Steady Mass Diffusion Through a Wall

14-42C The relations for steady one-dimensional heat conduction and mass diffusion through a plane wall are expressed as follows:

Heat conduction: $\dot{Q}_{cond} = -kA\frac{T_1 - T_2}{L}$

Mass diffusion: $\dot{m}_{\text{diff,A,wall}} = \rho D_{\text{AB}} A \frac{w_{\text{A},1} - w_{\text{A},2}}{L} = D_{\text{AB}} A \frac{\rho_{\text{A},1} - \rho_{\text{A},2}}{L}$

where A is the normal area and L is the thickness of the wall, and the other variables correspond to each other as follows:

rate of heat conduction $\dot{Q}_{\text{cond}} \longleftrightarrow \dot{m}_{\text{diff,A,wall}}$ rate of mass diffusion

thermal conductivity $k \longleftrightarrow D_{\text{AB}}$ mass diffusivity

temperature $T \longleftrightarrow \rho_{\text{A}}$ density of A

14-43C (*a*) T, (*b*) F, (*c*) T, (*d*) F

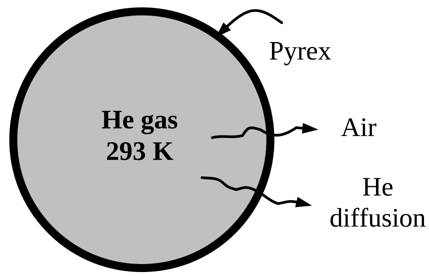
14-44C During one-dimensional mass diffusion of species A through a plane wall of thickness L, the concentration profile of species A in the wall will be a straight line when (1) steady operating conditions are established, (2) the concentrations of the species A at both sides are maintained constant, and (3) the diffusion coefficient is constant.

14-45C During one-dimensional mass diffusion of species A through a plane wall, the species A content of the wall will remain constant during steady mass diffusion, but will change during transient mass diffusion.

14-46 Pressurized helium gas is stored in a spherical container. The diffusion rate of helium through the container is to be determined.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the container is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the center of the container. **2** There are no chemical reactions in the pyrex shell that results in the generation or depletion of helium.

Properties The binary diffusion coefficient of helium in the pyrex at the specified temperature is 4.5×10^{-15} m^2/s (Table 14-3b). The molar mass of helium is M = 4 kg/kmol (Table A-1).



Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the container to be a *stationary* medium since there is no diffusion of pyrex molecules ($\dot{N}_B = 0$) and the concentration of the helium in the container is extremely low ($C_A << 1$). Then the molar flow rate of helium through the shell by diffusion can readily be determined from Eq. 14-28 to be

$$\begin{aligned}\dot{N}_{\text{diff}} &= 4\pi r_1 r_2 D_{AB}\frac{C_{A,1}-C_{A,2}}{r_2-r_1}\\ &= 4\pi(1.45\text{ m})(1.50\text{ m})(4.5\times10^{-15}\text{ m}^2/\text{s})\frac{(0.00073-0)\text{ kmol/m}^3}{1.50-1.45}\\ &= 1.80\times10^{-15}\text{ kmol/s}\end{aligned}$$

The mass flow rate is determined by multiplying the molar flow rate by the molar mass of helium,

$$\dot{m}_{\text{diff}} = M\dot{N}_{\text{diff}} = (4\text{ kg/kmol})(1.80\times10^{-15}\text{ kmol/s}) = \mathbf{7.2\times10^{-15}\ kg/s}$$

Therefore, helium will leak out of the container through the shell by diffusion at a rate of 7.2×10^{-15} kg/s or 0.00023 g/year.

Discussion Note that the concentration of helium in the pyrex at the inner surface depends on the temperature and pressure of the helium in the tank, and can be determined as explained in the previous example. Also, the assumption of zero helium concentration in pyrex at the outer surface is reasonable since there is only a trace amount of helium in the atmosphere (0.5 parts per million by mole numbers).

14-47 A thin plastic membrane separates hydrogen from air. The diffusion rate of hydrogen by diffusion through the membrane under steady conditions is to be determined.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentrations on both sides of the membrane are maintained constant. Also, there is symmetry about the center plane of the membrane. **2** There are no chemical reactions in the membrane that results in the generation or depletion of hydrogen.

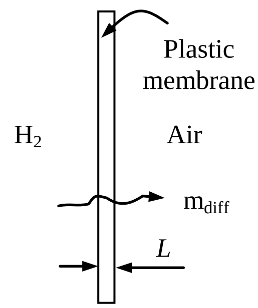
Properties The binary diffusion coefficient of hydrogen in the plastic membrane at the operation temperature is given to be 5.3×10^{-10} m^2/s. The molar mass of hydrogen is M = 2 kg/kmol (Table A-1).

Analysis (*a*) We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the plastic membrane to be a *stationary* medium since there is no diffusion of plastic molecules ($\dot{N}_B = 0$) and the concentration of the hydrogen in the membrane is extremely low (C_A << 1). Then the molar flow rate of hydrogen through the membrane by diffusion per unit area is determined from

$$\bar{j}_{\text{diff}} = \frac{\dot{N}_{\text{diff}}}{A} = D_{AB}\frac{C_{A,1}-C_{A,2}}{L}$$

$$= (5.3\times10^{-10}\ \text{m}^2/\text{s})\frac{(0.065-0.003)\,\text{kmol/m}^3}{2\times10^{-3}\,\text{m}}$$

$$= 1.64\times10^{-8}\ \text{kmol/m}^2.\text{s}$$



The mass flow rate is determined by multiplying the molar flow rate by the molar mass of hydrogen,

$$\dot{m}_{\text{diff}} = M\,\bar{j}_{\text{diff}} = (2\,\text{kg/kmol})(1.64\times10^{-8}\,\text{kmol/m}^2.\text{s})$$

$$= \mathbf{3.29\times10^{-8}\ kg/m^2.s}$$

(*b*) Repeating the calculations for a 0.5-mm thick membrane gives

$$\bar{j}_{\text{diff}} = \frac{\dot{N}_{\text{diff}}}{A} = D_{AB}\frac{C_{A,1}-C_{A,2}}{L}$$

$$= (5.3\times10^{-10}\ \text{m}^2/\text{s})\frac{(0.065-0.003)\,\text{kmol/m}^3}{0.5\times10^{-3}\,\text{m}}$$

$$= 6.57\times10^{-8}\ \text{kmol/m}^2.\text{s}$$

and

$$\dot{m}_{\text{diff}} = M\,\bar{j}_{\text{diff}} = (2\,\text{kg/kmol})(6.57\times10^{-8}\,\text{kmol/m}^2.\text{s}) = \mathbf{1.31\times10^{-7}\ kg/m^2.s}$$

The mass flow rate through the entire membrane can be determined by multiplying the mass flux value above by the membrane area.

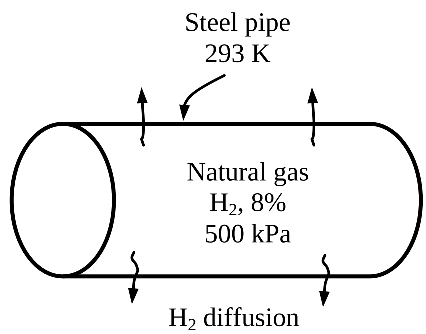
14-48 Natural gas with 8% hydrogen content is transported in an above ground pipeline. The highest rate of hydrogen loss through the pipe at steady conditions is to be determined.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentrations inside the pipe is constant, and in the atmosphere it is negligible. Also, there is symmetry about the centerline of the pipe. **2** There are no chemical reactions in the pipe that results in the generation or depletion of hydrogen. **3** Both H_2 and CH_4 are ideal gases.

Properties The binary diffusion coefficient of hydrogen in the steel pipe at the operation temperature is given to be 2.9×10^{-13} m^2/s. The molar masses of H_2 and CH_4 are 2 and 16 kg/kmol, respectively (Table A-1). The solubility of hydrogen gas in steel is given as $w_{H_2} = 2.09\times10^{-4}\exp(-3950/T)P_{H_2}^{0.5}$. The density of steel pipe is 7854 kg/m^3.

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the steel pipe to be a *stationary* medium since there is no diffusion of steel molecules ($\dot{N}_B = 0$) and the concentration of the hydrogen in the steel pipe is extremely low (C_A << 1). The molar mass of the H_2 and CH_4 mixture in the pipe is

$$M = \sum y_i M_i = (0.08)(2) + (0.92)(16) = 14.88 \text{ kg/kmol}$$



Noting that the mole fraction of hydrogen is 0.08, the partial pressure of hydrogen is

$$y_{H_2} = \frac{P_{H_2}}{P} \rightarrow P_{H_2} = (0.08)(500 \text{ kPa}) = 40 \text{ kPa} = 0.4 \text{ bar}$$

Then the mass fraction of hydrogen becomes

$$\begin{aligned} w_{H_2} &= 2.09\times10^{-4}\exp(-3950/T)P_{H_2}^{\ 0.5} \\ &= 2.09\times10^{-4}\exp(-3950/293)(0.4)^{0.5} \\ &= 1.85\times10^{-10} \end{aligned}$$

The hydrogen concentration in the atmosphere is practically zero, and thus in the limiting case the hydrogen concentration at the outer surface of pipe can be taken to be zero. Then the highest rate of hydrogen loss through a 100 m long section of the pipe at steady conditions is determined to be

$$\begin{aligned} \dot{m}_{\text{diff,A,cyl}} &= 2\pi L\rho D_{AB}\frac{w_{A,1} - w_{A,2}}{\ln(r_2/r_1)} \\ &= 2\pi(100\text{m})(7854\text{kg/m}^3)(2.9\times10^{-13})\frac{1.85\times10^{-10} - 0}{\ln(1.51/150)} \\ &= \mathbf{3.98\times10^{-14}\ kg/s} \end{aligned}$$

14-49 "!PROBLEM 14-49"

```
"GIVEN"
thickness=0.01 "[m]"
D_i=3 "[m]"
L=100 "[m]"
P=500 "[kPa]"
"y_H2=0.08 parameter to be varied"
T=293 "[K]"
D_AB=2.9E-13 "[m^2/s]"

"PROPERTIES"
MM_H2=molarmass(H2)
MM_CH4=molarmass(CH4)
R_u=8.314 "[kPa-m^3/kmol-K]"
rho=7854 "[kg/m^3]"

"ANALYSIS"
MM=y_H2*MM_H2+(1-y_H2)*MM_CH4
P_H2=y_H2*P*Convert(kPa, bar)
w_H2=2.09E-4*exp(-3950/T)*P_H2^0.5
m_dot_diff=2*pi*L*rho*D_AB*w_H2/ln(r_2/r_1)*Convert(kg/s, g/s)
r_1=D_i/2
r_2=r_1+thickness
```

y_{H2}	m_{diff} [g/s]
0.05	3.144E-11
0.06	3.444E-11
0.07	3.720E-11
0.08	3.977E-11
0.09	4.218E-11
0.1	4.446E-11
0.11	4.663E-11
0.12	4.871E-11
0.13	5.070E-11
0.14	5.261E-11
0.15	5.446E-11

14-50 Helium gas is stored in a spherical fused silica container. The diffusion rate of helium through the container and the pressure drop in the tank in one week as a result of helium loss are to be determined.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the container is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the container. **2** There are no chemical reactions in the fused silica that results in the generation or depletion of helium. **3** Helium is an ideal gas. **4** The helium concentration at the inner surface of the container is at the highest possible level (the solubility).

Properties The solubility of helium in fused silica (SiO_2) at 293 K and 500 kPa is 0.00045 kmol /m^3.bar (Table 14-7). The diffusivity of hydrogen in fused silica at 293 K (actually, at 298 K) is 4×10^{-14} m^2/s (Table 14-3b). The molar mass of helium is M = 4 kg/kmol (Table A-1).

Analysis (*a*) We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the container to be a *stationary* medium since there is no diffusion of silica molecules ($\dot{N}_B = 0$) and the concentration of the helium in the container is extremely low (C_A << 1). The molar concentration of helium at the inner surface of the container is determined from the solubility data to be

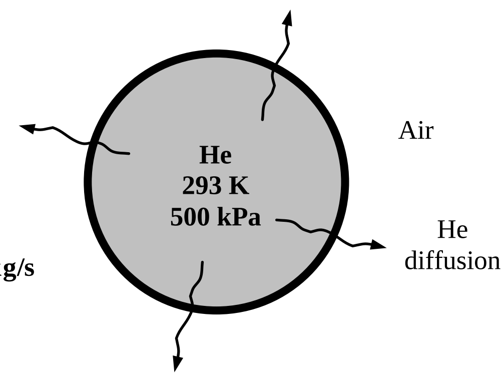
$$C_{A,1} = S \times P_{\text{He}} = (0.00045\text{kmol/m}^3\text{.bar})(5\text{bar}) = 2.25\times10^{-3}\text{ kmol/m}^3 = 0.00225\text{kmol/m}^3$$

The hydrogen concentration in the atmosphere and thus at the outer surface is taken to be zero since the tank is well ventilated. Then the molar flow rate of helium through the tank by diffusion becomes

$$\dot{N}_{\text{diff}} = 4\pi r_1 r_2 D_{AB}\frac{C_{A,1}-C_{A,2}}{r_2-r_1}$$

$$= 4\pi(1\text{m})(1.01\text{m})(4\times10^{-14}\text{ m}^2\text{/s})\frac{(0.00225\text{-}0)\text{kmol/m}^3}{(1.01\text{-}1)\text{m}}$$

$$= 1.14\times10^{-13}\text{ kmol/s}$$



The mass flow rate is determined by multiplying the molar flow rate by the molar mass of helium,

$$\dot{m}_{\text{diff}} = M\dot{N}_{\text{diff}} = (4\text{ kg/kmol})(1.14\times10^{-13}\text{ kmol/s}) = \mathbf{4.57\times10^{-13}\text{ kg/s}}$$

(*b*) Noting that the molar flow rate of helium is 1.14 $\times10^{-13}$ kmol / s, the amount of helium diffused through the shell in 1 week becomes

$$N_{\text{diff}} = N_{\text{diff}}\Delta t = (1.14\times10^{-13}\text{ kmol / s})(7\times24\times3600\text{ s / week})$$

$$= 6.908\times10^{-8}\text{ kmol / week}$$

The volume of the spherical tank and the initial amount of helium gas in the tank are

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1\text{m})^3 = 4.18\text{m}^3$$

$$N_{\text{initial}} = \frac{PV}{R_u T} = \frac{(500\text{kPa})(4.18\text{m}^3)}{(8.314\text{kPam}^3\text{/kmolK})(293\text{K})} = 0.85796\text{kmol}$$

Then the number of moles of helium remaining in the tank after one week becomes

$$N_{\text{final}} = N_{\text{initial}} - N_{\text{diff}} = 0.85796 - 6.908\times10^{-8} \cong 0.85796\text{ kPa}$$

which is the practically the same as the initial value (in 6 significant digits). Therefore, the amount of helium that leaves the tank by diffusion is negligible, and the final pressure in the tank is the same as the initial pressure of $P_2 = P_1$ = **500 kPa.**

14-51 A balloon is filled with helium gas. The initial rates of diffusion of helium, oxygen, and nitrogen through the balloon and the mass fraction of helium that escapes during the first 5 h are to be determined.

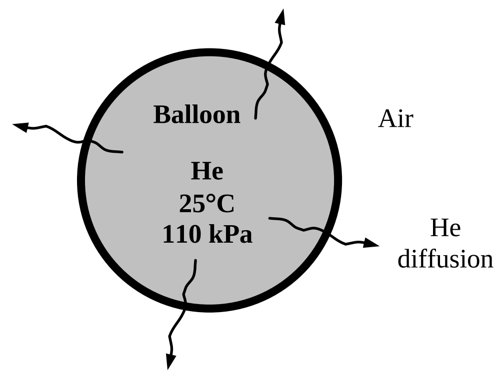
Assumptions **1** The pressure of helium inside the balloon remains nearly constant. **2** Mass diffusion is *steady* for the time period considered. **3** Mass diffusion is *one-dimensional* since the helium concentration in the balloon and thus at the inner surface is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the balloon. **4** There are no chemical reactions in the balloon that results in the generation or depletion of helium. **5** Both the helium and the air are ideal gases. **7** The curvature effects of the balloon are negligible so that the balloon can be treated as a plane layer.

Properties The permeability of rubber to helium, oxygen, and nitrogen at 25°C are given to be 9.4×10^{-13}, 7.05×10^{-13}, and 2.6×10^{-13} kmol/m.s.bars, respectively. The molar mass of helium is M = 4 kg/kmol and its gas constant is R = 2.0709 kPa.m^3/kg.K (Table A-1).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the balloon to be a *stationary* medium since there is no diffusion of rubber molecules ($\dot{N}_B = 0$) and the concentration of the helium in the balloon is extremely low (C_A << 1). The partial pressures of oxygen and nitrogen in the air are

$$P_{N_2} = y_{N_2}P = (0.79)(100 \text{ kPa}) = 79 \text{ kPa} = 0.79 \text{ bar}$$

$$P_{O_2} = y_{O_2}P = (0.21)(100 \text{ kPa}) = 21 \text{ kPa} = 0.21 \text{ bar}$$



The partial pressure of helium in the air is negligible. Since the balloon is filled with pure helium gas at 110 kPa, the initial partial pressure of helium in the balloon is 110 kPa, and the initial partial pressures of oxygen and nitrogen are zero.

When permeability data is available, the molar flow rate of a gas through a solid wall of thickness L under steady one-dimensional conditions can be determined from Eq. 14-29,

$$\dot{N}_{\text{diff,A,wall}} = \mathcal{P}_{AB}A\frac{P_{A,1} - P_{A,2}}{L} \qquad \text{(kmol/s)}$$

where $\mathcal{P}_{AB}$ is the permeability and $P_{A,1}$ and $P_{A,2}$ are the partial pressures of gas A on the two sides of the wall (Note that the balloon can be treated as a plain layer since its thickness is very small compared to its diameter). Noting that the surface area of the balloon is $A = \pi D^2 = \pi(0.15 \text{ m})^2 = 0.07069 \text{ m}^2$, the initial rates of diffusion of helium, oxygen, and nitrogen at 25ºC are determined to be

$$\dot{N}_{\text{diff,He}} = \mathcal{P}_{AB}A\frac{P_{He,1} - P_{He,2}}{L}$$

$$= (9.4\times10^{-13} \text{kmol/m.s.bar})(0.07069 \text{ m}^2)\frac{(1.1-0)\text{bar}}{0.1\times10^{-3}\text{m}} = \mathbf{0.731\times10^{-9} \text{ kmol/s}}$$

$$\dot{N}_{\text{diff},O_2} = \mathcal{P}_{AB}A\frac{P_{O_2,1} - P_{O_2,2}}{L}$$

$$= (7.05\times10^{-13} \text{kmol/m.s.bar})(0.07069 \text{ m}^2)\frac{(0-0.21)\text{bar}}{0.1\times10^{-3}\text{m}} = \mathbf{-0.105\times10^{-9} \text{ kmol/s}}$$

$$\dot{N}_{\text{diff},N_2} = \mathcal{P}_{AB}A\frac{P_{N_2,1} - P_{N_2,2}}{r_2 - r_1}$$

$$= (2.06\times10^{-13} \text{kmol/m.s.bar})(0.07069 \text{ m}^2)\frac{(0-0.79)\text{bar}}{0.1\times10^{-3}\text{m}} = \mathbf{-0.115\times10^{-9} \text{ kmol/s}}$$

The initial mass flow rate of helium and the amount of helium that escapes during the first 5 hours are

$$\dot{m}_{diff,\text{He}} = M\,\dot{N}_{diff,\text{He}} = (4 \text{ kg/kmol})(0.731\times10^{-9} \text{ kmol/s}) = 2.92\times10^{-9} \text{ kg/s}$$

$$m_{\text{diff,He}} = \dot{m}_{\text{diff,He}}\Delta t = (2.92\times10^{-9}\ \text{kg/s})(5\times3600\ \text{s}) = \mathbf{5.26\times10^{-5}\ kg = 0.0526\ g}$$

The initial mass of helium in the balloon is

$$m_{\text{initial}} = \frac{PV}{RT} = \frac{(110\,\text{kPa})[4\pi(0.075\,\text{m})^3/3]}{(2.0709\,\text{kPa.m}^3/\text{kg}\cdot\text{K})(298\,\text{K})} = 3.15\times10^{-4}\ \text{kg} = 0.315\,\text{g}$$

Therefore, the fraction of helium that escapes the balloon during the first 5 h is

$$\text{Fraction} = \frac{m_{\text{diff,He}}}{m_{\text{initial}}} = \frac{0.0526\,\text{g}}{0.315\ \text{g}} = \mathbf{0.167}\ (\text{or } \mathbf{16.7\%})$$

Discussion This is a significant amount of helium gas that escapes the balloon, and explains why the helium balloons do not last long. Also, our assumption of constant pressure for the helium in the balloon is obviously not very accurate since 16.7% of helium is lost during the process.

14-52 A balloon is filled with helium gas. A relation for the variation of pressure in the balloon with time as a result of mass transfer through the balloon material is to be obtained, and the time it takes for the pressure in the balloon to drop from 110 to 100 kPa is to be determined.

Assumptions **1** The pressure of helium inside the balloon remains nearly constant. **2** Mass diffusion is *transient* since the conditions inside the balloon change with time. **3** Mass diffusion is *one-dimensional* since the helium concentration in the balloon and thus at the inner surface is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the balloon. **4** There are no chemical reactions in the balloon material that results in the generation or depletion of helium. **5** Helium is an ideal gas. **6** The diffusion of air into the balloon is negligible. **7** The volume of the balloon is constant. **8** The curvature effects of the balloon are negligible so that the balloon material can be treated as a plane layer.

Properties The permeability of rubber to helium at 25°C is given to be 9.4×10^{-13} kmol/m.s.bar. The molar mass of helium is M = 4 kg/kmol and its gas constant is R = 2.0709 kPa.m^3/kg.K (Table A-1).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the balloon to be a *stationary* medium since there is no diffusion of rubber molecules ($\dot{N}_B = 0$) and the concentration of the helium in the balloon is extremely low (C_A << 1). The partial pressure of helium in the air is negligible. Since the balloon is filled with pure helium gas at 110 kPa, the initial partial pressure of helium in the balloon is 110 kPa.

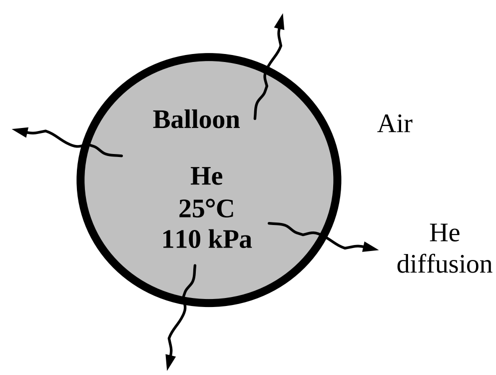
When permeability data is available, the molar flow rate of a gas through a solid wall of thickness L under steady one-dimensional conditions can be determined from Eq. 14-29,

$$\dot{N}_{\text{diff,A,wall}} = \mathcal{P}_{AB} A \frac{P_{A,1} - P_{A,2}}{L} = \mathcal{P}_{AB} A \frac{P}{L} \qquad \text{(kmol/s)}$$

where $\mathcal{P}_{AB}$ is the permeability and $P_{A,1}$ and $P_{A,2}$ are the partial pressures of helium on the two sides of the wall (note that the balloon can be treated as a plain layer since its thickness very small compared to its diameter, and $P_{A,1}$ is simply the pressure P of helium inside the balloon).

Noting that the amount of helium in the balloon can be expressed as $N = PV / R_u T$ and taking the temperature and volume to be constants,

$$N = \frac{PV}{R_u T} \;\rightarrow\; \frac{dN}{dt} = \frac{V}{R_u T}\frac{dP}{dt} \;\rightarrow\; \frac{dP}{dt} = \frac{R_u T}{V}\frac{dN}{dt} \qquad (1)$$



Conservation of mass dictates that the mass flow rate of helium from the balloon be equal to the rate of change of mass inside the balloon,

$$\frac{dN}{dt} = -\dot{N}_{\text{diff,A,wall}} = -\mathcal{P}_{AB} A \frac{P}{L} \qquad (2)$$

Substituting (2) into (1),

$$\frac{dP}{dt} = \frac{R_u T}{V}\frac{dN}{dt} = -\frac{R_u T}{V}\mathcal{P}_{AB} A \frac{P}{L} = -\frac{R_u T \mathcal{P}_{AB} A}{VL} P$$

Separating the variables and integrating gives

$$\frac{dP}{P} = -\frac{R_u T \mathcal{P}_{AB} A}{VL} dt \;\rightarrow\; \ln P\Big|_{P_0}^{P} = -\frac{R_u T \mathcal{P}_{AB} A}{VL}\, t\Big|_0^t \;\rightarrow\; \ln\frac{P}{P_0} = -\frac{R_u T \mathcal{P}_{AB} A}{VL} t$$

Rearranging, the desired relation for the variation of pressure in the balloon with time is determined to be

$$P = P_0 \exp\left(-\frac{R_u T \mathcal{P}_{AB} A}{VL} t\right) = P_0 \exp\left(-\frac{3 R_u T \mathcal{P}_{AB}}{rL} t\right) \qquad \text{since, for a sphere, } \frac{A}{V} = \frac{4\pi r^2}{4\pi r^3/3} = \frac{3}{r}$$

Then the time it takes for the pressure inside the balloon to drop from 110 kPa to 100 kPa becomes

$$\frac{100\text{ kPa}}{110\text{ kPa}} = \exp\left(-\frac{3(0.08314\text{ bar}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(298\text{ K})(9.4\times10^{-13}\text{ kmol/m}\cdot\text{s}\cdot\text{bar})}{(0.075\text{ m})(0.1\times10^{-3}\text{ m})} t\right) \rightarrow t = 10{,}230\text{ s} = \mathbf{2.84\,h}$$

Therefore, the balloon will lose 10% of its pressure in about 3 h.

14-53 Pure N_2 gas is flowing through a rubber pipe. The rate at which N_2 leaks out by diffusion is to be determined for the cases of vacuum and atmospheric air outside.

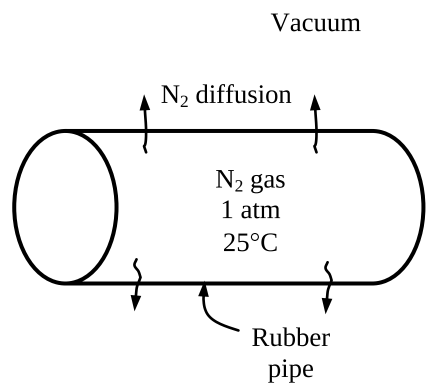
Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the nitrogen concentration in the pipe and thus at the inner surface of the pipe is practically constant, and the nitrogen concentration in the atmosphere also remains constant. Also, there is symmetry about the centerline of the pipe. **2** There are no chemical reactions in the pipe that results in the generation or depletion of nitrogen. **3** Both the nitrogen and air are ideal gases.

Properties The diffusivity and solubility of nitrogen in rubber at 25°C are 1.5×10^{-10} m^2/s and 0.00156 9 $kmol/m^3.bar$, respectively (Tables 14-3 and 14-7).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the container to be a *stationary* medium since there is no diffusion of silica molecules ($\dot{N}_B = 0$) and the concentration of the helium in the container is extremely low ($C_A << 1$). The partial pressures of oxygen and nitrogen in the air are

$$P_{N_2} = y_{N_2}P = (0.79)(100\text{ kPa}) = 79\text{ kPa} = 0.79\text{ bar}$$

$$P_{O_2} = y_{O_2}P = (0.21)(100\text{ kPa}) = 21\text{ kPa} = 0.21\text{ bar}$$



The partial pressure of helium in the air is negligible. Since the balloon is filled with pure helium gas at 110 kPa, the initial partial pressure of helium in the balloon is 110 kPa, and the initial partial pressures of oxygen and nitrogen are zero.

When solubility data is available, the molar flow rate of a gas through a solid can be determined by replacing the molar concentration by $C_{A,\text{ solid side}}(0) = \mathcal{S}_{AB}P_{A,\text{ gas side}}(0)$ where S_{AB} is the *solubility* and $P_{A,1}$ and $P_{A,2}$ are the partial pressures of gas A on the two sides of the wall. For a cylindrical pipe the molar rate of diffusion can be expressed in terms of solubility as

$$\dot{N}_{\text{diff,A,cyl}} = 2\pi L D_{AB} \mathcal{S}_{AB} \frac{P_{A,1} - P_{A,2}}{\ln(r_2 / r_1)}$$

(*a*) The pipe is in vacuum and thus $P_{A,2} = 0$:

$$\dot{N}_{\text{diff,A,cyl}} = 2\pi(10\text{ m})(1.5\times10^{-10}\text{ m}^2/\text{s})(0.00156\text{ kmol/m}^3\cdot\text{s}\cdot\text{bar})\frac{(1-0)\text{ bar}}{\ln(0.031/0.03)}$$

$$= \mathbf{4.483\times10^{-10}\ kmol/s}$$

(*b*) The pipe is in atmospheric air and thus $P_{A,2}$ = 0.79 bar:

$$\dot{N}_{\text{diff,A,cyl}} = 2\pi(10\text{ m})(1.5\times10^{-10}\text{ m}^2/\text{s})(0.00156\text{ kmol/m}^3\cdot\text{s}\cdot\text{bar})\frac{(1-0.79)\text{ bar}}{\ln(0.031/0.03)}$$

$$= \mathbf{9.416\times10^{-11}\ kmol/s}$$

Discussion In the case of a vacuum environment, the diffusion rate of nitrogen from the pipe is about 5 times the rate in atmospheric air. This is expected since mass diffusion is proportional to the concentration difference.

Water Vapor Migration in Buildings

14-54C A tank that contains moist air at 3 atm is located in moist air that is at 1 atm. The driving force for moisture transfer is the vapor pressure difference, and thus it is possible for the water vapor to flow into the tank from surroundings if the vapor pressure in the surroundings is greater than the vapor pressure in the tank.

14-55C The mass flow rate of water vapor through a wall of thickness L in therms of the partial pressure of water vapor on both sides of the wall and the permeability of the wall to the water vapor can be expressed as

$$\dot{m}_{\text{diff,A,wall}} = \text{M}\mathcal{P}_{\text{AB}} A \frac{P_{\text{A},1} - P_{\text{A},2}}{L}$$

where M is the molar mass of vapor, $\mathcal{P}_{\text{AB}}$ is the permeability, A is the normal area, and P_{A} is the partial pressure of the vapor.

14-56C The condensation or freezing of water vapor in the wall increases the thermal conductivity of the insulation material, and thus increases the rate of heat transfer through the wall. Similarly, the thermal conductivity of the soil increases with increasing amount of moisture.

14-57C Vapor barriers are materials that are impermeable to moisture such as sheet metals, heavy metal foils, and thick plastic layers, and they completely *eliminate* the vapor migration. Vapor retarders such as reinforced plastics or metals, thin foils, plastic films, treated papers and coated felts, on the other hand, *slow down* the flow of moisture through the structures. Vapor retarders are commonly used in residential buildings to control the vapor migration through the walls.

14-58C Excess moisture changes the *dimensions* of wood, and cyclic changes in dimensions weaken the joints, and can jeopardize the structural integrity of building components, causing "squeaking" at the minimum. Excess moisture can also cause *rotting* in woods, *mold* growth on wood surfaces, *corrosion* and *rusting* in metals, and *peeling of paint* on the interior and exterior wall surfaces.

14-59C Insulations on *chilled water lines* are always wrapped with *vapor barrier jackets* to eliminate the possibility of vapor entering the insulation. This is because moisture that migrates through the insulation to the cold surface will condense and remain there indefinitely with no possibility of vaporizing and moving back to the outside.

14-60C When the temperature, total pressure, and the relative humidity are given, the vapor pressure can be determined from the psychrometric chart or the relation $P_v = \phi P_{\text{sat}}$ where P_{sat} is the saturation (or boiling) pressure of water at the specified temperature and ϕ is the relative humidity.

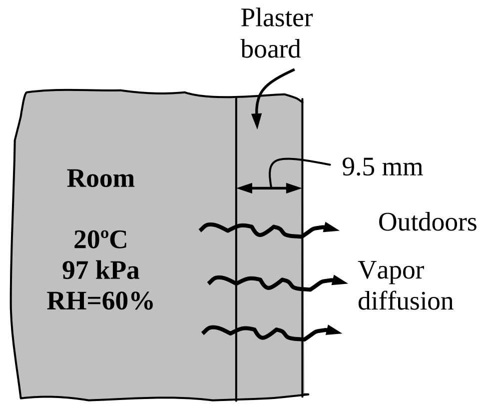
14-61 The inside wall of a house is finished with 9.5-mm thick gypsum wallboard. The maximum amount of water vapor that will diffuse through a 3 m × 8 m section of the wall in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeability of the wall is constant. **4** The vapor pressure at the outer side of the wallboard is zero.

Properties The permeance of the 9.5 mm thick gypsum wall board to water vapor is given to be 2.86×10^{-9} $kg/s.m^2.Pa$. (Table 14-10). The saturation pressure of water at 20ºC is 2339 Pa (Table 14-9).

Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given as (Eq. 14-31)

$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1}-P_{v,2}}{L} = \mathsf{P}A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{L} = \mathsf{M}A(\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2})$$



where P is the vapor permeability and M = P/L is the permeance of the material, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall.

Noting that the vapor pressure at the outer side of the wallboard is zero ($\phi_2 = 0$) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (2.86\times10^{-9}\ \text{kg/s.m}^2\text{.Pa})(3\times8\ \text{m}^2)[0.60(2339\ \text{Pa})-0] = 9.63\times10^{-5}\ \text{kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v\Delta t = (9.63\times10^{-5}\ \text{kg/s})(24\times3600\ \text{s}) = \mathbf{8.32\,kg}$$

Discussion This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero.

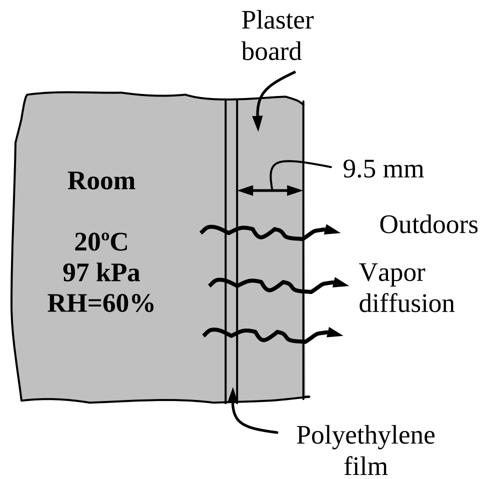
14-62 The inside wall of a house is finished with 9.5-mm thick gypsum wallboard with a 0.2-mm thick polyethylene film on one side. The maximum amount of water vapor that will diffuse through a 3 m × 8 m section of the wall in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeabilities of the wall and of the vapor barrier are constant. **4** The vapor pressure at the outer side of the wallboard is zero.

Properties The permeances of the 9.5 mm thick gypsum wall board and of the 0.2-mm thick polyethylene film are given to be 2.86×10^{-9} and 2.3×10^{-12} kg/s.m^2.Pa, respectively (Table 14-10). The saturation pressure of water at 20ºC is 2339 Pa (Table 14-9).

Analysis The mass flow rate of water vapor through a two-layer plain wall of normal area A is given as (Eqs. 14-33 and 14-35)

$$\dot{m}_v = A\frac{P_{v,1}-P_{v,2}}{R_{v,\text{total}}} = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}}$$



where $R_{v,\text{total}}$ is the total vapor resistance of the medium, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall. The total vapor resistance of the wallboard is

$$\begin{aligned} R_{v,\text{total}} &= R_{v,\text{wall}} + R_{v,\text{film}} \\ &= \frac{1}{2.86\times10^{-9}\ \text{kg/s.m}^2.\text{Pa}} + \frac{1}{2.3\times10^{-12}\ \text{kg/s.m}^2.\text{Pa}} \\ &= 4.35\times10^{11}\ \text{s.m}^2.\text{Pa/kg} \end{aligned}$$

Noting that the vapor pressure at the outer side of the wallboard is zero ($\phi_2 = 0$) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}} = (3\times8\ \text{m}^2)\frac{0.60(2339\ \text{Pa})-0}{4.35\times10^{11}\ \text{s.m}^2.\text{Pa/kg}} = 7.75\times10^{-8}\ \text{kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

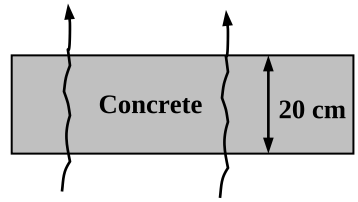
$$m_{v,24-h} = \dot{m}_v\Delta t = (7.75\times10^{-8}\ \text{kg/s})(24\times3600\ \text{s}) = \mathbf{0.00670\ kg = 6.7\ g}$$

Discussion This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero. Note that the vapor barrier reduced the amount of vapor migration to a negligible level.

14-63 The roof of a house is made of a 20-cm thick concrete layer. The amount of water vapor that will diffuse through a 15 m × 8 m section of the roof in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the roof is one-dimensional. **3** The vapor permeability of the roof is constant.

Properties The permeability of the roof to water vapor is given to be 24.7×10^{-12} kg/s.m.Pa. The saturation pressures of water are 768 Pa at 3ºC, and 3169 Pa at 25ºC (Table 14-9).



Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given as (Eq. 14-31)

$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1}-P_{v,2}}{L} = \mathsf{P}A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{L}$$

where P is the vapor permeability, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the roof. Substituting, the mass flow rate of water vapor through the roof is determined to be

$$\dot{m}_v = (24.7\times10^{-12}\ \text{kg/s.m.Pa})(15\times 8\ \text{m}^2)\frac{[0.50(3169\ \text{Pa})-0.30(768\ \text{Pa})]}{(0.20\ \text{m})} = 2.011\times10^{-5}\ \text{kg/s}$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v\Delta t = (2.011\times10^{-5}\ \text{kg/s})(24\times 3600\ \text{s}) = \mathbf{1.738\ kg}$$

Discussion The moisture migration through the roof can be reduced significantly by covering the roof with a vapor barrier or vapor retarder.

14-64 "!PROBLEM 14-64"

```
"GIVEN"
A=15*8 "[m^2]"
L=0.20 "[m]"
T_1=25 "[C], parameter to be varied"
"phi_1=0.50 parameter to be varied"
P_atm=100 "[kPa]"
time=24*3600 "[s]"
T_2=3 "[C]"
phi_2=0.30
Permeability=24.7E-12 "[kg/s-m-Pa]"

"PROPERTIES"
Fluid$='steam_NBS'
P_sat1=Pressure(Fluid$, T=T_1, x=1)*Convert(kPa, Pa)
P_sat2=Pressure(Fluid$, T=T_2, x=1)*Convert(kPa, Pa)

"ANALYSIS"
m_dot_v=Permeability*A*(phi_1*P_sat1-phi_2*P_sat2)/L
m_v=m_dot_v*time
```

T_1 [C]	m_v [kg]
15	0.8007
16	0.8731
17	0.9496
18	1.03
19	1.116
20	1.206
21	1.301
22	1.402
23	1.508
24	1.62
25	1.738
26	1.862
27	1.992
28	2.13
29	2.275
30	2.427

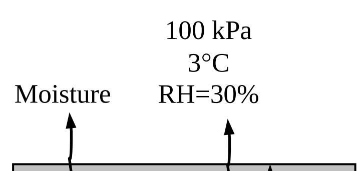
ϕ_1	m_v [kg]
0.3	0.9261
0.32	1.007
0.34	1.088
0.36	1.17
0.38	1.251
0.4	1.332
0.42	1.413
0.44	1.494
0.46	1.575
0.48	1.657
0.5	1.738
0.52	1.819
0.54	1.9
0.56	1.981
0.58	2.062

0.6	2.143
0.62	2.225
0.64	2.306
0.66	2.387
0.68	2.468
0.7	2.549

14-65 The roof of a house is made of a 20-cm thick concrete layer painted with a vapor retarder paint. The amount of water vapor that will diffuse through a 15 m × 8 m section of the roof in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the roof is one-dimensional. **3** The vapor permeabilities of the roof and of the vapor barrier are constant.

Properties The permeability of concrete to water vapor and the permeance of the vapor retarder to water vapor are given to be



24.7×10^{-12} kg/s.m.Pa and 26×10^{-12} kg/s.m^2.Pa, respectively. The saturation pressures of water are 768 Pa at 3ºC, and 3169 Pa at 25ºC (Table 14-9).

Analysis The mass flow rate of water vapor through a two-layer plain roof of normal area *A* is given as (Eqs. 14-33 and 14-35)

$$\dot{m}_v = A\frac{P_{v,1}-P_{v,2}}{R_{v,\text{total}}} = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}}$$

where $R_{\text{v,total}}$ is the total vapor resistance of the medium, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the roof. The total vapor resistance of the roof is

$$R_{v,\text{total}} = R_{v,\text{roof}} + R_{v,\text{film}} = \frac{L}{P}+\frac{1}{M} = \frac{0.20\text{ m}}{24.7\times10^{-12}\text{ kg/s.m.Pa}} + \frac{1}{26\times10^{-12}\text{ kg/s.m}^2\text{.Pa}}$$
$$= 4.66\times10^{10}\text{ s.m}^2\text{.Pa/kg}$$

Substituting, the mass flow rate of water vapor through the roof is determined to be

$$\dot{m}_v = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}} = (15\times8\text{ m}^2)\frac{0.50(3169\text{ Pa})-0.30(768\text{ Pa})}{4.66\times10^{10}\text{ s.m}^2\text{.Pa/kg}} = 3.49\times10^{-6}\text{ kg/s}$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v\Delta t = (3.49\times10^{-6}\text{ kg/s})(24\times3600\text{s}) = \mathbf{0.302\,kg = 302\,g}$$

14-66 A glass of milk left on top of a counter is tightly sealed by a sheet of 0.009-mm thick aluminum foil. The drop in the level of the milk in the glass in 12 h due to vapor migration through the foil is to be determined.

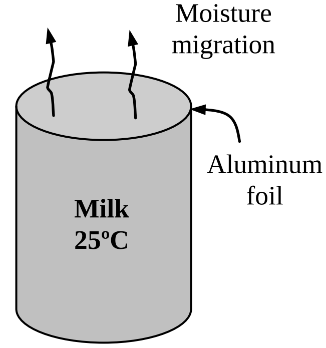
Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the foil is one-dimensional. **3** The vapor permeability of the foil is constant.

Properties The permeance of the foil to water vapor is given to be 2.9×10^{-12} kg/s.m^2.Pa. The saturation pressure of water at 25ºC is 3169 Pa (Table 14-9). We take the density of milk to be 1000 kg/m^3.

Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given as (Eq. 14-31)

$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1}-P_{v,2}}{L} = \mathsf{P}A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{L} = \mathsf{M}A(\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2})$$

where P is the vapor permeability and M = P/L is the permeance of the material, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the foil.



The diffusion area of the foil is $A=\pi r^2=\pi(0.06\text{ m})=0.0113\text{ m}^2$. Substituting, the mass flow rate of water vapor through the foil becomes

$$\begin{aligned}\dot{m}_v &= (2.9\times10^{-12}\text{ kg/s.m}^2\text{.Pa})(0.0113\text{m}^2)[1(3169\text{Pa})-0.5(768\text{Pa})]\\ &= 5.19\times10^{-11}\text{ kg/s}\end{aligned}$$

Then the total amount of moisture that flows through the foil during a 12-h period becomes

$$m_{v,48-h}=\dot{m}_v\Delta t=(5.19\times10^{-11}\text{ kg/s})(48\times3600\text{ s})=8.97\times10^{-6}\text{ kg}$$

$$V=m/\rho=(8.97\times10^{-6}\text{ kg})/(1000\text{ kg/m}^3)=8.97\times10^{-9}\text{ m}^3$$

Then the drop in the level of the milk becomes

$$\Delta h=\frac{V}{A}=\frac{8.97\times10^{-9}\text{ m}^3}{0.0113\text{ m}^2}=7.9\times10^{-7}\text{ m}=\mathbf{0.00079\text{ mm}}$$

Discussion The drop in the level of the milk in 48 h is much less than 1 mm, and thus it is not noticeable.

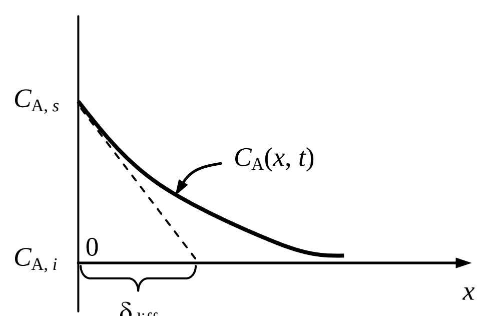
Transient Diffusion

14-67C The diffusion of a solid species into another solid of finite thickness can usually be treated as a diffusion process in a semi-infinite medium regardless of the shape and thickness of the solid since the diffusion process affects a very thin layer at the surface.

14-68C The penetration depth is defined as the location where the tangent to the concentration profile at the surface (x = 0) intercepts the $C_A=C_{A,i}$ line, and it represents the depth of diffusion at a given time. The penetration depth can be determined to be

$$\delta_{diff}=\sqrt{\pi D_{AB}t}$$

where D_{AB} is the diffusion coefficient and t is the time.



14-69C When the density of a species A in a semi-infinite medium is known initially and at the surface, the concentration of the species A at a specified location and time can be determined from

$$\frac{C_A(x,t)-C_{A,i}}{C_{A,s}-C_{A,i}}=\text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

where $C_{A,i}$ is the initial concentration of species *A* at time $t = 0$, $C_{A,s}$ is the concentration at the inner side of the exposed surface of the medium, and erfc is the complementary error function.

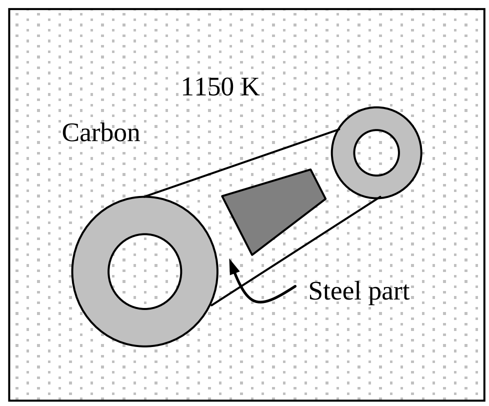
14-70 A steel component is to be surface hardened by packing it in a carbonaceous material in a furnace at 1150 K. The length of time the component should be kept in the furnace is to be determined.

Assumptions **1** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial carbon concentration in the steel component is uniform. **3** The carbon concentration at the surface remains constant.

Properties The relevant properties are given in the problem statement.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x,t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$



Substituting the specified quantities gives

$$\frac{0.0032 - 0.0012}{0.011 - 0.0012} = 0.204 = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The argument whose complementary error function is 0.204 is determined from Table 4-3 to be 0.742. That is,

$$\frac{x}{2\sqrt{D_{AB}\ t}} = 0.742$$

Then solving for the time *t* gives

$$t = \frac{x^2}{4D_{AB}(0.742)^2} = \frac{(0.0007\text{m})^2}{4 \times (7.2 \times 10^{-12}\ \text{m}^2/\text{s})(0.742)^2} = 32{,}458\text{s} \cong \mathbf{9h}$$

Therefore, the steel component must be held in the furnace for 9 h to achieve the desired level of hardening.

Discussion The diffusion coefficient of carbon in steel increases exponentially with temperature, and thus this process is commonly done at high temperatures to keep the diffusion time to a reasonable level.

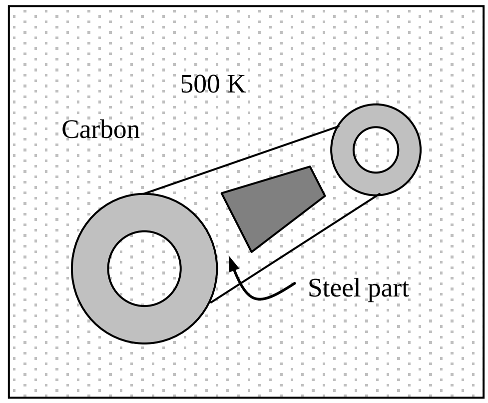
14-71 A steel component is to be surface hardened by packing it in a carbonaceous material in a furnace at 5000 K. The length of time the component should be kept in the furnace is to be determined.

Assumptions **1** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial carbon concentration in the steel component is uniform. **3** The carbon concentration at the surface remains constant.

Properties The relevant properties are given in the problem statement.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x,t)-w_{A,i}}{w_{A,s}-w_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$



Substituting the specified quantities gives

$$\frac{0.0032-0.0012}{0.011-0.0012} = 0.204 = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The argument whose complementary error function is 0.204 is determined from Table 4-3 to be 0.742. That is,

$$\frac{x}{2\sqrt{D_{AB}\ t}} = 0.742$$

Solving for the time t gives

$$t = \frac{x^2}{4D_{AB}(0.742)^2} = \frac{(0.0007\text{m})^2}{4\times(2.1\times10^{-20}\ \text{m}^2/\text{s})(0.742)^2} = \mathbf{1.06\times10^{13}\ s = 336{,}000\ years}$$

Therefore, the steel component must be held in the furnace forever to achieve the desired level of hardening.

Discussion The diffusion coefficient of carbon in steel increases exponentially with temperature, and thus this process is commonly done at high temperatures to keep the diffusion time to a reasonable level.

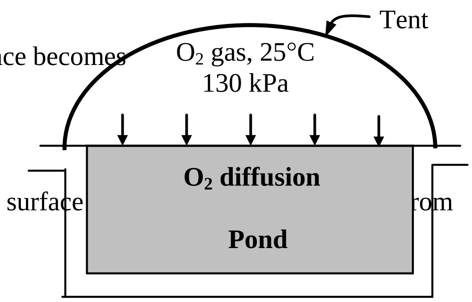
14-72 A pond is to be oxygenated by forming a tent over the water surface and filling the tent with oxygen gas. The molar concentration of oxygen at a depth of 2 cm from the surface after 12 h is to be determined.

Assumptions **1** The oxygen in the tent is saturated with water vapor. **2** Oxygen penetrates into a thin layer at the pond surface, and thus the pond can be modeled as a semi-infinite medium. **3** Both the water vapor and oxygen are ideal gases. **4** The initial oxygen content of the pond is zero.

Properties The diffusion coefficient of oxygen in water at 25ºC is D_{AB} = 2.4 $\times 10^{-9}$ m^2/s (Table 14-3a). Henry's constant for oxygen dissolved in water at 300 K (≅ 25ºC) is given in Table 14-6 to be H = 43,600 bar. The saturation pressure of water at 25ºC is 3.2 kPa (Table 14-9).

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. The vapor pressure in the tent is the saturation pressure of water at 25ºC since the oxygen in the tent is saturated, and thus the partial pressure of oxygen in the tank is

$$P_{O_2} = P - P_v = 130 - 3.17 = 126.83 \text{ kPa}$$



Then the mole fraction of oxygen in the water at the pond surface becomes

$$y_{O_2,\text{liquid side}}(0) = \frac{P_{O_2,\text{gas side}}(0)}{H} = \frac{1.2683 \text{ bar}}{43{,}600 \text{ bar}} = 2.91\times 10^{-5}$$

The molar concentration of oxygen at a depth of 2 cm from the surface … from

$$\frac{y_{O_2}(x,t) - y_{O_2,i}}{y_{O_2,s} - y_{O_2,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting,

$$\frac{y_{O_2}(x,t) - 0}{2.88\times 10^{-5} - 0} = \text{erfc}\left(\frac{0.02\text{m}}{2\sqrt{(2.4\times 10^{-9}\text{ m}^2/\text{s})(12\times 3600\text{s})}}\right) \longrightarrow y_{O_2}(0.02\text{m},12\text{h}) = \mathbf{4.77\times 10^{-6}}$$

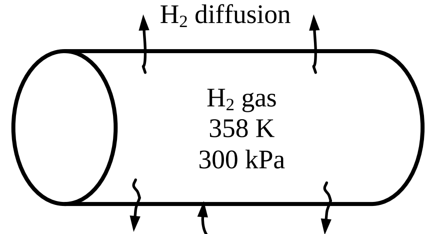
Therefore, there will be 4.77 moles of oxygen per million at a depth of 2 cm from the surface in 12 h.

14-73 A long cylindrical nickel bar saturated with hydrogen is taken into an area that is free of hydrogen. The length of time for the hydrogen concentration at the center of the bar to drop by half is to be determined.

Assumptions **1** The bar can be treated as an infinitely long cylinder since it is very long and there is symmetry about the centerline. **2** The initial hydrogen concentration in the steel bar is uniform. **3** The hydrogen concentration at the surface remains constant at zero at all times. **4** The Fourier number is $\tau > 0.2$ so that the one-term transient solutions are valid.

Properties The molar mass of hydrogen H_2 is M = 2 kg / kmol (Table A-1). The solubility of hydrogen in nickel at 358 K is 0.00901 kmol / m^3.bar (Table 14-7). The diffusion coefficient of hydrogen in nickel at 358 K is $D_{AB} = 1.2\times10^{-12}$ m^2/s (Table 14-3b).

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in an infinitely long cylinder with specified surface temperature, and thus can be solved accordingly. Noting that 300 kPa = 3 bar, the molar density of hydrogen in the nickel bar before it is taken out of the storage room is



$$C_{\mathrm{H_2,solid\,side}}(0) = S \times P_{\mathrm{H_2,gas\,side}}$$
$$= (0.00901\,\mathrm{kmol/m^3.bar})(3\,\mathrm{bar})$$
$$= 0.027\,\mathrm{kmol/m^3}$$

The molar concentration of hydrogen at the center of the bar can be calculated from

$$\frac{C_{\mathrm{H_2},o} - C_{\mathrm{H_2},\infty}}{C_{\mathrm{H_2},i} - C_{\mathrm{H_2},\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

The Biot number in this case can be taken to be infinity since the bar is in a well-ventilated area during the transient case. The constants A_1 and λ_1 for the infinite Bi are determined from Table 4-1 to be 1.6021 and 2.4048, respectively. Noting that the concentration of hydrogen at the outer surface is zero, and the concentration of hydrogen at the center of the bar is one half of the initial concentration, the Fourier number, τ, can be determined from

$$\frac{(0.027/2) - 0}{0.027 - 0} = 1.6021 e^{-(2.4048)^2 \tau} \longrightarrow \tau = 0.2014$$

Using the definition of the Fourier number, the time required to drop the concentration of hydrogen by half is determined to be

$$\tau = \frac{D_{AB}\,t}{r_o^{\,2}} \longrightarrow t = \frac{\tau r_o^{\,2}}{D_{AB}} = \frac{(0.2014)(0.025)^2}{1.2\times10^{-12}} = 1.049\times10^8\ \mathrm{s} = 1214\,\mathrm{days} = \mathbf{3.33\,years}$$

Therefore, it will take years for this nickel bar to be free of hydrogen.