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سایت آموزش مهندسی مکانیک

Diffusion in a Moving Medium

14-74C The **mass-average velocity** of a medium at some location is the average velocity of the mass at that location relative to an external reference point. It is the velocity that would be measured by a velocity sensor such as a pitot tube, a turbine device, or a hot wire anemometer inserted into the flow. The **diffusion velocity** at a location is the average velocity of a group of molecules at that location moving under the influence of concentration gradient. A **stationary medium** is a medium whose mass average velocity is zero. A **moving medium** is a medium that involves a bulk fluid motion caused by an external force.

14-75C The **diffusion velocity** at a location is the average velocity of a group of molecules at that location moving under the influence of concentration gradient. The average velocity of a species in a moving medium is equal to the sum of the bulk flow velocity and the diffusion velocity. Therefore, the diffusion velocity can increase or decrease the average velocity, depending on the direction of diffusion relative to the direction of bulk flow. The velocity of a species in the moving medium relative to a fixed reference point **will be zero** when the diffusion velocity of the species and the bulk flow velocity are equal in magnitude and opposite in direction.

14-76 C The **mass-average velocity** of a medium at some location is the average velocity of the mass at that location relative to an external reference point. The **molar-average velocity** of a medium at some location is the average velocity of the molecules at that location, regardless of their mass, relative to an external reference point. If one of these velocities are zero, the other will not necessarily be zero. The mass-average and molar-average velocities of a binary mixture will be the same when the molar masses of the two constituents are equal to each other. The mass and mole fractions of each species in this case will be the same.

14-77C (a) T, (b) T, (c) F, (d) F

14-78C The diffusion of a vapor through a stationary gas column is called the **Stefan flow**. The **Stefan's law** can be expressed as

$$\bar{j}_A = \dot{N}_A / A = \frac{CD_{AB}}{L} \ln \frac{1 - y_{A,L}}{1 - y_{A,o}}$$

where C is the average concentration of the mixture, D_{AB} is the diffusion coefficient of A in B , L is the height of the gas column, $y_{A,L}$ is the molar concentration of a species at $x = L$, and $y_{A,o}$ is the molar concentration of the species A at $x = 0$.

14-79E The pressure in a helium pipeline is maintained constant by venting to the atmosphere through a long tube. The mass flow rates of helium and air, and the net flow velocity at the bottom of the tube are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Helium and atmospheric air are ideal gases. 3 No chemical reactions occur in the tube. 4 Air concentration in the pipeline and helium concentration in the atmosphere are negligible so that the mole fraction of the helium is 1 in the pipeline, and 0 in the atmosphere (we will check this assumption later).

Properties The diffusion coefficient of helium in air (or air in helium) at normal atmospheric conditions is $D_{AB} = 7.75 \times 10^{-4} \text{ ft}^2/\text{s}$ (Table 14-2). The molar mass of helium is $M = 4 \text{ lbm} / \text{lbmol}$, and the molar mass of air is $29 \text{ lbm} / \text{lbmol}$ (Table A-1E).

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant.

(a) The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2 / 4 = \pi (0.25 / 12 \text{ ft})^2 / 4 = 3.41 \times 10^{-4} \text{ ft}^2$$

Noting that the pressure of helium is 14.5 psia at the bottom of the tube ($x = 0$) and 0 at the top ($x = L$), its molar flow rate is

$$\begin{aligned} \dot{N}_{\text{helium}} &= \dot{N}_{\text{diff,A}} = \frac{D_{AB} A}{R_u T} \frac{P_{A,0} - P_{A,L}}{L} \\ &= \frac{(7.75 \times 10^{-4} \text{ ft}^2/\text{s})(3.41 \times 10^{-4} \text{ ft}^2)}{(10.73 \text{ psia} \cdot \text{ft}^3/\text{lbmol} \cdot \text{R})(540 \text{ R})} \frac{(14.5 - 0) \text{ psia}}{30 \text{ ft}} \\ &= 2.20 \times 10^{-11} \text{ lbmol/s} \end{aligned}$$

Therefore, the mass flow rate of helium through the tube is

$$\dot{m}_{\text{helium}} = (\dot{N}M)_{\text{helium}} = (2.20 \times 10^{-11} \text{ lbmol/s})(4 \text{ lbm/lbmol}) = 8.80 \times 10^{-11} \text{ lbm/s}$$

which corresponds to 0.00278 lbm per year.

(b) Noting that $\dot{N}_B = -\dot{N}_A$ during an equimolar counterdiffusion process, the molar flow rate of air into the helium pipeline is equal to the molar flow rate of helium. Thus the mass flow rate of air into the pipeline is

$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-2.20 \times 10^{-11} \text{ lbmol/s})(29 \text{ lbm/lbmol}) = -6.38 \times 10^{-10} \text{ lbm/s}$$

The mass fraction of air in helium pipeline is

$$w_{\text{air}} = \frac{|\dot{m}_{\text{air}}|}{\dot{m}_{\text{total}}} = \frac{6.38 \times 10^{-11} \text{ lbm/s}}{(5 + 6.38 \times 10^{-10}) \text{ lbm/s}} = 1.28 \times 10^{-10} \approx 0$$

which validates our original assumption of negligible air in the pipeline.

(c) The net mass flow rate through the tube is

$$\dot{m}_{\text{net}} = \dot{m}_{\text{helium}} + \dot{m}_{\text{air}} = 8.80 \times 10^{-11} - 6.38 \times 10^{-10} = -5.50 \times 10^{-10} \text{ lbm/s}$$

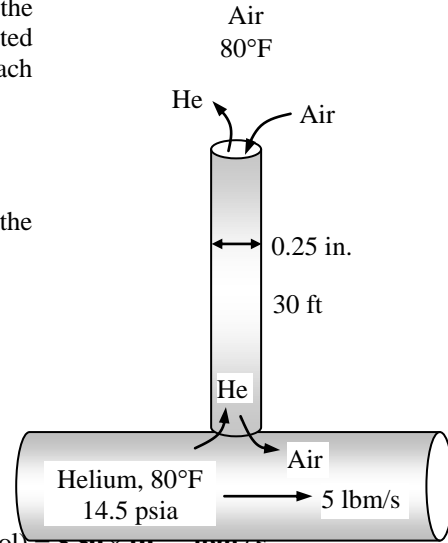
The mass fraction of air at the bottom of the tube is very small, as shown above, and thus the density of the mixture at $x = 0$ can simply be taken to be the density of helium which is

$$\rho \cong \rho_{\text{helium}} = \frac{P}{RT} = \frac{14.5 \text{ psia}}{(2.6805 \text{ psia} \cdot \text{ft}^3 / \text{lbm} \cdot \text{R})(540 \text{ R})} = 0.01002 \text{ lbm/ft}^3$$

Then the average flow velocity at the bottom part of the tube becomes

$$V = \frac{\dot{m}_{\text{net}}}{\rho A} = \frac{-5.50 \times 10^{-10} \text{ lbm/s}}{(0.01002 \text{ lbm/ft}^3)(3.41 \times 10^{-4} \text{ ft}^2)} = -1.61 \times 10^{-4} \text{ ft/s}$$

Discussion This flow rate is difficult to measure by even the most sensitive velocity measurement devices. The negative sign indicates flow in the negative x direction (towards the pipeline).



14-80E The pressure in a carbon dioxide pipeline is maintained constant by venting to the atmosphere through a long tube. The mass flow rates of carbon dioxide and air, and the net flow velocity at the bottom of the tube are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Carbon dioxide and atmospheric air are ideal gases. 3 No chemical reactions occur in the tube. 4 Air concentration in the pipeline and carbon dioxide concentration in the atmosphere are negligible so that the mole fraction of the carbon dioxide is 1 in the pipeline, and 0 in the atmosphere (we will check this assumption later).

Properties The diffusion coefficient of carbon dioxide in air (or air in carbon dioxide) at normal atmospheric conditions is $D_{AB} = 1.72 \times 10^{-4} \text{ ft}^2/\text{s}$ (Table 14-2). The molar mass of carbon dioxide is $M = 44 \text{ lbm/lbmol}$, and the molar mass of air is 29 lbm/lbmol (Table A-1E).

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant.

(a) The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2 / 4 = \pi (0.25 / 12 \text{ ft})^2 / 4 = 3.41 \times 10^{-4} \text{ ft}^2$$

Noting that the pressure of carbon dioxide is 14.5 psia at the bottom of the tube ($x = 0$) and 0 at the top ($x = L$), its molar flow rate is determined from Eq. 14-64 to be

$$\begin{aligned} \dot{N}_{\text{helium}} = \dot{N}_{\text{diff,A}} &= \frac{D_{AB} A}{R_u T} \frac{P_{A,0} - P_{A,L}}{L} \\ &= \frac{(1.72 \times 10^{-4} \text{ ft}^2/\text{s})(3.41 \times 10^{-4} \text{ ft}^2)}{(10.73 \text{ psia} \cdot \text{ft}^3/\text{lbmol} \cdot \text{R})(540 \text{ R})} \frac{(14.5 - 0) \text{ psia}}{30 \text{ ft}} \\ &= 4.89 \times 10^{-12} \text{ lbmol/s} \end{aligned}$$

Therefore, the mass flow rate of carbon dioxide through the tube is

$$\dot{m}_{\text{CO}_2} = (\dot{N}M)_{\text{CO}_2} = (4.89 \times 10^{-12} \text{ lbmol/s})(44 \text{ lbm/lbmol}) = \mathbf{2.15 \times 10^{-10} \text{ lbm/s}}$$

which corresponds to 0.00678 lbm per year.

(b) Noting that $\dot{N}_B = -\dot{N}_A$ during an equimolar counter diffusion process, the molar flow rate of air into the CO_2 pipeline is equal to the molar flow rate of CO_2 . Thus the mass flow rate of air into the pipeline is

$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-4.89 \times 10^{-12} \text{ lbmol/s})(29 \text{ lbm/lbmol}) = \mathbf{-1.42 \times 10^{-10} \text{ lbm/s}}$$

The mass fraction of air in carbon dioxide pipeline is

$$w_{\text{air}} = \frac{|\dot{m}_{\text{air}}|}{\dot{m}_{\text{total}}} = \frac{1.42 \times 10^{-10} \text{ lbm/s}}{(5 + 1.42 \times 10^{-10}) \text{ lbm/s}} = 2.84 \times 10^{-11} \approx 0$$

which validates our original assumption of negligible air in the pipeline.

(c) The net mass flow rate through the tube is

$$\dot{m}_{\text{net}} = \dot{m}_{\text{CO}_2} + \dot{m}_{\text{air}} = 2.15 \times 10^{-10} - 1.42 \times 10^{-10} = \mathbf{-7.3 \times 10^{-11} \text{ lbm/s}}$$

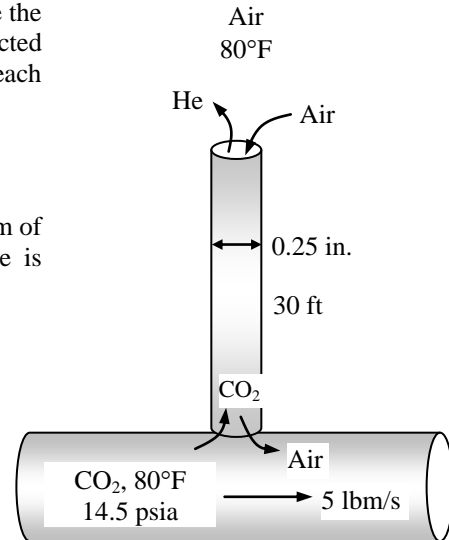
The mass fraction of air at the bottom of the tube is very small, as shown above, and thus the density of the mixture at $x = 0$ can simply be taken to be the density of carbon dioxide which is

$$\rho \cong \rho_{\text{CO}_2} = \frac{P}{RT} = \frac{14.5 \text{ psia}}{(0.2438 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(540 \text{ R})} = 0.110 \text{ lbm/ft}^3$$

Then the average flow velocity at the bottom part of the tube becomes

$$V = \frac{\dot{m}_{\text{net}}}{\rho A} = \frac{-7.30 \times 10^{-11} \text{ lbm/s}}{(0.110 \text{ lbm/ft}^3)(3.41 \times 10^{-4} \text{ ft}^2)} = \mathbf{-1.95 \times 10^{-6} \text{ ft/s}}$$

Discussion This flow rate is difficult to measure by even the most sensitive velocity measurement devices. The negative sign indicates flow in the negative x direction (towards the pipeline).



14-81 A hydrogen tank is maintained at atmospheric temperature and pressure by venting to the atmosphere through the charging valve. The initial mass flow rate of hydrogen out of the tank is to be determined.

Assumptions 1 Steady operating conditions at initial conditions exist. 2 Hydrogen and atmospheric air are ideal gases. 3 No chemical reactions occur in the valve. 4 Air concentration in the tank and hydrogen concentration in the atmosphere are negligible so that the mole fraction of the hydrogen is 1 in the tank, and 0 in the atmosphere (we will check this assumption later).

Properties The molar mass of hydrogen is $M = 2 \text{ kg/kmol}$ (Table A-1). The diffusion coefficient of hydrogen in air (or air in hydrogen) at 1 atm and 25°C is $D_{AB} = 7.2 \times 10^{-5} \text{ m}^2/\text{s}$ (Table 14-2). However, the pressure in the tank is 90 kPa = 0.88 atm. The diffusion coefficient at 25°C and 0.88 atm is determined from

$$D_{AB} = \frac{D_{AB,1\text{atm}}}{P(\text{in atm})} = \frac{7.2 \times 10^{-5}}{0.88} = 8.11 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant. The cross-sectional area of the valve is

$$A = \pi D^2 / 4 = \pi (0.03 \text{ m})^2 / 4 = 7.069 \times 10^{-4} \text{ m}^2$$

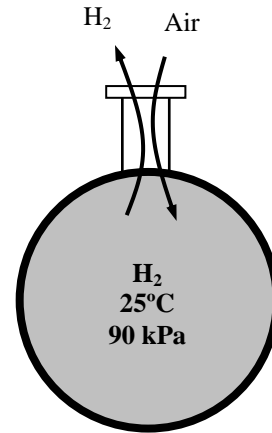
Noting that the pressure of hydrogen is 90 kPa at the bottom of the charging valve ($x = 0$) and 0 kPa at the top ($x = L$), its molar flow rate is determined from Eq. 14-64 to be

$$\begin{aligned} \dot{N}_{\text{H}_2} = \dot{N}_{\text{diff},A} &= \frac{D_{AB} A}{R_u T} \frac{P_{A,0} - P_{A,L}}{L} \\ &= \frac{(8.11 \times 10^{-5} \text{ m}^2/\text{s})(7.069 \times 10^{-4} \text{ m}^2)}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(298 \text{ K})} \frac{(90 - 0) \text{ kPa}}{0.1 \text{ m}} \\ &= 2.081 \times 10^{-8} \text{ kmol/s} \end{aligned}$$

Then the mass flow rate of hydrogen becomes

$$\dot{m}_{\text{H}_2} = (\dot{N}M)_{\text{H}_2} = (2.081 \times 10^{-8} \text{ kmol/s})(2 \text{ kg/kmol}) = 4.2 \times 10^{-8} \text{ kg/s}$$

Discussion This is the highest mass flow rate. It will decrease during the process as air diffuses into the tank and the concentration of hydrogen in tank drops.



14-82 "PROBLEM 14-82"

"GIVEN"

thickness=0.02 "[m]"

T=25+273 "[K]"

P_atm=90 "[kPa]"

"D=3 [cm], parameter to be varied"

extension=0.08 "[m]"

L=0.10 "[m]"

"PROPERTIES"

MM_H2=Molar mass(H2)

D_AB_1atm=7.2E-5 "[m^2/s], from Table 14-2 of the text at 1 atm and 25 C"

D_AB=D_AB_1atm*P_1atm/(P_atm*Convert(kPa, atm)) "at 90 kPa and 25 C"

P_1atm=1 "[atm]"

R_u=8.314 "[kPa-m^3/kmol-K]"

"ANALYSIS"

A=pi*D^2/4*Convert(cm^2, m^2)

N_dot_H2=(D_AB*A)/(R_u*T)*(P_atm-0)/L

m_dot_H2=N_dot_H2*MM_H2

D [cm]	m _{H2} [kg/s]
1	4.662E-09
1.2	6.714E-09
1.4	9.138E-09
1.6	1.193E-08
1.8	1.511E-08
2	1.865E-08
2.2	2.257E-08
2.4	2.686E-08
2.6	3.152E-08
2.8	3.655E-08
3	4.196E-08
3.2	4.774E-08
3.4	5.390E-08
3.6	6.043E-08
3.8	6.733E-08
4	7.460E-08
4.2	8.225E-08
4.4	9.026E-08
4.6	9.866E-08
4.8	1.074E-07
5	1.165E-07

14-83E The amount of water that evaporates from a Stefan tube at a specified temperature and pressure over a specified time period is measured. The diffusion coefficient of water vapor in air is to be determined.

Assumptions **1** Water vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid water is negligible. **3** Heat is transferred to the water from the surroundings to make up for the latent heat of vaporization so that the temperature of water remains constant at 70°F.

Properties The saturation pressure of water at 70°F is 0.3632 psia (Table A-9E).

Analysis The vapor pressure at the air-water interface is the saturation pressure of water at 70°F, and the mole fraction of water vapor (species A) is determined from

$$y_{\text{vapor},o} = y_{A,o} = \frac{P_{\text{vapor},o}}{P} = \frac{0.3632 \text{ psia}}{13.8 \text{ psia}} = 0.0263$$

Dry air is blown on top of the tube and thus $y_{\text{vapor},L} = y_{A,L} = 0$. Also, the total molar density throughout the tube remains constant because of the constant temperature and pressure conditions, and is determined to be

$$C = \frac{P}{R_u T} = \frac{13.8 \text{ psia}}{(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})(530 \text{ R})} = 2.42 \text{ lbmol/ft}^3$$

The cross-sectional area of the valve is

$$A = \pi D^2 / 4 = \pi (1/12 \text{ ft})^2 / 4 = 5.45 \times 10^{-3} \text{ ft}^2$$

The evaporation rate is given to be 0.0015 lbm per 10 days. Then the molar flow rate of vapor is determined to be

$$\dot{N}_A = \dot{N}_{\text{vapor}} = \frac{m_{\text{vapor}}}{M_{\text{vapor}}} = \frac{0.0015 \text{ lbm}}{(10 \times 24 \times 3600 \text{ s})(18 \text{ lbm/lbmol})} = 9.65 \times 10^{-11} \text{ lbm/s}$$

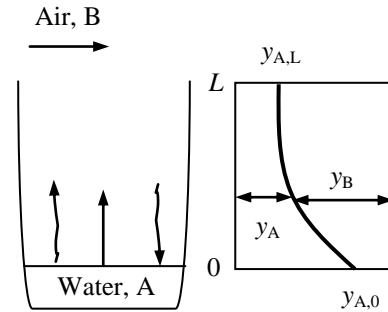
Finally, substituting the information above into Eq. 14-59 we get

$$\frac{\dot{N}_A}{A} = \frac{C D_{AB}}{L} \ln \left(\frac{1 - y_{A,L}}{1 - y_{A,o}} \right) \longrightarrow \frac{9.65 \times 10^{-11} \text{ lbm/s}}{5.45 \times 10^{-3} \text{ ft}^2} = \frac{(2.42 \text{ lbm/ft}^3) D_{AB}}{10/12 \text{ ft}} \ln \left(\frac{1 - 0}{1 - 0.0263} \right)$$

It gives

$$D_{AB} = 2.29 \times 10^{-7} \text{ ft}^2/\text{s}$$

for the binary diffusion coefficient of water vapor in air at 70°F and 13.8 psia.



14-84 A pitcher that is half filled with water is left in a room with its top open. The time it takes for the entire water in the pitcher to evaporate is to be determined.

Assumptions 1 Water vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid water is negligible. **3** Heat is transferred to the water from the surroundings to make up for the latent heat of vaporization so that the temperature of water remains constant at 15°C.

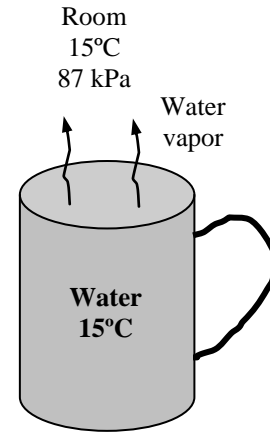
Properties The saturation pressure of water at 15°C is 1.705 kPa (Table A-9). The density of water in the pitcher can be taken to be 1000 kg/m³. The diffusion coefficient of water vapor in air at 15°C (= 288 K) and 87 kPa (0.86 atm) can be determined from

$$D_{AB} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288\text{K})^{2.072}}{0.86} = 2.71 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The flow area, which is the cross-sectional area of the pitcher, is

$$A = \pi D^2 / 4 = \pi (0.08\text{m})^2 / 4 = 5.026 \times 10^{-3} \text{ m}^2$$

The vapor pressure at the air-water interface is the saturation pressure of water at 15°C, which is 1.705 kPa. The air at the top of the pitcher ($x = L$) can be assumed to be dry ($P_{A,L} = 0$). The distance between the water surface and the top of the pitcher is initially 15 cm, and will be 30 cm at the end of the process when all the water is evaporated. Therefore, we can take the average height of the air column above the water surface to be $(15+30)/2 = 22.5$ cm. Then the molar flow rate is determined from



$$\begin{aligned} \dot{N}_A &= \frac{D_{AB} A}{R_u T} \left(\frac{P_{A,o} - P_{A,L}}{L} \right) = \frac{(2.71 \times 10^{-5} \text{ m}^2/\text{s})(5.026 \times 10^{-3} \text{ m}^2) (1.705 - 0) \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(288 \text{ K}) 0.225 \text{ m}} \\ &= 4.31 \times 10^{-10} \text{ kmol/s} \end{aligned}$$

The initial mass of water in the pitcher is

$$m_{\text{water}} = \rho \frac{\pi D^2}{4} L = (1000 \text{ kg/m}^3) \frac{\pi (0.08 \text{ m})^2}{4} (0.15 \text{ m}) = 0.754 \text{ kg}$$

Then the time required to evaporate the water completely becomes

$$\begin{aligned} \dot{N}_{\text{vapor}} &= \frac{m_{\text{vapor}}}{\Delta t \times M_{\text{vapor}}} \\ \Delta t &= \frac{m_{\text{vapor}}}{\dot{N}_{\text{vapor}} \times M_{\text{vapor}}} = \frac{0.754 \text{ kg}}{(4.31 \times 10^{-10} \text{ kmol/s})(18 \text{ kg/kmol})} = \mathbf{97,190,000 \text{ s}} \end{aligned}$$

which is equivalent to 1125 days. Therefore, it will take the water in the pitcher about 3 years to evaporate completely.

14-85 A large ammonia tank is vented to the atmosphere. The rate of loss of ammonia and the rate of air infiltration into the tank are to be determined.

Assumptions **1** Ammonia vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid ammonia is negligible. **3** Heat is transferred to the ammonia from the surroundings to make up for the latent heat of vaporization so that the temperature of ammonia remains constant at 25°C.

Properties The molar mass of ammonia is $M = 17$ kg/kmol, and the molar mass of air is $M = 29$ kg/kmol (Table A-1). The diffusion coefficient of ammonia in air (or air in ammonia) at 1 atm and 25°C is $D_{AB} = 7.2 \times 10^{-5}$ m²/s (Table 14-2).

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the tank and the atmosphere) remain constant. The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2 / 4 = \pi (0.01 \text{ m})^2 / 4 = 7.86 \times 10^{-5} \text{ m}^2$$

Noting that the pressure of ammonia is 1 atm = 101.3 kPa at the bottom of the tube ($x = 0$) and 0 at the top ($x = L$), its molar flow rate is determined from Eq. 14-64 to be

$$\begin{aligned} \dot{N}_{\text{ammonia}} &= \dot{N}_{\text{diff,A}} = \frac{D_{AB} A}{R_u T} \frac{P_{A,o} - P_{A,L}}{L} \\ &= \frac{(2.6 \times 10^{-5} \text{ m}^2/\text{s})(7.86 \times 10^{-5} \text{ m}^2)}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(298 \text{ K})} \frac{(101.3 - 0) \text{ kPa}}{3 \text{ m}} \\ &= \mathbf{2.78 \times 10^{-11} \text{ kmol/s}} \end{aligned}$$

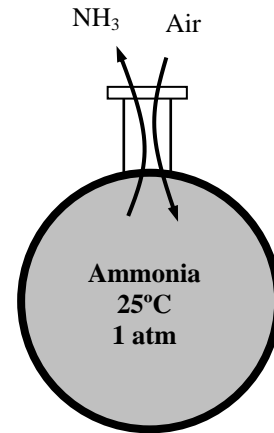
Therefore, the mass flow rate of ammonia through the tube is

$$\dot{m}_{\text{NH}_3} = (\dot{N}M)_{\text{NH}_3} = (2.78 \times 10^{-11} \text{ kmol/s})(17 \text{ kg/kmol}) = \mathbf{4.73 \times 10^{-10} \text{ kg/s}}$$

which corresponds to 0.0149 kg per year.

Note that $\dot{N}_B = -\dot{N}_A$ during an equimolar counter diffusion process. Therefore, the molar flow rate of air into the ammonia tank is equal to the molar flow rate of ammonia out of the tank. Then the mass flow rate of air into the pipeline becomes

$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-2.78 \times 10^{-11} \text{ kmol/s})(29 \text{ kg/kmol}) = \mathbf{-8.06 \times 10^{-10} \text{ kg/s}}$$



Mass Convection

14-86C Mass convection is expressed on a mass basis in an analogous manner to heat transfer as

$$\dot{m}_{\text{conv}} = h_{\text{mass}} A_s (\rho_{A,s} - \rho_{A,\infty})$$

where h_{mass} is the average mass transfer coefficient in m / s, A_s is the surface area in m², and $\rho_{A,s}$ and $\rho_{A,\infty}$ are the densities of species A at the surface (on the fluid side) and the free stream, respectively.

14-87C The region of the fluid near the surface in which concentration gradients exist is called the **concentration boundary layer**. In **flow over a plate**, the thickness of the concentration boundary layer δ_c for a species A at a specified location on the surface is defined as the normal distance y from the surface at which

$$\frac{\rho_{A,s} - \rho_A}{\rho_{A,s} - \rho_\infty} = 0.99$$

where $\rho_{A,s}$ and $\rho_{A,\infty}$ are the densities of species A at the surface (on the fluid side) and the free stream, respectively.

14-88C The dimensionless **Schmidt number** is defined as the ratio of momentum diffusivity to mass diffusivity $Sc = \nu / D_{AB}$, and it represents the relative magnitudes of momentum and mass diffusion at molecular level in the velocity and concentration boundary layers, respectively. The Schmidt number corresponds to the *Prandtl number* in heat transfer. A Schmidt number of *unity* indicates that momentum and mass transfer by diffusion are comparable, and velocity and concentration boundary layers almost coincide with each other.

14-89C The dimensionless **Sherwood number** is defined as $Sh = h_{\text{mass}} L / D_{AB}$ where L is the characteristic length, h_{mass} is the mass transfer coefficient and D_{AB} is the mass diffusivity. The Sherwood number represents the effectiveness of mass convection at the surface, and serves as the dimensionless mass transfer coefficient. The Sherwood number corresponds to the *Nusselt number* in heat transfer. A Sherwood number of unity for a plain fluid layer indicates mass transfer by pure diffusion in a fluid.

14-90C The dimensionless **Lewis number** is defined as the ratio of thermal diffusivity to mass diffusivity ($Le = \alpha / D_{AB}$), and it represents the relative magnitudes of heat and mass diffusion at molecular level in the thermal and concentration boundary layers, respectively. A Lewis number of unity indicates that heat and mass diffuse at the same rate, and the thermal and concentration boundary layers coincide.

14-91C Yes, the Grashof number evaluated using density difference instead of temperature difference can also be used in natural convection heat transfer calculations. In natural convection heat transfer, the term $\Delta\rho / \rho$ is replaced by $\beta\Delta T$ for convenience in calculations.

14-92C Using the analogy between heat and mass transfer, the mass transfer coefficient can be determined from the relations for heat transfer coefficient using the **Chilton-Colburn Analogy** expressed as

$$\frac{h_{\text{heat}}}{h_{\text{mass}}} = \rho C_p \left(\frac{\text{Sc}}{\text{Pr}} \right)^{2/3} = \rho C_p \left(\frac{\alpha}{D_{AB}} \right)^{2/3} = \rho C_p \text{Le}^{2/3}$$

Once the heat transfer coefficient h_{heat} is available, the mass transfer coefficient h_{mass} can be obtained from the relation above by substituting the values of the properties.

14-93C The molar mass of gasoline (C_8H_{18}) is 114 kg/kmol, which is much larger than the molar mass of air, which is 29 kg/kmol. Therefore, the gasoline vapor will settle down instead of rising even if it is at a much higher temperature than the surrounding air. As a result, the warm mixture of air and gasoline on top of an open gasoline will most likely settle down instead of rising in a cooler environment

14-94C Of the two identical cups of coffee, the one with no sugar will cool much faster than the one with plenty of sugar at the bottom. This is because in the case of no sugar, the coffee at the top will cool relatively fast and it will settle down while the warmer coffee at the bottom will rise to the top and cool off. When there is plenty of sugar at the bottom, however, the warmer coffee at the bottom will be heavier and thus it will not rise to the top. The elimination of natural convection currents and limiting heat transfer in water to conduction only will slow down the heat loss from the coffee considerably. In solar ponds, the rise of warm water at the bottom to the top is prevented by planting salt to the bottom of the pond.

14-95C The normalized velocity, thermal, and concentration boundary layers coincide during flow over a plate when the molecular diffusivity of momentum, heat, and mass are identical. That is, $\nu = \alpha = D_{AB}$ or $\text{Pr} = \text{Sc} = \text{Le} = 1$.

14-96C The relation $f \text{Re} / 2 = \text{Nu} = \text{Sh}$ is known as the **Reynolds analogy**. It is valid under the conditions that the Prandtl, Schmidt, and Lewis numbers are equal to unity. That is, $\nu = \alpha = D_{AB}$ or $\text{Pr} = \text{Sc} = \text{Le} = 1$. Reynolds analogy enables us to determine the seemingly unrelated friction, heat transfer, and mass transfer coefficients when only one of them is known or measured.

14-97C The relation $f / 2 = \text{St} \text{Pr}^{2/3} = \text{St}_{\text{mass}} \text{Sc}^{2/3}$ is known as the **Chilton-Colburn analogy**. Here St is the Stanton number, Pr is the Prandtl number, St_{mass} is the Stanton number in mass transfer, and Sc is the Schmidt number. The relation is valid for $0.6 < \text{Pr} < 60$ and $0.6 < \text{Sc} < 3000$. Its importance in engineering is that Chilton-Colburn analogy enables us to determine the seemingly unrelated friction, heat transfer, and mass transfer coefficients when only one of them is known or measured.

14-98C The relation $h_{\text{heat}} = \rho C_p h_{\text{mass}}$ is the result of the Lewis number $\text{Le} = 1$, and is known as the **Lewis relation**. It is valid for air-water vapor mixtures in the temperature range encountered in heating and air-conditioning applications. The Lewis relation is commonly used in air-conditioning practice. It asserts that the wet-bulb and adiabatic saturation temperatures of moist air are nearly identical. The Lewis relation can be used for heat and mass transfer in turbulent flow even when the Lewis number is not unity.

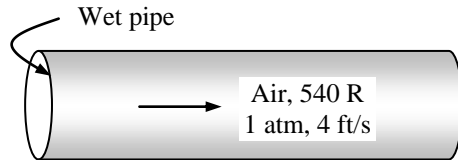
14-99C A convection mass transfer is referred to as the **low mass flux** when the flow rate of species undergoing mass flow is low relative to the total flow rate of the liquid or gas mixture so that the mass transfer between the fluid and the surface does not affect the *flow velocity*. The evaporation of water into air from lakes, rivers, etc. can be treated as a low mass-flux process since the mass fraction of water vapor in the air in such cases is just a few percent.

14-100E The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

Assumptions 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 540 R). **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 540 R and 1 atm, for which $\nu = 0.17 \times 10^{-3} \text{ ft}^2/\text{s}$ (Table A-15E). The mass diffusivity of water vapor in air at 540 R is determined from Eq. 14-15 to be

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(540/1.8)^{2.072}}{1} = 2.54 \times 10^{-5} \text{ m}^2/\text{s} \\ &= 2.73 \times 10^{-4} \text{ ft}^2/\text{s} \end{aligned}$$



The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(4 \text{ ft/s})(0.5/12 \text{ ft})}{0.17 \times 10^{-3} \text{ ft}^2/\text{s}} = 980$$

which is less than 2300 and thus the flow is laminar. Therefore, based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case are $\text{Nu} = \text{Sh} = 3.66$. Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(3.66)(2.73 \times 10^{-4} \text{ ft}^2/\text{s})}{0.5/12 \text{ ft}} = \mathbf{0.024 \text{ ft/s}}$$

Discussion The mass transfer rate (or the evaporation rate) in this case can be determined by defining logarithmic mean concentration difference in an analogous manner to the logarithmic mean temperature difference.

14-101 Air is blown over a body covered with a layer of naphthalene, and the rate of sublimation is measured. The heat transfer coefficient under the same flow conditions over the same geometry is to be determined.

Assumptions 1 The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable (will be verified). **2** Both air and naphthalene vapor are ideal gases.

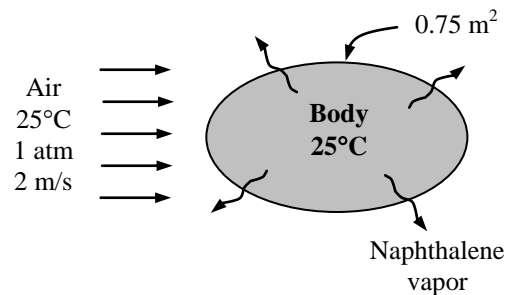
Properties The molar mass of naphthalene is 128.2 kg/kmol. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 1 atm, at which $\rho = 1.184 \text{ kg/m}^3$, $C_p = 1007 \text{ J/kg} \cdot \text{K}$, and $\alpha = 2.141 \times 10^{-5} \text{ m}^2/\text{s}$ (Table A-15).

Analysis The incoming air is free of naphthalene, and thus the mass fraction of naphthalene at free stream conditions is zero, $w_{A,\infty} = 0$. Noting that the vapor pressure of naphthalene at the surface is 11 Pa, the surface mass fraction is determined to be

$$w_{A,s} = \frac{P_{A,s}}{P} \left(\frac{M_A}{M_{air}} \right) = \frac{11 \text{ Pa}}{101,325 \text{ Pa}} \left(\frac{128.2 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 4.8 \times 10^{-4}$$

which confirms that the low mass flux approximation is valid. The rate of evaporation of naphthalene in this case is

$$\dot{m}_{\text{evap}} = \frac{m}{\Delta t} = \frac{0.1 \text{ kg}}{(45 \times 60 \text{ s})} = 3.703 \times 10^{-5} \text{ kg/s}$$



Then the mass convection coefficient becomes

$$h_{\text{mass}} = \frac{\dot{m}}{\rho A (w_{A,s} - w_{A,\infty})} = \frac{3.703 \times 10^{-5} \text{ kg/s}}{(1.184 \text{ kg/m}^3)(0.75 \text{ m}^2)(4.8 \times 10^{-4} - 0)} = 0.0869 \text{ m/s}$$

Using the analogy between heat and mass transfer, the average heat transfer coefficient is determined from Eq. 14-89 to be

$$\begin{aligned} h_{\text{heat}} &= \rho C_p h_{\text{mass}} \left(\frac{\alpha}{D_{AB}} \right)^{2/3} \\ &= (1.184 \text{ kg/m}^3)(1007 \text{ J/kg} \cdot \text{K})(0.0869 \text{ m/s}) \left(\frac{2.141 \times 10^{-5} \text{ m}^2/\text{s}}{0.61 \times 10^{-5} \text{ m}^2/\text{s}} \right)^{2/3} \\ &= \mathbf{239 \text{ W/m}^2 \cdot \text{°C}} \end{aligned}$$

Discussion Naphthalene has been commonly used in heat transfer studies to determine convection heat transfer coefficients because of the convenience it offers.

14-102 The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

Assumptions 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 1 atm, for which $\nu = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$ (Table A-15). The mass diffusivity of water vapor in air at 288 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 187 \times 10^{-10} \frac{T^{2.072}}{P}$$

$$= 187 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{1} = 2.33 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(3 \text{ m/s})(0.15 \text{ m})}{1.47 \times 10^{-5} \text{ m}^2/\text{s}} = 30,612$$

which is greater than 10,000 and thus the flow is turbulent. The Schmidt number in this case is

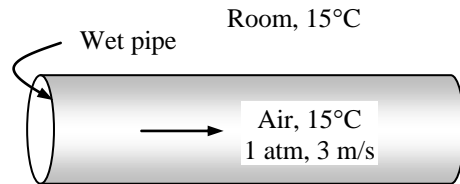
$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.47 \times 10^{-5} \text{ m}^2/\text{s}}{2.33 \times 10^{-5} \text{ m}^2/\text{s}} = 0.631$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.023 \text{Re}^{0.8} \text{Sc}^{0.4} = 0.023(30,612)^{0.8} (0.631)^{0.4} = 74.2$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\text{Sh}D_{AB}}{D} = \frac{(74.2)(2.33 \times 10^{-5} \text{ m}^2/\text{s})}{0.15 \text{ m}} = \mathbf{0.0115 \text{ m/s}}$$



14-103 "PROBLEM 14-103"

"GIVEN"

D=0.15 "[m]"

L=10 "[m]"

P=101.3 "[kPa]"

T=15+273 "[K]"

"Vel=3 [m/s], parameter to be varied"

"PROPERTIES"

Fluid\$='air'

rho=Density(Fluid\$, T=T, P=P)

mu=Viscosity(Fluid\$, T=T)

nu=mu/rho

D_AB=1.87E-10*T^2.072/(P*Convert(kPa, atm)) "from the text"

"ANALYSIS"

Re=Vel*D/nu

"Re is calculated to be greater than 10,000, and thus the flow is turbulent."

Sc=nu/D_AB

Sh=0.023*Re^0.8*Sc^0.4

h_mass=Sh*D_AB/D

Vel [m/s]	h _{mass} [m/s]
1	0.00479
1.5	0.006625
2	0.00834
2.5	0.00997
3	0.01154
3.5	0.01305
4	0.01452
4.5	0.01595
5	0.01736
5.5	0.01873
6	0.02008
6.5	0.02141
7	0.02272
7.5	0.02401
8	0.02528

14-104 A wet flat plate is dried by blowing air over it. The mass transfer coefficient is to be determined.

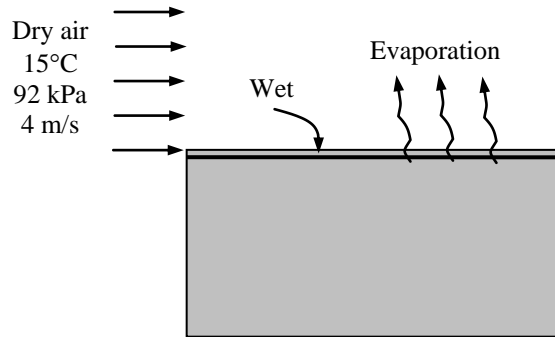
Assumptions 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 92 kPa = 92/101.325 = 0.908 atm, for which (Table A-15)

$$\nu = \nu_{1\text{atm}} / P(\text{atm}) = (1.47 \times 10^{-5} \text{ m}^2/\text{s}) / 0.908 \text{ atm} = 1.62 \times 10^{-5} \text{ m}^2 / \text{s}$$

Analysis The mass diffusivity of water vapor in air at 288 K is determined from Eq. 14-15 to be

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} \\ &= 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{0.908 \text{ atm}} \\ &= 2.57 \times 10^{-5} \text{ m}^2 / \text{s} \end{aligned}$$



The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(4 \text{ m/s})(2 \text{ m})}{1.62 \times 10^{-5} \text{ m}^2/\text{s}} = 493,827$$

which is less than 500,000, and thus the flow is laminar. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.62 \times 10^{-5} \text{ m}^2/\text{s}}{2.57 \times 10^{-5} \text{ m}^2/\text{s}} = 0.630$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664 \text{Re}^{0.5} \text{Sc}^{1/3} = 0.664(493,827)^{0.5} (0.630)^{1/3} = 400.1$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{L} = \frac{(400.1)(2.57 \times 10^{-5} \text{ m}^2/\text{s})}{2 \text{ m}} = \mathbf{0.00514 \text{ m/s}}$$

14-105 A wet concrete patio is to be dried by winds. The time it takes for the patio to dry is to be determined.

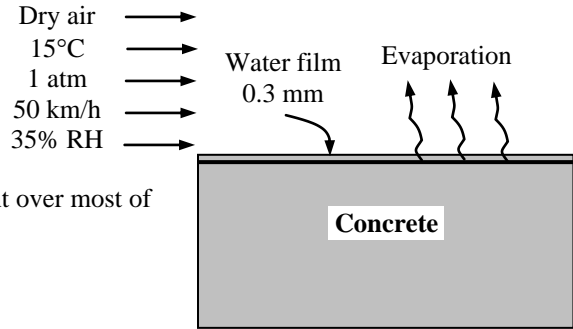
Assumptions 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as air.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 1 atm, for which $\nu = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$ and $\rho = 1.225 \text{ kg/m}^3$ (Table A-15). The saturation pressure of water at 15°C is 1.705 kPa. The mass diffusivity of water vapor in air at 15°C = 288 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{1 \text{ atm}} = 2.33 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The Reynolds number of the flow is

$$\begin{aligned} \text{Re} &= \frac{VL}{\nu} \\ &= \frac{(50 \text{ km/h})(5 \text{ m})}{1.47 \times 10^{-5} \text{ m}^2/\text{s}} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) \\ &= 4.724 \times 10^6 \end{aligned}$$



which is more than 500,000, and thus the flow is turbulent over most of the surface. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.47 \times 10^{-5} \text{ m}^2/\text{s}}{2.33 \times 10^{-5} \text{ m}^2/\text{s}} = 0.631$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.037 \text{Re}^{0.8} \text{Sc}^{1/3} = 0.037(4.724 \times 10^6)^{0.8} (0.631)^{1/3} = 6934$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{L} = \frac{(6934)(2.33 \times 10^{-5} \text{ m}^2/\text{s})}{5 \text{ m}} = 0.0323 \text{ m/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 15°C is 1.705 kPa, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s} \frac{M_A}{M} = \frac{P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(1.705 \text{ kPa})}{101.325 \text{ kPa}} \left(\frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 0.01044$$

$$w_{A,\infty} = y_{A,\infty} \frac{M_A}{M_{\text{air}}} = \frac{\phi P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(0.35)(1.705 \text{ kPa})}{101.325 \text{ kPa}} \left(\frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 0.00365$$

Then the rate of mass transfer to the air becomes

$$\begin{aligned} \dot{m}_{\text{evap.}} &= h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty}) \\ &= (0.0323 \text{ m/s})(1.225 \text{ kg/m}^3)(5 \text{ m} \times 5 \text{ m})(0.01044 - 0.00365) \\ &= 0.00671 \text{ kg/s} \end{aligned}$$

The total mass of water on the concrete patio is

$$m_{\text{water}} = \rho V = (1000 \text{ kg/m}^3)(5 \text{ m} \times 5 \text{ m} \times 0.3 \times 10^{-3} \text{ m}) = 7.5 \text{ kg}$$

Then the time required to evaporate the water on the concrete patio becomes

$$\Delta t = \frac{m_{\text{water}}}{\dot{m}_{\text{evap}}} = \frac{7.5 \text{ kg}}{0.00671 \text{ kg/s}} = 1117 \text{ s} = \mathbf{18.6 \text{ min}}$$

14-106E A spherical naphthalene ball is suspended in a room where it is subjected to forced air flow. The average mass transfer coefficient between the naphthalene and the air is to be determined.

Assumptions 1 The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable (will be verified). **2** Both air and naphthalene vapor are ideal gases. **3** Both the ball and the room are at the same temperature.

Properties The Schmidt number of naphthalene in air at room temperature is given to be 2.35. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 80°F and 1 atm from Table A-15E,

$$k = 0.015 \text{ Btu/h.ft.}^\circ\text{F} \quad \nu = 0.17 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\mu = 1.250 \times 10^{-5} \text{ lbm/ft.s} \quad \text{Pr} = 0.72$$

Analysis Noting that the Schmidt number for naphthalene in air is 2.35, the mass diffusivity of naphthalene in air is determined from

$$\text{Sc} = \frac{\nu}{D_{AB}} \longrightarrow D_{AB} = \frac{\nu}{\text{Sc}} = \frac{0.17 \times 10^{-3} \text{ ft}^2/\text{s}}{2.35} = 7.234 \times 10^{-5} \text{ ft}^2/\text{s}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(15 \text{ ft/s})(2/12 \text{ ft})}{(0.17 \times 10^{-3} \text{ ft}^2/\text{s})} = 14,706$$

Noting that $\mu_\infty = \mu_s$ for air in this case since the air and the ball are assumed to be at the same temperature, the Sherwood number can be determined from the forced heat convection relation for a sphere by replacing Pr by the Sc number to be

$$\text{Sh} = \frac{h_{mass}D}{D_{AB}} = 2 + \left[0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3} \right] \text{Sc}^{0.4} \left(\frac{\mu_\infty}{\mu_s} \right)^{1/4}$$

$$= 2 + \left[0.4(14,706)^{1/2} + 0.06(14,706)^{2/3} \right] (2.35)^{0.4} = 121$$

Then the mass transfer coefficient becomes

$$h_{mass} = \frac{\text{Sh}D_{AB}}{D} = \frac{(121)(7.234 \times 10^{-5} \text{ ft}^2/\text{s})}{0.166 \text{ ft}} = \mathbf{0.0525 \text{ ft/s}}$$

Discussion Note that the Nusselt number relations in heat transfer can be used to determine the Sherwood number in mass transfer by replacing Prandtl number by the Schmidt number.

