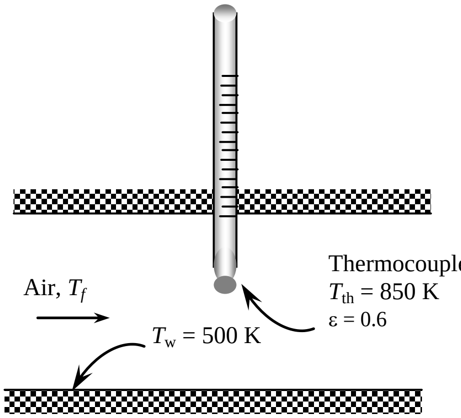
## Review Problems

**12-88** The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

***Assumptions*** The surfaces are opaque, diffuse, and gray.

***Properties*** The emissivity of thermocouple is given to be ε=0.6.

***Analysis*** The actual temperature of the air can be determined from




$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

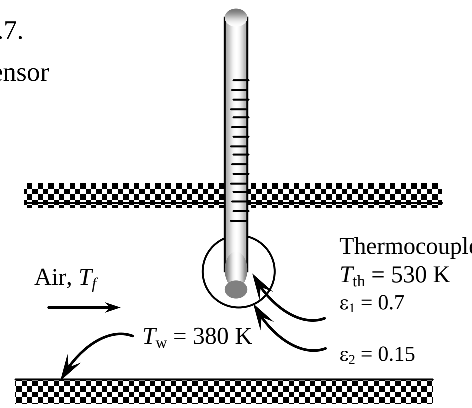
$$= 850\,\text{K} + \frac{(0.6)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(850\,\text{K})^4 - (500\,\text{K})^4]}{60\,\text{W/m}^2\cdot{}^\circ\text{C}} = \mathbf{1111\,K}$$

**12-89** The temperature of hot gases in a duct is measured by a thermocouple. The actual temperature of the gas is to be determined, and compared with that without a radiation shield.

***Assumptions*** The surfaces are opaque, diffuse, and gray.

***Properties*** The emissivity of the thermocouple is given to be ε =0.7.

***Analysis*** Assuming the area of the shield to be very close to the sensor of the thermometer, the radiation heat transfer from the sensor is determined from




$$\dot{Q}_{\text{rad,fromsensor}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} - 1\right) + \left(2\frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(530\,\text{K})^4 - (380\,\text{K})^4]}{\left(\frac{1}{0.7} - 1\right) + \left(2\frac{1}{0.15} - 1\right)}$$

$$= 257.9\ \text{W/m}^2$$

Then the actual temperature of the gas can be determined from a heat transfer balance to be

$$\dot{q}_{\text{conv,to sensor}} = \dot{q}_{\text{conv,fromsensor}}$$

$$h(T_f - T_{th}) = 257.9\ \text{W/m}^2$$

$$120\,\text{W/m}^2\cdot{}^\circ\text{C}(T_f - 530) = 257.9\ \text{W/m}^2 \longrightarrow T_f = \mathbf{532\ K}$$

Without the shield the temperature of the gas would be

$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

$$= 530\,\text{K} + \frac{(0.7)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(530\,\text{K})^4 - (380\,\text{K})^4]}{120\,\text{W/m}^2\cdot{}^\circ\text{C}} = \mathbf{549.2\ K}$$

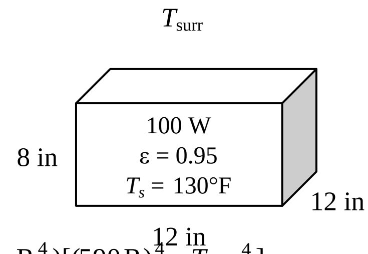
**12-90E** A sealed electronic box is placed in a vacuum chamber. The highest temperature at which the surrounding surfaces must be kept if this box is cooled by radiation alone is to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** Heat transfer from the bottom surface of the box is negligible.

***Properties*** The emissivity of the outer surface of the box is ε = 0.95.

***Analysis*** The total surface area is

$$A_s = 4\times(8\times1/12)+(1\times1) = 3.67\,\text{ft}^2$$




Then the temperature of the surrounding surfaces is determined to be

$$\dot{Q}_{rad} = \varepsilon A_s \sigma (T_s^{\,4} - T_{surr}^{\,4})$$

$$(100\times3.41214)\ \text{Btu/h} = (0.95)(3.67\,\text{m}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)[(590\,\text{R})^4 - T_{surr}^{\,4}]$$

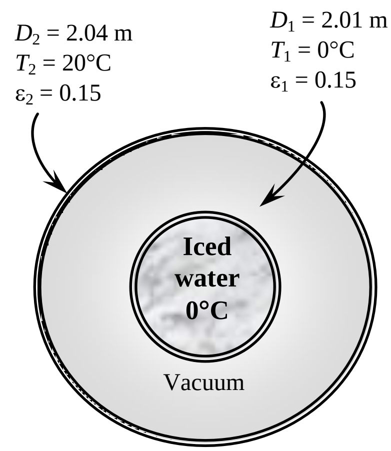
$$\longrightarrow T_{surr} = 503\,\text{R} = \mathbf{43°F}$$

**12-91** A double-walled spherical tank is used to store iced water. The air space between the two walls is evacuated. The rate of heat transfer to the iced water and the amount of ice that melts a 24-h period are to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

***Properties*** The emissivities of both surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.15$.

***Analysis*** (*a*) Assuming the conduction resistance s of the walls to be negligible, the rate of heat transfer to the iced water in the tank is determined to be




$$A_1 = \pi D_1^{\,2} = \pi(2.01\ \text{m})^2 = 12.69\ \text{m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^{\,4} - T_1^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

$$= \frac{(12.69\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(20+273\text{K})^4 - (0+273\text{K})^4]}{\dfrac{1}{0.15} + \dfrac{1-0.15}{0.15}\left(\dfrac{2.01}{2.04}\right)^2}$$

$$= \mathbf{107.4\ W}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.1074\,\text{kJ/s})(24\times3600\text{s}) = 9275\,\text{kJ}$$

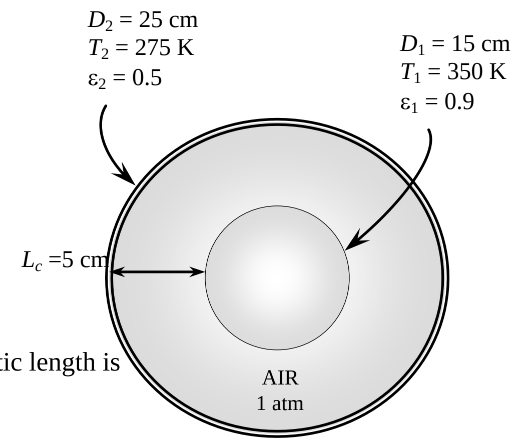
The amount of ice that melts during this period then becomes

$$Q = m h_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{9275\,\text{kJ}}{333.7\ \text{kJ/kg}} = \mathbf{27.8\ kg}$$

**12-92** Two concentric spheres which are maintained at uniform temperatures are separated by air at 1 atm pressure. The rate of heat transfer between the two spheres by natural convection and radiation is to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

***Properties*** The emissivities of the surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.5$. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K = 39.5°C are (Table A-15)




$$k = 0.02658 \text{ W/m.°C}$$

$$\upsilon = 1.697\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.7256$$

$$\beta = \frac{1}{312.5 \text{ K}} = 0.0032 \text{ K}^{-1}$$

***Analysis*** (*a*) Noting that $D_i = D_1$ and $D_o = D_2$ , the characteristic length is

$$L_c = \frac{1}{2}(D_o - D_i) = \frac{1}{2}(0.25\text{ m} - 0.15\text{ m}) = 0.05\text{ m}$$

Then

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003200 \text{ K}^{-1})(350-275 \text{ K})(0.05 \text{ m})^3}{(1.697\times10^{-5} \text{ m}^2/\text{s})^2}(0.7256) = 7.415\times10^5$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} = \frac{0.05 \text{ m}}{[(0.15 \text{ m})(0.25 \text{ m})]^4 [(0.15 \text{ m})^{-7/5} + (0.25 \text{ m})^{-7/5}]^5} = 0.005900$$

$$k_{\text{eff}} = 0.74k\left(\frac{\Pr}{0.861+\Pr}\right)^{1/4}(F_{sph}Ra)^{1/4}$$

$$= 0.74(0.02658 \text{ W/m.°C})\left(\frac{0.7256}{0.861+0.7256}\right)^{1/4}\left[(0.00590)(7.415\times10^5)\right]^{1/4} = 1315 \text{ W/m.°C}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o) = (0.1315 \text{ W/m.°C})\pi\left[\frac{(0.15 \text{ m})(0.25 \text{ m})}{(0.05 \text{ m})}\right](350-275)\text{K} = \textbf{23.3 W}$$

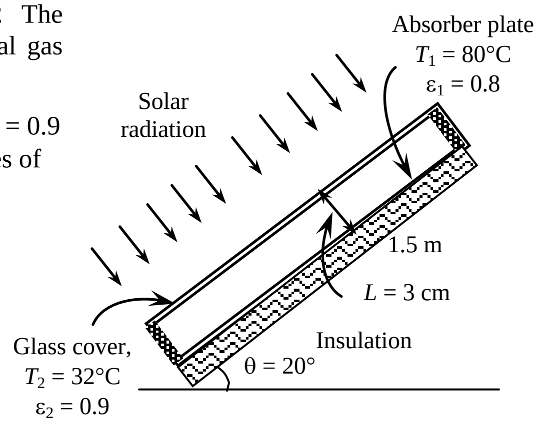
(*b*) The rate of heat transfer by radiation is determined from

$$A_1 = \pi D_1^2 = \pi(0.15 \text{ m})^2 = 0.0707 \text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2} = \frac{(0.0707 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(350 \text{ K})^4 - (275 \text{ K})^4]}{\dfrac{1}{0.9} + \dfrac{1-0.9}{0.9}\left(\dfrac{0.15}{0.25}\right)^2} = \textbf{32.3 W}$$

**12-93** A solar collector is considered. The absorber plate and the glass cover are maintained at uniform temperatures, and are separated by air. The rate of heat loss from the absorber plate by natural convection and radiation is to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.




***Properties*** The emissivities of surfaces are given to be $\varepsilon_1 = 0.9$ for glass and $\varepsilon_2$ = 0.8 for the absorber plate. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2$ = (80+32)/2 = 56°C are (Table A-15)

$$k = 0.02779 \text{ W/m.°C}$$
$$\upsilon = 1.857\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7212$$
$$\beta = \frac{1}{T_f} = \frac{1}{(56+273)\text{K}} = 0.003040 \text{K}^{-1}$$

***Analysis*** For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L$ = 0.03 m Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00304 \text{ K}^{-1})(80-32 \text{ K})(0.03 \text{ m})^3}{(1.857\times10^{-5} \text{ m}^2\text{/s})^2}(0.7212) = 8.083\times10^4$$

$$A_s = H\times W = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}\cos\theta}\right]^+\left[1-\frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right]+\left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18}-1\right]^+$$

$$= 1+1.44\left[1-\frac{1708}{(8.083\times10^4)\cos(20)}\right]^+\left[1-\frac{1708[\sin(1.8\times20)]^{1.6}}{(8.083\times10^4)\cos(20)}\right]+\left[\frac{[(8.083\times10^4)\cos(20)]^{1/3}}{18}-1\right]^+$$

$$= 3.747$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(3.747)(4.5 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{750 \text{ W}}$$

Neglecting the end effects, the rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^4-T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(80+273 \text{K})^4-(32+273 \text{K})^4]}{\frac{1}{0.8}+\frac{1}{0.9}-1} = \mathbf{1289 \text{ W}}$$

***Discussion*** The rates of heat loss by natural convection for the horizontal and vertical cases would be as follows (Note that the Ra number remains the same):

*Horizontal*:

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18}-1\right]^+ = 1+1.44\left[1-\frac{1708}{8.083\times10^4}\right]^+ + \left[\frac{(8.083\times10^4)^{1/3}}{18}-1\right]^+ = 3.812$$

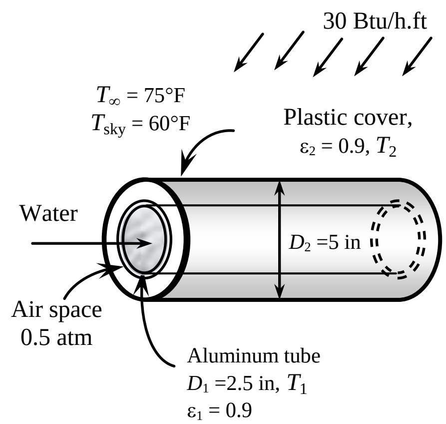
$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(3.812)(6 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{1017 \text{ W}}$$

*Vertical*:

$$Nu = 0.42Ra^{1/4}\ \mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(8.083\times10^4)^{1/4}(0.7212)^{0.012}\left(\frac{2\ \mathrm{m}}{0.03\ \mathrm{m}}\right)^{-0.3} = 2.001$$

$$\dot{Q} = kNuA_s\,\frac{T_1 - T_2}{L} = (0.02779\ \mathrm{W/m.°C})(2.001)(6\ \mathrm{m}^2)\frac{(80-32)\mathrm{°C}}{0.03\ \mathrm{m}} = \mathbf{534\ W}$$

**12-94E** The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.




***Assumptions*** **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** The surfaces are opaque, diffuse, and gray for infrared radiation. **5** The glass cover is transparent to solar radiation.

***Properties*** The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 85°F, and use properties at an anticipated average temperature of (75+85)/2 =80°F (Table A-15E),

$$k = 0.01481\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F}$$

$$\upsilon = 0.6110\ \text{ft}^2/\text{h} = 1.697\times10^{-4}\ \text{ft}^2/s$$

$$\Pr = 0.7290$$

$$\beta = \frac{1}{T_{\text{ave}}} = \frac{1}{540\,\text{R}}$$

***Analysis*** We have a horizontal cylindrical enclosure filled with air at 0.5 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30\ \text{Btu/h} \quad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{glass} = (\pi D_o W) = \pi(5/12\ \text{ft})(1\ \text{ft}) = 1.309\ \text{ft}^2 \quad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, solution will require a trial-and-error approach. Assuming the glass cover temperature to be 85°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\text{Ra}_{Do} = \frac{g\beta(T_o - T_\infty)D_o^3}{\nu^2}\Pr$$

$$= \frac{(32.2\ \text{ft/s}^2)[1/(540\,\text{R})](85-75\,\text{R})(5/12\ \text{ft})^3}{(1.675\times10^{-4}\ \text{ft}^2/\text{s})^2}(0.7290) = 1.092\times10^6$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.092\times10^6)^{1/6}}{\left[1+(0.559/0.7290)^{9/16}\right]^{8/27}}\right\}^2$$

$$= 14.95$$

$$h_o = \frac{k}{D_0}\text{Nu} = \frac{0.01481\,\text{Btu/h}\cdot\text{ft}\cdot°\text{F}}{5/12\ \text{ft}}(14.95) = 0.5315\ \text{Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

$$\dot{Q}_{o,\text{conv}} = h_o A_o(T_o - T_\infty) = (0.5315\,\text{Btu/h}\cdot\text{ft}^2\cdot°\text{F})(1.309\ \text{ft}^2)(85-75)°\text{F} = 6.96\ \text{Btu/h}$$

Also,

$$\dot{Q}_{o,\text{rad}} = \varepsilon_o\sigma A_o(T_o^4 - T_{\text{sky}}^4)$$

$$= (0.9)(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(1.309\ \text{ft}^2)\left[(545\,\text{R})^4 - (535\,\text{R})^4\right]$$

$$= 30.5\ \text{Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{o,\text{total}} = \dot{Q}_{o,\text{conv}} + \dot{Q}_{o,\text{rad}} = 7.0 + 30.5 = 37.5\ \text{Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 85°F for the glass cover is high. Repeating the calculations with lower temperatures (including the evaluation of properties), the glass cover temperature corresponding to 30 Btu/h is determined to be **81.5°F**.

The temperature of the aluminum tube is determined in a similar manner using the natural convection and radiation relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (5-2.5)/2 = 1.25\,\text{in} = 1.25/12\,\text{ft}$$

Also,

$$A_i = A_{tube} = (\pi D_i W) = \pi(2.5/12\,\text{ft})(1\,\text{ft}) = 0.6545\ \text{ft}^2 \quad \text{(per foot of tube)}$$

We start the calculations by assuming the tube temperature to be 118.5°F, and thus an average temperature of (81.5+118.5)/2 = 100°F=640 R. Using properties at 100°F,

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr} = \frac{(32.2\,\text{ft/s}^2)[1/(640\,\text{R})](118.5-81.5\,\text{R})(1.25/12\,\text{ft})^3}{(1.809\times10^{-4}\ \text{ft}^2/\text{s})^2}(0.726) = 1.334\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyc}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{[\ln(5/2.5)]^4}{(1.25/12\text{ft})^3[(2.5/12\,\text{ft})^{-3/5}+(5/12\,\text{ft})^{-3/5}]^5} = 0.1466$$

$$\begin{aligned} k_{\text{eff}} &= 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyc}}\text{Ra}_L)^{1/4} \\ &= 0.386(0.01529\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})\left(\frac{0.726}{0.861+0.726}\right)(0.1466\times1.334\times10^4)^{1/4} \\ &= 0.03227\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F} \end{aligned}$$

Then the rate of heat transfer between the cylinders by convection becomes

$$\dot{Q}_{i,\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.03227\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})}{\ln(5/2.5)}(118.5-81.5)^\circ\text{F} = 10.8\,\text{Btu/h}$$

Also,

$$\dot{Q}_{i,\text{rad}} = \frac{\sigma A_i(T_i^4 - T_o^4)}{\dfrac{1}{\varepsilon_i}+\dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{D_i}{D_o}\right)} = \frac{(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(0.6545\text{ft}^2)\left[(578.5\ \text{R})^4-(541.5\,\text{R})^4\right]}{\dfrac{1}{0.9}+\dfrac{1-0.9}{0.9}\left(\dfrac{2.5\,\text{in}}{5\,\text{in}}\right)} = 25.0\,\text{Btu/h}$$

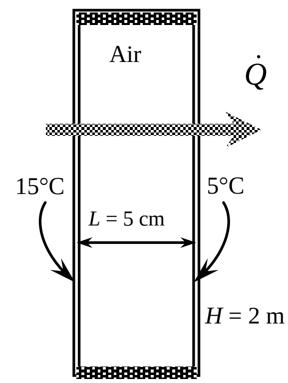
Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{i,\text{total}} = \dot{Q}_{i,\text{conv}} + \dot{Q}_{i,\text{rad}} = 10.8 + 25.0 = 35.8\,\text{Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 118.5°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **113.2°F**. Therefore, the tube will reach an equilibrium temperature of 113.2°F when the pump fails.

**12-95** A double-pane window consists of two sheets of glass separated by an air space. The rates of heat transfer through the window by natural convection and radiation are to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats. **4** Heat transfer through the window is one-dimensional and the edge effects are negligible.




***Properties*** The emissivities of glass surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air at 0.3 atm and the average temperature of $(T_1+T_2)/2 = (15+5)/2 = 10°C$ are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$

$$\upsilon = \upsilon_{1atm} / 0.3 = 1.426\times10^{-5}/0.3 = 4.753\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7336$$

$$\beta = \frac{1}{(10+273)\text{ K}} = 0.003534 \text{K}^{-1}$$

***Analysis*** The characteristic length in this case is the distance between the glasses, $L_c = L = 0.05 \text{ m}$

$$Ra = \frac{g\beta(T_1 - T_2)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003534 \text{K}^{-1})(15-5)\text{K}(0.05\text{ m})^3}{(4.753\times10^{-5} \text{ m}^2/\text{s})^2}(0.7336) = 1.918\times10^4$$

$$Nu = 0.197 Ra^{1/4}\left(\frac{H}{L}\right)^{-1/9} = 0.197(1.918\times10^4)^{1/4}\left(\frac{2}{0.05}\right)^{-1/9} = 1.539$$

$$A_s = (2 \text{ m})(3 \text{ m}) = 6 \text{ m}^2$$

Then the rate of heat transfer by natural convection becomes

$$\dot{Q}_{conv} = kNuA_s\frac{T_1 - T_2}{L} = (0.02439 \text{ W/m.°C})(1.539)(6 \text{ m}^2)\frac{(15-5)°\text{C}}{0.05 \text{ m}} = \mathbf{45.0\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{(6 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(15+273\text{K})^4 - (5+273\text{K})^4]}{\frac{1}{0.9} + \frac{1}{0.9} - 1}$$

$$= \mathbf{252\ W}$$

Then the rate of total heat transfer becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 45 + 252 = \mathbf{297\ W}$$

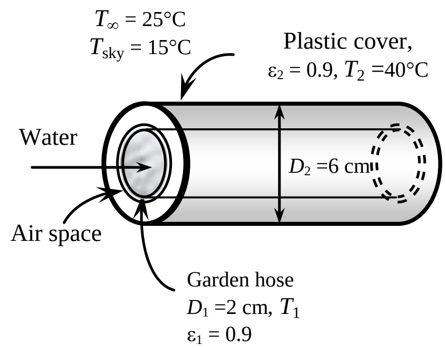
***Discussion*** Note that heat transfer through the window is mostly by radiation.

**12-96** A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose by natural convection and radiation is to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

***Properties*** The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air are at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+25)/2 = 32.5$°C are (Table A-15)

$$k = 0.02607 \text{ W/m.°C}$$
$$\upsilon = 1.632\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7275$$
$$\beta = \frac{1}{(32.5+273)\text{ K}} = 0.003273 \text{K}^{-1}$$




***Analysis*** Under steady conditions, the heat transfer rate from the water in the hose equals to the rate of heat loss from the clear plastic tube to the surroundings by natural convection and radiation. The characteristic length in this case is the diameter of the plastic tube, $L_c = D_{plastic} = D_2 = 0.06 \text{ m}$.

$$Ra = \frac{g\beta(T_s - T_\infty)D_2^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003273 \text{K}^{-1})(40-25)\text{K}(0.06 \text{ m})^3}{(1.632\times10^{-5} \text{ m}^2\text{/s})^2}(0.7275) = 2.842\times10^5$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.842\times10^5)^{1/6}}{\left[1+(0.559/0.7241)^{9/16}\right]^{8/27}}\right\}^2 = 10.30$$

$$h = \frac{k}{D_2}Nu = \frac{0.02607 \text{ W/m.°C}}{0.06 \text{ m}}(10.30) = 4.475 \text{ W/m}^2\text{.°C}$$

$$A_{plastic} = A_2 = \pi D_2 L = \pi(0.06 \text{ m})(1 \text{ m}) = 0.1885 \text{ m}^2$$

Then the rate of heat transfer from the outer surface by natural convection becomes

$$\dot{Q}_{conv} = hA_2(T_s - T_\infty) = (4.475 \text{ W/m}^2\text{.°C})(0.1885 \text{ m}^2)(40-25)\text{°C} = \mathbf{12.7\ W}$$

The rate of heat transfer by radiation from the outer surface is determined from

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_2\sigma(T_s{}^4 - T_{sky}{}^4) \\ &= (0.90)(0.1885 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(40+273\text{K})^4 - (15+273\text{K})^4] \\ &= \mathbf{26.2\ W}\end{aligned}$$

Finally,

$$\dot{Q}_{total,loss} = 12.7 + 26.2 = 38.9 \text{ W}$$

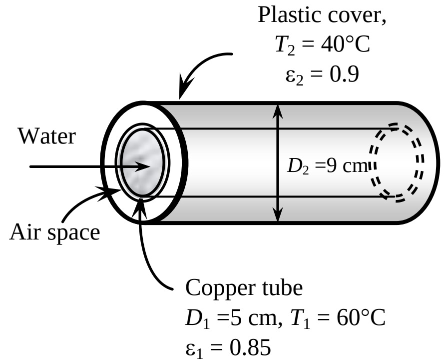
***Discussion*** Note that heat transfer is mostly by radiation.

**12-97** A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. The annular space between the copper and the glass tubes is filled with air at 1 atm. The rate of heat loss from the collector by natural convection and radiation is to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

***Properties*** The emissivities of surfaces are given to be $\varepsilon_1 = 0.85$ for the tube surface and $\varepsilon_2 = 0.9$ for glass cover. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (60+40)/2 = 50$°C are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\upsilon = 1.798\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7228$$
$$\beta = \frac{1}{(50+273)\text{ K}} = 0.003096 \text{K}^{-1}$$




***Analysis*** The characteristic length in this case is

$$L_c = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.09\text{ m} - 0.05\text{ m}) = 0.02\text{ m}$$

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003096\text{ K}^{-1})(60-40)\text{K}(0.02\text{ m})^3}{(1.798\times10^{-5}\text{ m}^2\text{/s})^2}(0.7228) = 10{,}850$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{[\ln(0.09/0.05)]^4}{(0.02\text{ m})^3[(0.09\text{ m})^{-3/5} + (0.05\text{ m})^{-3/5}]^5} = 0.1303$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$
$$= 0.386(0.02735\text{ W/m.°C})\left(\frac{0.7228}{0.861+0.7228}\right)^{1/4}[(0.1303)(10{,}850)]^{1/4} = 0.05321\text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q}_{\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.05321\text{ W/m.°C})}{\ln(0.09/0.05)}(60-40)\text{°C} = \mathbf{11.4\ W} \qquad \text{(Eq. 1)}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$
$$= \frac{(0.1571\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(60+273\text{ K})^4 - (40+273\text{ K})^4]}{\dfrac{1}{0.85} + \dfrac{1-0.9}{0.9}\left(\dfrac{5}{9}\right)}$$
$$= \mathbf{13.4\ W}$$

Finally,

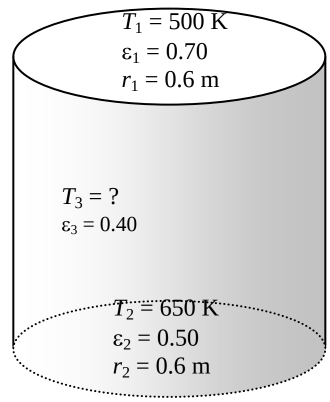
$$\dot{Q}_{total,loss} = 11.4 + 13.4 = 24.8\text{ W} \quad \text{(per m length)}$$

**12-98** A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The temperature of the side surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

***Properties*** The emissivities of the top, bottom, and side surfaces are 0.70, 0.50, and 0.40, respectively.

***Analysis*** We consider the top surface to be surface 1, the bottom surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from




$$\left.\begin{array}{l}\dfrac{L}{r}=\dfrac{1.2}{0.6}=2\\[2ex]\dfrac{r}{L}=\dfrac{0.6}{1.2}=0.5\end{array}\right\}F_{12}=0.17 \qquad \text{(Fig. 12-7)}$$

The surface areas are

$$A_1 = A_2 = \pi D^2/4 = \pi(1.2\text{ m})^2/4 = 1.131\text{m}^2$$

$$A_3 = \pi DL = \pi(1.2\text{ m})(1.2\text{ m}) = 4.524\text{ m}^2$$

Then other view factors are determined to be

$$F_{12} = F_{21} = 0.17$$

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + 0.17 + F_{13} = 1 \longrightarrow F_{13} = 0.83 \quad \text{(summation rule)}, \quad F_{23} = F_{13} = 0.83$$

$$A_1F_{13} = A_3F_{31} \longrightarrow (1.131)(0.83) = (4.524)F_{31} \longrightarrow F_{31} = 0.21 \quad \text{(reciprocity rule)}, \qquad F_{32} = F_{31} = 0.21$$

We now apply Eq. 12-35 to each surface

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4 = J_1 + \frac{1-0.70}{0.70}\left[0.17(J_1 - J_2) + 0.83(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4 = J_2 + \frac{1-0.50}{0.50}\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^4 = J_3 + \frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3 - J_1) + F_{32}(J_3 - J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)T_3^4 = J_3 + \frac{1-0.40}{0.40}\left[0.21(J_1 - J_2) + 0.21(J_1 - J_3)\right]$$

We now apply Eq. 12-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (1.131\text{m}^2)\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$T_3 = \mathbf{631 K}, \quad J_1 = 4974\text{W/m}^2, \quad J_2 = 8883\text{W/m}^2, \quad J_3 = 8193\text{W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$\dot{Q}_{21} = A_2F_{21}(J_2 - J_1) = (1.131\text{m}^2)(0.17)(8883 - 4974)\text{W/m}^2 = \mathbf{751.6 W}$$

The rate of heat transfer between the bottom and the side surface is

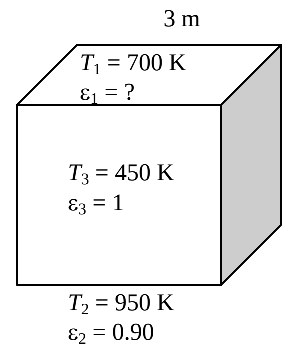
$$\dot{Q}_{23} = A_2F_{23}(J_2 - J_3) = (1.131\text{m}^2)(0.83)(8883 - 8197)\text{W/m}^2 = \mathbf{644.0 W}$$

***Discussion*** The sum of these two heat transfer rates are 751.6 + 644 = 1395.6 W, which is practically equal to 1400 W heat supply rate from surface 2. This must be satisfied to maintain the surfaces at the specified temperatures under steady operation. Note that the difference is due to round-off error.

**12-99** A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

***Properties*** The emissivity of the bottom surface is 0.90.




***Analysis*** We consider the top surface to be surface 1, the base surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from the base to the top surface of the cube is $F_{12} = 0.2$. The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Other view factors are

$$F_{21} = F_{12} = 0.20, \qquad F_{23} = F_{13} = 0.80, \qquad F_{31} = F_{32} = 0.20$$

We now apply Eq. 9-35 to each surface

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)(700\text{ K})^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[0.20(J_1 - J_2) + 0.80(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)(950\text{ K})^4 = J_2 + \frac{1-0.90}{0.90}\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^4 = J_3$$

$$(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)(450\text{ K})^4 = J_3$$

We now apply Eq. 9-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (9\text{ m}^2)\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$\varepsilon_1 = \mathbf{0.44}, \quad J_1 = 11{,}736\text{ W/m}^2, \quad J_2 = 41{,}985\text{ W/m}^2, \quad J_3 = 2325\text{ W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$A_1 = A_2 = (3\text{ m})^2 = 9\text{ m}^2$$

$$\dot{Q}_{21} = A_2 F_{21}(J_2 - J_1) = (9\text{ m}^2)(0.20)(41{,}985 - 11{,}736)\text{W/m}^2 = \mathbf{54.4\text{ kW}}$$

The rate of heat transfer between the bottom and the side surface is

$$A_3 = 4A_1 = 4(9\text{ m}^2) = 36\text{ m}^2$$

$$\dot{Q}_{23} = A_2 F_{23}(J_2 - J_3) = (9\text{ m}^2)(0.8)(41{,}985 - 2325)\text{W/m}^2 = \mathbf{285.6\text{ kW}}$$

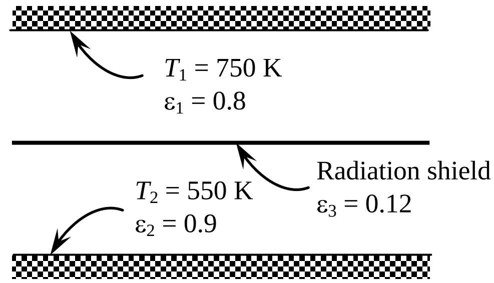
***Discussion*** The sum of these two heat transfer rates are 54.4 + 285.6 = 340 kW, which is equal to 340 kW heat supply rate from surface 2.

**12-100** A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates and the temperature of the radiation shield are to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

***Properties*** The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.9$, and $\varepsilon_3 = 0.12$.

***Analysis*** The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is




$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\,\text{K})^4 - (550\,\text{K})^4]}{\left(\frac{1}{0.8} + \frac{1}{0.9} - 1\right) + \left(\frac{1}{0.12} + \frac{1}{0.12} - 1\right)}$$

$$= \mathbf{748.9\,W/m^2}$$

The equilibrium temperature of the radiation shield is determined from

$$\dot{Q}_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right)}$$

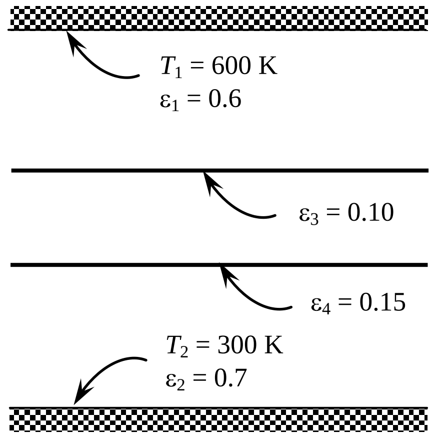
$$748.9\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\,\text{K})^4 - T_3^4]}{\left(\frac{1}{0.8} + \frac{1}{0.12} - 1\right)} \longrightarrow T_3 = \mathbf{671.3\,K}$$

**12-101** Two thin radiation shields are placed between two large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates with and without the shields, and the temperatures of radiation shields are to be determined.

***Assumptions*** **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

***Properties*** The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.7$, $\varepsilon_3 = 0.10$, and $\varepsilon_4 = 0.15$.

***Analysis*** The net rate of radiation heat transfer without the shields per unit area of the plates is




$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\,\text{K})^4 - (300\,\text{K})^4]}{\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1} = \mathbf{3288\ W/m^2}$$

The net rate of radiation heat transfer with two thin radiation shields per unit area of the plates is

$$\dot{Q}_{12,\text{two-shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_4} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\,\text{K})^4 - (300\,\text{K})^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1\right) + \left(\dfrac{1}{0.10} + \dfrac{1}{0.10} - 1\right) + \left(\dfrac{1}{0.15} + \dfrac{1}{0.15} - 1\right)} = \mathbf{206\ W/m^2}$$

The equilibrium temperatures of the radiation shields are determined from

$$\dot{Q}_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right)}$$

$$206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\,\text{K})^4 - T_3^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.10} - 1\right)} \longrightarrow T_3 = \mathbf{549\ K}$$

$$\dot{Q}_{42} = \frac{\sigma(T_4^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

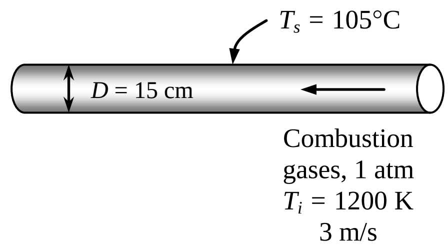
$$206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[T_4^4 - (300\,\text{K})^4]}{\left(\dfrac{1}{0.15} + \dfrac{1}{0.7} - 1\right)} \longrightarrow T_4 = \mathbf{429\ K}$$

**12-102** Combustion gases flow inside a tube in a boiler. The rates of heat transfer by convection and radiation and the rate of evaporation of water are to be determined.

***Assumptions*** **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Combustion gases are assumed to have the properties of air, which is an ideal gas with constant properties.

***Properties*** The properties of air at 1200 K = 927°C and 1 atm are (Table A-15)

$$\rho = 0.2944\ \text{kg/m}^3 \qquad C_p = 1173\ \text{J/kg.°C}$$
$$k = 0.07574\ \text{W/m.°C} \qquad \text{Pr} = 0.7221$$
$$\upsilon = 1.586\times10^{-5}\ \text{m}^2/\text{s}$$




***Analysis*** (*a*) The Reynolds number is

$$\text{Re} = \frac{\mathbf{V}_m D}{\upsilon} = \frac{(3\ \text{m/s})(0.15\ \text{m})}{1.586\times10^{-5}\ \text{m}^2/\text{s}} = 28{,}373$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.15\ \text{m}) = 1.5\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.3} = 0.023(28{,}373)^{0.8}(0.7221)^{0.3} = 76.14$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.07574\ \text{W/m.°C}}{0.15\ \text{m}}(76.14) = 38.45\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.15\ \text{m})(6\ \text{m}) = 2.827\ \text{m}^2$$
$$A_c = \pi D^2/4 = \pi(0.15\ \text{m})^2/4 = 0.01767\ \text{m}^2$$
$$\dot{m} = \rho V A_c = (0.2944\ \text{kg/m}^3)(3\ \text{m/s})(0.01767\ \text{m}^2) = 0.01561\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 105 - (105 - 927)e^{-\frac{(38.45)(2.827)}{(0.01561)(1173)}} = 107.2°\text{C}$$

Then the rate of heat transfer by convection becomes

$$\dot{Q}_{\text{conv}} = \dot{m}C_p(T_i - T_e) = (0.01561\ \text{kg/s})(1173\ \text{J/kg.°C})(927 - 107.2)°\text{C} = \mathbf{15{,}010\ W}$$

Next, we determine the emissivity of combustion gases. First, the mean beam length for an infinite circular cylinder is, from Table 12-4,

$$L = 0.95(0.15\ \text{m}) = 0.1425\ \text{m}$$

Then,

$$P_c L = (0.08\ \text{atm})(0.1425\ \text{m}) = 0.0114\ \text{m}\cdot\text{atm} = 0.037\ \text{ft}\cdot\text{atm}$$
$$P_w L = (0.16\ \text{atm})(0.1425\ \text{m}) = 0.0228\ \text{m}\cdot\text{atm} = 0.075\ \text{ft}\cdot\text{atm}$$

The emissivities of $CO_2$ and $H_2O$ corresponding to these values at the average gas temperature of $T_g$=($T_g$+$T_g$)/2 = (927+107.2)/2 = 517.1°C = 790 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.055 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.062$$

Both $CO_2$ and $H_2O$ are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 800 K is, from Fig. 12-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.037 + 0.075 = 0.112 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.16}{0.16 + 0.08} = 0.67 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1 \times 0.055 + 1 \times 0.062 - 0.0 = 0.117$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of $T_s$ = 105°C = 378 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.08\,\text{atm})(0.1425\,\text{m}) \frac{378\,\text{K}}{790\,\text{K}} = 0.00545\,\text{m} \cdot \text{atm} = 0.018\,\text{ft} \cdot \text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16\,\text{atm})(0.1425\,\text{m}) \frac{378\,\text{K}}{790\,\text{K}} = 0.0109\,\text{m} \cdot \text{atm} = 0.036\,\text{ft} \cdot \text{atm}$$

The emissivities of $CO_2$ and $H_2O$ corresponding to these values at a temperature of $T_s$ = 378 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.037 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.062$$

Then the absorptivities of $CO_2$ and $H_2O$ become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{790\,\text{K}}{378\,\text{K}}\right)^{0.65} (0.037) = 0.0597$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{790\,\text{K}}{378\,\text{K}}\right)^{0.45} (0.062) = 0.0864$$

Also Δα = Δε, but the emissivity correction factor is to be evaluated from Fig. 12-38 at $T = T_s$ = 378 K instead of $T_g$ = 790 K. We use the chart for 400 K. At $P_w/(P_w + P_c)$ = 0.67 and $P_cL + P_wL$ = 0.112 we read Δε = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.0597 + 0.0864 - 0.0 = 0.146$$

The emissivity of the inner surface s of the tubes is 0.9. Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned} \dot{Q}_\text{rad} &= \frac{\varepsilon_s + 1}{2} A_s \sigma (\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= \frac{0.9 + 1}{2} (2.827\,\text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[0.117(790\,\text{K})^4 - 0.146(378\,\text{K})^4] \\ &= \mathbf{6486\ W} \end{aligned}$$

(*b*) The heat of vaporization of water at 1 atm is 2257 kJ/kg (Table A-9). Then rate of evaporation of water becomes

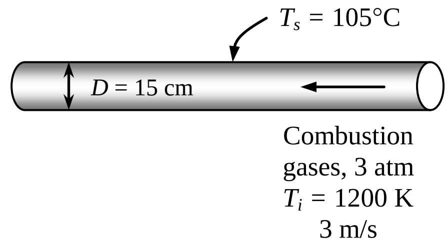
$$\dot{Q}_\text{conv} + \dot{Q}_\text{rad} = \dot{m}_\text{evap} h_{fg} \longrightarrow \dot{m}_\text{evap} = \frac{\dot{Q}_\text{conv} + \dot{Q}_\text{rad}}{h_{fg}} = \frac{(15{,}010 + 6486)\ \text{W}}{333.7 \times 10^3\ \text{J/kg}} = \mathbf{0.0644\ kg/s}$$

**12-103** Combustion gases flow inside a tube in a boiler. The rates of heat transfer by convection and radiation and the rate of evaporation of water are to be determined.

***Assumptions*** **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Combustion gases are assumed to have the properties of air, which is an ideal gas with constant properties.

***Properties*** The properties of air at 1200 K = 927°C and 3 atm are (Table A-15)

$$\rho = 0.2944\ \text{kg/m}^3 \qquad C_p = 1173\ \text{J/kg.°C}$$
$$k = 0.07574\ \text{W/m.°C} \qquad \text{Pr} = 0.7221$$
$$\upsilon = (1.586\times10^{-5}\ \text{m}^2\text{/s})/3 = 0.5287\times10^{-5}\ \text{m}^2\text{/s}$$




***Analysis*** (*a*) The Reynolds number is

$$\text{Re} = \frac{\mathbf{V}_m D}{\upsilon} = \frac{(3\ \text{m/s})(0.15\ \text{m})}{0.5287\times10^{-5}\ \text{m}^2\text{/s}} = 85{,}114$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.15\ \text{m}) = 1.5\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.3} = 0.023(85{,}114)^{0.8}(0.7221)^{0.3} = 183.4$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.07574\ \text{W/m.°C}}{0.15\ \text{m}}(183.4) = 92.59\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.15\ \text{m})(6\ \text{m}) = 2.827\ \text{m}^2$$
$$A_c = \pi D^2/4 = \pi(0.15\ \text{m})^2/4 = 0.01767\ \text{m}^2$$
$$\dot{m} = \rho VA_c = (0.2944\ \text{kg/m}^3)(3\ \text{m/s})(0.01767\ \text{m}^2) = 0.01561\ \text{kg/s}$$
$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 105 - (105 - 927)e^{-\frac{(92.59)(2.827)}{(0.01561)(1173)}} = 105.0°\text{C}$$

Then the rate of heat transfer by convection becomes

$$\dot{Q}_{\text{conv}} = \dot{m}C_p(T_i - T_e) = (0.01561\ \text{kg/s})(1173\ \text{J/kg.°C})(927 - 105.0)°\text{C} = \mathbf{15{,}050\ W}$$

Next, we determine the emissivity of combustion gases. First, the mean beam length for an infinite circular cylinder is, from Table 12-4,

$$L = 0.95(0.15\ \text{m}) = 0.1425\ \text{m}$$

Then,

$$P_c L = (0.08\ \text{atm})(0.1425\ \text{m}) = 0.0114\ \text{m}\cdot\text{atm} = 0.037\ \text{ft}\cdot\text{atm}$$
$$P_w L = (0.16\ \text{atm})(0.1425\ \text{m}) = 0.0228\ \text{m}\cdot\text{atm} = 0.075\ \text{ft}\cdot\text{atm}$$

The emissivities of $CO_2$ and $H_2O$ corresponding to these values at the average gas temperature of $T_g$=($T_g$+$T_g$)/2 = (927+105)/2 = 516°C = 790 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\text{atm}} = 0.055 \quad \text{and} \quad \varepsilon_{w,1\text{atm}} = 0.062$$

These are the base emissivity values at 1 atm, and they need to be corrected for the 3 atm total pressure. Noting that ($P_w$+$P$)/2 = (0.16+3)/2 = 1.58 atm, the pressure correction factors are, from Fig. 12-37,

$$C_c = 1.5 \quad \text{and} \quad C_w = 1.8$$

Both $CO_2$ and $H_2O$ are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 800 K is, from Fig. 12-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.037 + 0.075 = 0.112 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.16}{0.16 + 0.08} = 0.67 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1.5\times 0.055 + 1.8\times 0.062 - 0.0 = 0.194$$

For a source temperature of $T_s$ = 105°C = 378 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.08\,\text{atm})(0.1425\,\text{m})\frac{378\,\text{K}}{790\,\text{K}} = 0.00545\,\text{m}\cdot\text{atm} = 0.018\,\text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16\,\text{atm})(0.1425\,\text{m})\frac{378\,\text{K}}{790\,\text{K}} = 0.0109\,\text{m}\cdot\text{atm} = 0.036\,\text{ft}\cdot\text{atm}$$

The emissivities of $CO_2$ and $H_2O$ corresponding to these values at a temperature of $T_s$ = 378 K and 1atm are, from Fig. 12-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.037 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.062$$

Then the absorptivities of $CO_2$ and $H_2O$ become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1.5)\left(\frac{790\,\text{K}}{378\,\text{K}}\right)^{0.65}(0.037) = 0.0896$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1.8)\left(\frac{790\,\text{K}}{378\,\text{K}}\right)^{0.45}(0.062) = 0.1555$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 12-38 at $T = T_s$ = 378 K instead of $T_g$ = 790 K. We use the chart for 400 K. At $P_w/(P_w + P_c)$ = 0.67 and $P_cL + P_wL$ = 0.112 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.0896 + 0.1555 - 0.0 = 0.245$$

The emissivity of the inner surfaces of the tubes is 0.9. Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \frac{\varepsilon_s + 1}{2} A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= \frac{0.9+1}{2}(2.827\,\text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.194(790\,\text{K})^4 - 0.245(378\,\text{K})^4] \\ &= \mathbf{10{,}745\ W}\end{aligned}$$

(*b*) The heat of vaporization of water at 1 atm is 2257 kJ/kg (Table A-9). Then rate of evaporation of water becomes

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = \dot{m}_{\text{evap}} h_{fg} \longrightarrow \dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}}{h_{fg}} = \frac{(15{,}050 + 10{,}745)\ \text{W}}{333.7\times 10^3\ \text{J/kg}} = \mathbf{0.0773\ kg/s}$$

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**12-104 ….. 12-106 Design and Essay Problems**

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