coated with a material of high reflectivity (low emissivity).

12-49C A person who feels fine in a room at a specified temperature may feel chilly in another room at the same temperature as a result of radiation effect if the walls of second room are at a considerably lower temperature. For example most people feel comfortable in a room at 22°C if the walls of the room are also roughly at that temperature. When the wall temperature drops to 5°C for some reason, the interior temperature of the room must be raised to at least 27°C to maintain the same level of comfort. Also, people sitting near the windows of a room in winter will feel colder because of the radiation exchange between the person and the cold windows.

12-50 The rate of heat loss from a person by radiation in a large room whose walls are maintained at a uniform temperature is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the person is given to be ε_1 = 0.7.



Analysis (*a*) Noting that the view factor from the person to the walls $F_{12} = 1$, the rate of heat loss from that person to the walls at a large room which are at a temperature of 300 K is

$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^{\,4} - T_2^{\,4}) \\ &= (0.85)(1)(1.7\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\text{K})^4 - (300\text{K})^4] \\ &= \mathbf{26.9\ W}\end{aligned}$$

(*b*) When the walls are at a temperature of 280 K,

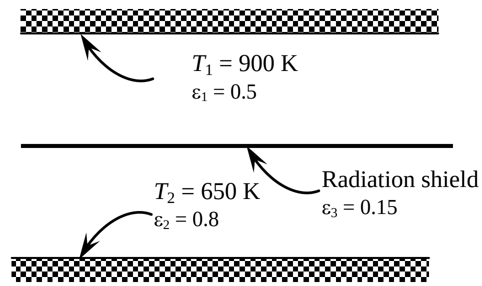
$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^{\,4} - T_2^{\,4}) \\ &= (0.85)(1)(1.7\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\text{K})^4 - (280\text{K})^4] \\ &= \mathbf{187\ W}\end{aligned}$$

12-51 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined for the cases of with and without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is



$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\,\text{K})^4 - (650\,\text{K})^4]}{\left(\frac{1}{0.5} + \frac{1}{0.8} - 1\right) + \left(\frac{1}{0.15} + \frac{1}{0.15} - 1\right)}$$

$$= \mathbf{1857\ W/m^2}$$

The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\,\text{shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\,\text{K})^4 - (650\,\text{K})^4]}{\left(\frac{1}{0.5} + \frac{1}{0.8} - 1\right)} = 12{,}035\,\text{W/m}^2$$

Then the ratio of radiation heat transfer for the two cases becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{1857\ \text{W}}{12{,}035\ \text{W}} \cong \mathbf{\frac{1}{6}}$$

12-52 "!PROBLEM 12-52"

```
"GIVEN"
"epsilon_3=0.15 parameter to be varied"
T_1=900 "[K]"
T_2=650 "[K]"
epsilon_1=0.5
epsilon_2=0.8
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-
1)+(1/epsilon_3+1/epsilon_3-1))
```

ε_3	$Q_{12,1\ shield}$ [W/m^2]
0.05	656.5
0.06	783
0.07	908.1
0.08	1032
0.09	1154
0.1	1274
0.11	1394
0.12	1511
0.13	1628
0.14	1743
0.15	1857
0.16	1969
0.17	2081
0.18	2191
0.19	2299
0.2	2407
0.21	2513
0.22	2619
0.23	2723
0.24	2826
0.25	2928

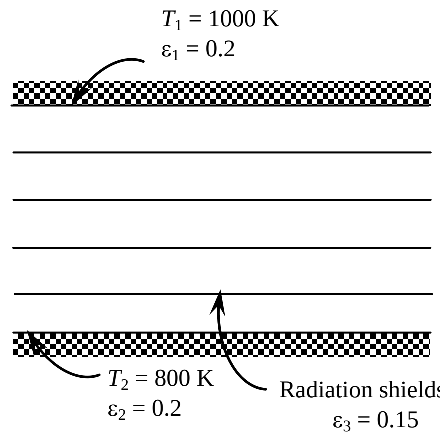
12-53 Two very large plates are maintained at uniform temperatures. The number of thin aluminum sheets that will reduce the net rate of radiation heat transfer between the two plates to one-fifth is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\,\text{shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\,\text{K})^4 - (800\,\text{K})^4]}{\left(\dfrac{1}{0.2} + \dfrac{1}{0.2} - 1\right)} = 3720\,\text{W/m}^2$$



The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to onefifth can be determined from

$$\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

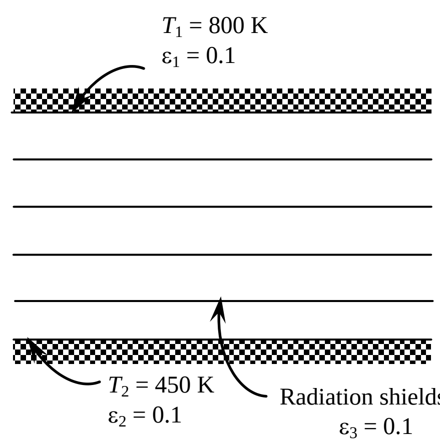
$$\frac{1}{5}(3720\,\text{W/m}^2) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\,\text{K})^4 - (800\,\text{K})^4]}{\left(\dfrac{1}{0.2} + \dfrac{1}{0.2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{0.15} + \dfrac{1}{0.15} - 1\right)} \longrightarrow N_{\text{shield}} = 2.92 \cong \mathbf{3}$$

12-54 Five identical thin aluminum sheets are placed between two very large parallel plates which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.1$ and $\varepsilon_3 = 0.1$.

Analysis Since the plates and the sheets have the same emissivity value, the net rate of radiation heat transfer with 5 thin aluminum shield can be determined from



$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\text{ no shield}} = \frac{1}{N+1}\frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)} = \frac{1}{5+1}\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\,\text{K})^4 - (450\,\text{K})^4]}{\left(\dfrac{1}{0.1} + \dfrac{1}{0.1} - 1\right)} = \mathbf{183\ W/m^2}$$

The net rate of radiation heat transfer without the shield is

$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\text{ no shield}} \longrightarrow \dot{Q}_{12,\text{ no shield}} = (N+1)\dot{Q}_{12,5\,\text{shield}} = 6\times183\,\text{W} = \mathbf{1098\ W}$$

12-55 "!PROBLEM 12-55"

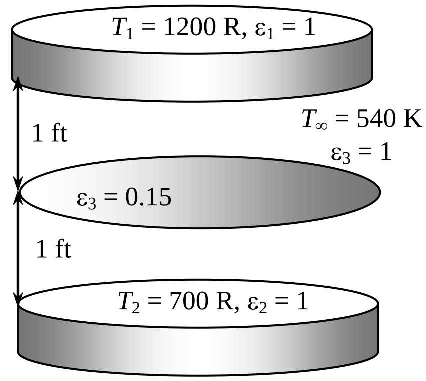
```
"GIVEN"
N=5 "parameter to be varied"
epsilon_3=0.1
"epsilon_1=0.1 parameter to be varied"
epsilon_2=epsilon_1
T_1=800 "[K]"
T_2=450 "[K]"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
Q_dot_12_shields=1/(N+1)*Q_dot_12_NoShield
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

N	$Q_{12,shields}$ [W/m^2]
1	550
2	366.7
3	275
4	220
5	183.3
6	157.1
7	137.5
8	122.2
9	110
10	100

ε_1	$Q_{12,shields}$ [W/m^2]
0.1	183.3
0.15	282.4
0.2	387
0.25	497.6
0.3	614.7
0.35	738.9
0.4	870.8
0.45	1011
0.5	1161
0.55	1321
0.6	1493
0.65	1677
0.7	1876
0.75	2090
0.8	2322
0.85	2575
0.9	2850

12-56E A radiation shield is placed between two parallel disks which are maintained at uniform temperatures. The net rate of radiation heat transfer through the shields is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 1$ and $\varepsilon_3 = 0.15$.

Analysis From Fig. 12-44 we have $F_{32} = F_{13} = 0.52$. Then $F_{34} = 1 - 0.52 = 0.48$. The disk in the middle is surrounded by black surfaces on both sides. Therefore, heat transfer between the top surface of the middle disk and its black surroundings can expressed as

$$\begin{aligned}\dot{Q}_3 &= \varepsilon A_3 \sigma[F_{31}(T_3^4 - T_1^4)] + \varepsilon A_3 \sigma[F_{32}(T_3^4 - T_2^4)] \\ &= 0.15(7.069\,\text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.52[(T_3^4 - (1200\,\text{R})^4] + 0.48[T_3^4 - (540\,\text{K})^4]\}\end{aligned}$$

Similarly, for the bottom surface of the middle disk, we have

$$\begin{aligned}-\dot{Q}_3 &= \varepsilon A_3 \sigma[F_{34}(T_3^4 - T_4^4)] + \varepsilon A_3 \sigma[F_{35}(T_3^4 - T_5^4)] \\ &= 0.15(7.069\,\text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.48[(T_3^4 - (700\,\text{R})^4] + 0.52[T_3^4 - (540\,\text{K})^4]\}\end{aligned}$$

Combining the equations above, the rate of heat transfer between the disks through the radiation shield (the middle disk) is determined to be

$$\dot{Q} = \mathbf{866\ Btu/h} \quad \text{and} \quad T_3 = 895\ \text{K}$$

12-57 A radiation shield is placed between two large parallel plates which are maintained at uniform temperatures. The emissivity of the radiation shield is to be determined if the radiation heat transfer between the plates is reduced to 15% of that without the radiation shield.

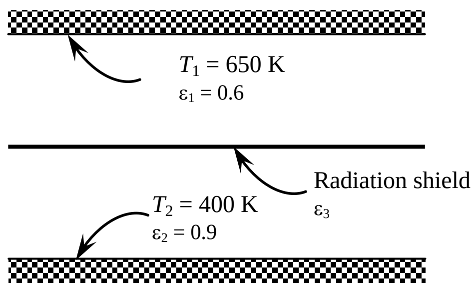
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.9$.

Analysis First, the net rate of radiation heat transfer between the two large parallel plates per unit area without a shield is

$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1}+\dfrac{1}{\varepsilon_2}-1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\,\text{K})^4-(400\,\text{K})^4]}{\dfrac{1}{0.6}+\dfrac{1}{0.9}-1} = 4877\,\text{W/m}^2$$

The radiation heat transfer in the case of one shield is



$$\dot{Q}_{12,\text{one shield}} = 0.15\times\dot{Q}_{12,\text{no shield}}$$
$$= 0.15\times 4877\,\text{W/m}^2 = 731.6\ \text{W/m}^2$$

Then the emissivity of the radiation shield becomes

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1}+\dfrac{1}{\varepsilon_2}-1\right)+\left(\dfrac{1}{\varepsilon_{3,1}}+\dfrac{1}{\varepsilon_{3,2}}-1\right)}$$

$$731.6\,\text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\,\text{K})^4-(400\,\text{K})^4]}{\left(\dfrac{1}{0.6}+\dfrac{1}{0.9}-1\right)+\left(\dfrac{2}{\varepsilon_3}-1\right)}$$

which gives $\varepsilon_3 = \mathbf{0.18}$

12-58 "!PROBLEM 12-58"

```
"GIVEN"
T_1=650 "[K]"
T_2=400 "[K]"
epsilon_1=0.6
epsilon_2=0.9
"PercentReduction=85 [%], parameter to be varied"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-
1)+(1/epsilon_3+1/epsilon_3-1))
Q_dot_12_1shield=(1-PercentReduction/100)*Q_dot_12_NoShield
```

Percent Reduction [%]	ε_3
40	0.9153
45	0.8148
50	0.72
55	0.6304
60	0.5455
65	0.4649
70	0.3885
75	0.3158
80	0.2466
85	0.1806
90	0.1176
95	0.05751

12-59 A coaxial radiation shield is placed between two coaxial cylinders which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.4$. and $\varepsilon_3 = 0.2$.

Analysis The surface areas of the cylinders and the shield per unit length are

$$A_{\text{pipe,inner}} = A_1 = \pi D_1 L = \pi(0.2\text{ m})(1\text{ m}) = 0.628\text{ m}^2$$

$$A_{\text{pipe,outer}} = A_2 = \pi D_2 L = \pi(0.1\text{ m})(1\text{ m}) = 0.314\text{ m}^2$$

$$A_{\text{shield}} = A_3 = \pi D_3 L = \pi(0.3\text{ m})(1\text{ m}) = 0.942\text{ m}^2$$

The net rate of radiation heat transfer between the two cylinders with a shield per unit length is

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{13}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3F_{3,2}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(750\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1-0.7}{(0.314)(0.7)} + \dfrac{1}{(0.314)(1)} + 2\dfrac{1-0.2}{(0.628)(0.2)} + \dfrac{1}{(0.628)(1)} + \dfrac{1-0.4}{(0.942)(0.4)}}$$

$$= \mathbf{703\ W}$$

If there was no shield,

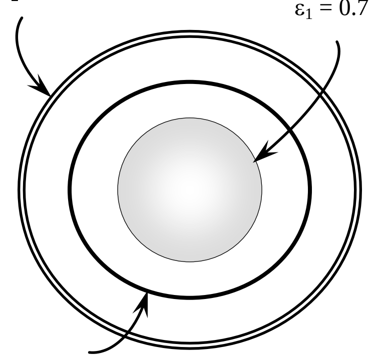
$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(750\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1}{0.7} + \dfrac{1-0.4}{0.4}\left(\dfrac{0.1}{0.3}\right)}$$

$$= \mathbf{7465\ W}$$

Then their ratio becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{703\text{ W}}{7465\text{ W}} = \mathbf{0.094}$$



12-60 "!PROBLEM 12-60"

```
"GIVEN"
D_1=0.10 "[m]"
D_2=0.30 "[m], parameter to be varied"
D_3=0.20 "[m]"
epsilon_1=0.7
epsilon_2=0.4
epsilon_3=0.2 "parameter to be varied"
T_1=750 "[K]"
T_2=500 "[K]"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
L=1 "[m], a unit length of the cylinders is considered"
A_1=pi*D_1*L
A_2=pi*D_2*L
A_3=pi*D_3*L
F_13=1
F_32=1
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1-
epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_13)+(1-epsilon_3)/(A_3*epsilon_3)+(1-
epsilon_3)/(A_3*epsilon_3)+1/(A_3*F_32)+(1-epsilon_2)/(A_2*epsilon_2))
```

D_2 [m]	$Q_{12,1\ shield}$ [W]
0.25	692.8
0.275	698.6
0.3	703.5
0.325	707.8
0.35	711.4
0.375	714.7
0.4	717.5
0.425	720
0.45	722.3
0.475	724.3
0.5	726.1

ε_3	$Q_{12,1\ shield}$ [W]
0.05	211.1
0.07	287.8
0.09	360.7
0.11	429.9
0.13	495.9
0.15	558.7
0.17	618.6
0.19	675.9
0.21	730.6
0.23	783
0.25	833.1
0.27	881.2
0.29	927.4
0.31	971.7
0.33	1014
0.35	1055

Radiation Exchange with Absorbing and Emitting Gases

12-61C A nonparticipating medium is completely transparent to thermal radiation, and thus it does not emit, absorb, or scatter radiation. A participating medium, on the other hand, emits and absorbs radiation throughout its entire volume.

12-62C Spectral transmissivity of a medium of thickness L is the ratio of the intensity of radiation leaving the medium to that entering the medium, and is expressed as $\tau_\lambda = \dfrac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L}$ and $\tau_\lambda = 1 - \alpha_\lambda$.

12-63C Using Kirchhoff's law, the spectral emissivity of a medium of thickness L in terms of the spectral absorption coefficient is expressed as $\varepsilon_\lambda = \alpha_\lambda = 1 - e^{-\kappa_\lambda L}$.

12-64C Gases emit and absorb radiation at a number of narrow wavelength bands. The emissivity-wavelength charts of gases typically involve various peaks and dips together with discontinuities, and show clearly the band nature of absorption and the strong nongray characteristics. This is in contrast to solids, which emit and absorb radiation over the entire spectrum.

12-65 An equimolar mixture of CO_2 and O_2 gases at 500 K and a total pressure of 0.5 atm is considered. The emissivity of the gas is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis Volumetric fractions are equal to pressure fractions. Therefore, the partial pressure of CO_2 is

$$P_c = y_{\mathrm{CO_2}} P = 0.5(0.5\,\mathrm{atm}) = 0.25\,\mathrm{atm}$$

Then,

$$P_c L = (0.25\,\mathrm{atm})(1.2\mathrm{m}) = 0.30\,\mathrm{m\cdot atm} = 0.98\,\mathrm{ft\cdot atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 500 K and 1 atm is, from Fig. 12-36,

$$\varepsilon_{c,1\,\mathrm{atm}} = 0.14$$

This is the base emissivity value at 1 atm, and it needs to be corrected for the 0.5 atm total pressure. The pressure correction factor is, from Fig. 12-37,

$$C_c = 0.90$$

Then the effective emissivity of the gas becomes

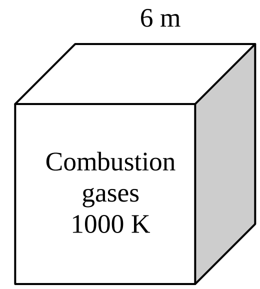
$$\varepsilon_g = C_c \varepsilon_{c,1\,\mathrm{atm}} = 0.90 \times 0.14 = \mathbf{0.126}$$

12-66 The temperature, pressure, and composition of a gas mixture is given. The emissivity of the mixture is to be determined.

Assumptions **1** All the gases in the mixture are ideal gases. **2** The emissivity determined is the mean emissivity for radiation emitted to all surfaces of the cubical enclosure.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$
$$P_w = y_{H_2O} P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for a cube of side length 6 m for radiation emitted to all surfaces is, from Table 12-4,

$$L = 0.66(6\text{ m}) = 3.96\text{ m}$$

Then,

$$P_c L = (0.10\,\text{atm})(3.96\text{m}) = 0.396\,\text{m}\cdot\text{atm} = 1.30\,\text{ft}\cdot\text{atm}$$
$$P_w L = (0.09\,\text{atm})(3.96\text{m}) = 0.48\,\text{m}\cdot\text{atm} = 1.57\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1000 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.17 \quad \text{and} \quad \varepsilon_{w,1\text{atm}} = 0.26$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1000 K is, from Fig. 12-38,

$$\left.\begin{aligned} P_c L + P_w L &= 1.30 + 1.57 = 2.87 \\ \frac{P_w}{P_w + P_c} &= \frac{0.09}{0.09 + 0.10} = 0.474 \end{aligned}\right\} \Delta\varepsilon = 0.039$$

Note that we obtained the average of the emissivity correction factors from the two figures for 800 K and 1200 K. Then the effective emissivity of the combustion gases becomes

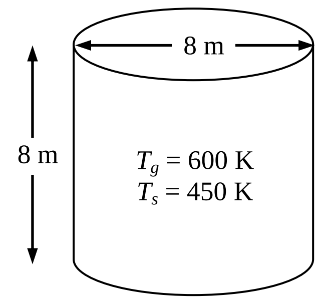
$$\varepsilon_g = C_c \varepsilon_{c,1\text{atm}} + C_w \varepsilon_{w,1\text{atm}} - \Delta\varepsilon = 1\times 0.17 + 1\times 0.26 - 0.039 = \mathbf{0.391}$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm.

12-67 A mixture of CO_2 and N_2 gases at 600 K and a total pressure of 1 atm are contained in a cylindrical container. The rate of radiation heat transfer between the gas and the container walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 12-4



$L = 0.60D = 0.60(8 \text{ m}) = 4.8 \text{ m}$

Then,

$$P_c L = (0.15 \text{ atm})(4.8 \text{m}) = 0.72 \text{ m} \cdot \text{atm} = 2.36 \text{ ft} \cdot \text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 600 K and 1 atm is, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.24$$

For a source temperature of T_s = 450 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.15 \text{ atm})(4.8 \text{m}) \frac{450 \text{ K}}{600 \text{ K}} = 0.54 \text{ m} \cdot \text{atm} = 1.77 \text{ ft} \cdot \text{atm}$$

The emissivity of CO_2 corresponding to this value at a temperature of T_s = 450 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.14$$

The absorptivity of CO_2 is determined from

$$\alpha_c = C_c \left(\frac{T_g}{T_s} \right)^{0.65} \varepsilon_{c,1\text{atm}} = (1) \left(\frac{600 \text{ K}}{450 \text{ K}} \right)^{0.65} (0.14) = 0.17$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2 \frac{\pi D^2}{4} = \pi(8 \text{ m})(8 \text{ m}) + 2 \frac{\pi(8 \text{ m})^2}{4} = 301.6 \text{ m}^2$$

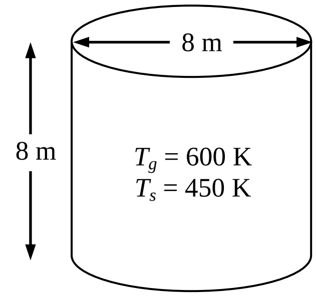
Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

$$\begin{aligned} \dot{Q}_{\text{net}} &= A_s \sigma (\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (301.6 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[0.14(600 \text{ K})^4 - 0.17(450 \text{ K})^4] \\ &= \mathbf{1.91 \times 10^5 \text{ W}} \end{aligned}$$

12-68 A mixture of H_2O and N_2 gases at 600 K and a total pressure of 1 atm are contained in a cylindrical container. The rate of radiation heat transfer between the gas and the container walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 12-4



$$L = 0.60D = 0.60(8 \text{ m}) = 4.8 \text{ m}$$

Then,

$$P_w L = (0.15\,\text{atm})(4.8\text{m}) = 0.72\,\text{m}\cdot\text{atm} = 2.36\,\text{ft}\cdot\text{atm}$$

The emissivity of H_2O corresponding to this value at the gas temperature of T_g = 600 K and 1 atm is, from Fig. 12-36,

$$\varepsilon_{w1\,\text{atm}} = 0.36$$

For a source temperature of T_s = 450 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_w L \frac{T_s}{T_g} = (0.15\,\text{atm})(4.8\text{m})\frac{450\,\text{K}}{600\,\text{K}} = 0.54\,\text{m}\cdot\text{atm} = 1.77\,\text{ft}\cdot\text{atm}$$

The emissivity of H_2O corresponding to this value at a temperature of T_s = 450 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{w,1\,\text{atm}} = 0.34$$

The absorptivity of H_2O is determined from

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{600\,\text{K}}{450\,\text{K}}\right)^{0.45} (0.34) = 0.39$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(8\,\text{m})(8\,\text{m}) + 2\frac{\pi(8\,\text{m})^2}{4} = 301.6\,\text{m}^2$$

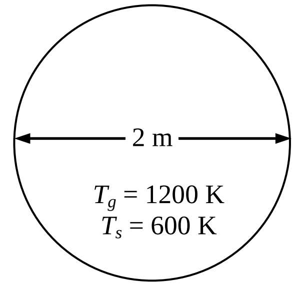
Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (301.6\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.36(600\,\text{K})^4 - 0.39(450\,\text{K})^4] \\ &= \mathbf{5.244\times10^5\ W}\end{aligned}$$

12-69 A mixture of CO_2 and N_2 gases at 1200 K and a total pressure of 1 atm are contained in a spherical furnace. The net rate of radiation heat transfer between the gas mixture and furnace walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 12-4



$$L = 0.65D = 0.65(2\text{ m}) = 1.3\text{ m}$$

The mole fraction is equal to pressure fraction. Then,

$$P_c L = (0.15\,\text{atm})(1.3\text{m}) = 0.195\,\text{m}\cdot\text{atm} = 0.64\,\text{ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 1200 K and 1 atm is, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.14$$

For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.15\,\text{atm})(1.3\text{m})\frac{600\,\text{K}}{1200\,\text{K}} = 0.0975\,\text{m}\cdot\text{atm} = 0.32\,\text{ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at a temperature of T_s = 600 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.092$$

The absorptivity of CO_2 is determined from

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{0.65}(0.092) = 0.144$$

The surface area of the sphere is

$$A_s = \pi D^2 = \pi(2\,\text{m})^2 = 12.57\,\text{m}^2$$

Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

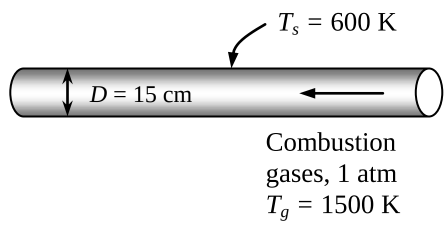
$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (12.57\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.14(1200\,\text{K})^4 - 0.144(600\,\text{K})^4]\\ &= \mathbf{1.936\times10^5\ W}\end{aligned}$$

12-70 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2}P = 0.06(1\,\text{atm}) = 0.06\,\text{atm}$$
$$P_w = y_{H_2O}P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for an infinite cicrcular cylinder is, from Table 12-4,

$$L = 0.95(0.15\text{ m}) = 0.1425\text{ m}$$

Then,

$$P_c L = (0.06\,\text{atm})(0.1425\,\text{m}) = 0.00855\,\text{m}\cdot\text{atm} = 0.028\,\text{ft}\cdot\text{atm}$$
$$P_w L = (0.09\,\text{atm})(0.1425\,\text{m}) = 0.0128\,\text{m}\cdot\text{atm} = 0.042\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1500 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.034 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.016$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1500 K is, from Fig. 12-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.028 + 0.042 = 0.07 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.09}{0.09 + 0.06} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c\varepsilon_{c,1\,\text{atm}} + C_w\varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.034 + 1\times 0.016 - 0.0 = 0.05$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L\frac{T_s}{T_g} = (0.06\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00342\,\text{m}\cdot\text{atm} = 0.011\,\text{ft}\cdot\text{atm}$$
$$P_w L\frac{T_s}{T_g} = (0.09\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00513\,\text{m}\cdot\text{atm} = 0.017\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.031 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.027$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.65}(0.031) = 0.056$$
$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45}\varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.45}(0.027) = 0.041$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 12-38 at $T = T_s = 600$ K instead of $T_g = 1500$ K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c) = 0.6$ and $P_cL + P_wL = 0.07$ we read $\Delta\varepsilon = 0.0$. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.056 + 0.041 - 0.0 = 0.097$$

The surface area of the pipe per m length of tube is

$$A_s = \pi DL = \pi(0.15\,\text{m})(1\,\text{m}) = 0.4712\,\text{m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

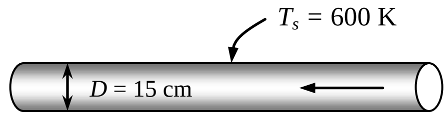
$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (0.4712\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.05(1500\,\text{K})^4 - 0.097(600\,\text{K})^4]\\ &= \mathbf{6427\ W}\end{aligned}$$

12-71 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.06(1\,\text{atm}) = 0.06\,\text{atm}$$
$$P_w = y_{H_2O} P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for an infinite cicrcular cylinder is, from Table 12-4,

$$L = 0.95(0.15\text{ m}) = 0.1425\text{ m}$$

Then,

$$P_c L = (0.06\,\text{atm})(0.1425\,\text{m}) = 0.00855\,\text{m}\cdot\text{atm} = 0.028\,\text{ft}\cdot\text{atm}$$
$$P_w L = (0.09\,\text{atm})(0.1425\,\text{m}) = 0.0128\,\text{m}\cdot\text{atm} = 0.042\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1500 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.034 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.016$$

These are base emissivity values at 1 atm, and they need to be corrected for the 3 atm total pressure. Noting that $(P_w+P)/2$ = (0.09+3)/2 = 1.545 atm, the pressure correction factors are, from Fig. 12-37,

$$C_c = 1.5 \quad \text{and} \quad C_w = 1.8$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1500 K is, from Fig. 12-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.028 + 0.042 = 0.07 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.09}{0.09 + 0.06} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1.5\times0.034 + 1.8\times0.016 - 0.0 = 0.080$$

For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L\frac{T_s}{T_g} = (0.06\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00342\,\text{m}\cdot\text{atm} = 0.011\text{ft}\cdot\text{atm}$$
$$P_w L\frac{T_s}{T_g} = (0.09\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00513\,\text{m}\cdot\text{atm} = 0.017\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.031 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.027$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1.5)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.65}(0.031) = 0.084$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1.8)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.45}(0.027) = 0.073$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 12-38 at $T = T_s = 600$ K instead of $T_g = 1500$ K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c) = 0.6$ and $P_cL + P_wL = 0.07$ we read $\Delta\varepsilon = 0.0$. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.084 + 0.073 - 0.0 = 0.157$$

The surface area of the pipe per m length of tube is

$$A_s = \pi DL = \pi(0.15\,\text{m})(1\,\text{m}) = 0.4712\,\text{m}^2$$

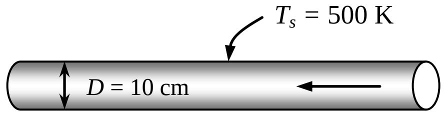
Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_\text{net} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (0.4712\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.08(1500\,\text{K})^4 - 0.157(600\,\text{K})^4]\\ &= \mathbf{10{,}276\ W}\end{aligned}$$

12-72 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length for an infinite cicrcular cylinder is, from Table 12-4,



$$L = 0.95(0.10 \text{ m}) = 0.095 \text{ m}$$

Then,

$$P_c L = (0.12\,\text{atm})(0.095\text{m}) = 0.0114\,\text{m}\cdot\text{atm} = 0.037\,\text{ft}\cdot\text{atm}$$

$$P_w L = (0.18\,\text{atm})(0.095\text{m}) = 0.0171\,\text{m}\cdot\text{atm} = 0.056\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 800 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.055 \quad \text{and} \quad \varepsilon_{w,1\text{atm}} = 0.050$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 800 K is, from Fig. 12-38,

$$\left.\begin{aligned} P_c L + P_w L &= 0.037 + 0.056 = 0.093 \\ \frac{P_w}{P_w + P_c} &= \frac{0.18}{0.18 + 0.12} = 0.6 \end{aligned}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\text{atm}} + C_w \varepsilon_{w,1\text{atm}} - \Delta\varepsilon = 1\times 0.055 + 1\times 0.050 - 0.0 = 0.105$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 500 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.12\,\text{atm})(0.095\text{m})\frac{500\,\text{K}}{800\,\text{K}} = 0.007125\text{m}\cdot\text{atm} = 0.023\text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.18\,\text{atm})(0.095\text{m})\frac{500\,\text{K}}{800\,\text{K}} = 0.01069\text{m}\cdot\text{atm} = 0.035\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 500 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.042 \quad \text{and} \quad \varepsilon_{w,1\text{atm}} = 0.050$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\text{atm}} = (1)\left(\frac{800\,\text{K}}{500\,\text{K}}\right)^{0.65}(0.042) = 0.057$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\text{atm}} = (1)\left(\frac{800\,\text{K}}{500\,\text{K}}\right)^{0.45}(0.050) = 0.062$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 12-38 at $T = T_s$ = 500 K instead of T_g = 800 K. There is no chart for 500 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and interpolate. At $P_w/(P_w + P_c)$ = 0.6 and $P_cL + P_wL$ = 0.093 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.057 + 0.062 - 0.0 = 0.119$$

The surface area of the pipe is

$$A_s = \pi DL = \pi(0.10\,\text{m})(6\,\text{m}) = 1.885\,\text{m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned} \dot{Q}_\text{net} &= A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (1.885\,\text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.105(800\,\text{K})^4 - 0.119(500\,\text{K})^4] \\ &= \mathbf{3802\ W} \end{aligned}$$

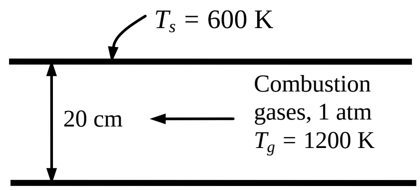
12-73 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$

$$P_w = y_{H_2O} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$



The mean beam length for this geometry is, from Table 12-4,

$$L = 3.6V/A_s = 1.8D = 1.8(0.20\text{ m}) = 0.36\text{ m}$$

where D is the distance between the plates. Then,

$$P_c L = P_w L = (0.10\,\text{atm})(0.36\text{m}) = 0.036\,\text{m}\cdot\text{atm} = 0.118\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1200 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.080 \quad \text{and} \quad \varepsilon_{w,1\text{atm}} = 0.055$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1200 K is, from Fig. 12-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.118 + 0.118 = 0.236 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.10}{0.10 + 0.10} = 0.5 \end{array}\right\} \Delta\varepsilon = 0.0025$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\text{atm}} + C_w \varepsilon_{w,1\text{atm}} - \Delta\varepsilon = 1\times 0.080 + 1\times 0.055 - 0.0025 = 0.1325$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = P_w L \frac{T_s}{T_g} = (0.10\,\text{atm})(0.36\text{m}) \frac{600\,\text{K}}{1200\,\text{K}} = 0.018\,\text{m}\cdot\text{atm} = 0.059\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.065 \quad \text{and} \quad \varepsilon_{w,1\text{atm}} = 0.067$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\text{atm}} = (1)\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{0.65} (0.065) = 0.102$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\text{atm}} = (1)\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{0.45} (0.067) = 0.092$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 12-38 at $T = T_s$ = 600 K instead of T_g = 1200 K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c)$ = 0.5 and $P_cL + P_wL$ = 0.236 we read $\Delta\varepsilon$ = 0.00125. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.102 + 0.092 - 0.00125 = 0.1928$$

Then the net rate of radiation heat transfer from the gas to each plate per unit surface area becomes

$$\begin{aligned} \dot{Q}_{\text{net}} &= A_s \sigma (\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (1\,\text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.1325(1200\,\text{K})^4 - 0.1928(600\,\text{K})^4] \\ &= \mathbf{1.42 \times 10^4\ W} \end{aligned}$$