14-107 A raindrop is falling freely in atmospheric air. The terminal velocity of the raindrop at which the drag force equals the weight of the drop and the average mass transfer coefficient are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The raindrop is spherical in shape. **3** The reduction in the diameter of the raindrop due to evaporation when the terminal velocity is reached is negligible.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture. The properties of air at 1 atm and the free-stream temperature of 25ºC (and the dynamic viscosity at the surface temperature of 9ºC) are (Table A-15)

$$\rho = 1.184\ \text{kg/m}^3 \qquad \mu_\infty = 1.849\times10^{-5}\ \text{kg/m.s}$$

$$\upsilon = 1.562\times10^{-5}\ \text{m}^2/\text{s} \qquad \mu_{s,@\,282\,\text{K}} = 1.759\times10^{-5}\ \text{kg/m.s}$$

At 1 atm and the film temperature of (25+9)/2 = 17ºC = 290 K, the kinematic viscosity of air is, from Table A-11, $\nu = 1.488\times10^{-5}\ \text{m}^2/\text{s}$, while the mass diffusivity of water vapor in air is, Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(290\ \text{K})^{2.072}}{1\ \text{atm}} = 2.37\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis The weight of the raindrop before any evaporation occurs is

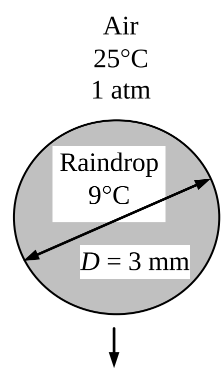
$$F_D = mg = \rho V g = (1000\ \text{kg/m}^3)\left[\frac{\pi(0.003\text{m})^3}{6}\right](9.8\ \text{m/s}^2) = 1.38\times10^{-4}\ \text{N}$$

The drag force is determined from $F_D = C_D A_N \frac{\rho u_\infty^2}{2}$ where drag coefficient C_D is to be determined using Fig. 10-20 which requires the Reynolds number. Since we do not know the velocity we cannot determine the Reynolds number. Therefore, the solution requires a trial-error approach. We choose a velocity and perform calculations to obtain the drag force. After a couple trial we choose a velocity of 8 m/s. Then the Reynolds number becomes

$$\text{Re} = \frac{\mathbf{V}_\infty D}{\upsilon} = \frac{(8\ \text{m/s})(0.003\text{m})}{1.562\times10^{-5}\ \text{m}^2/\text{s}} = 1536$$

The corresponding drag coefficient from Fig. 10-20 is 0.5. Then,

$$F_D = C_D A_N \frac{\rho u_\infty^2}{2} = (0.5)\left[\frac{\pi(0.003\text{m})^2}{4}\right]\frac{(1.184\ \text{kg/m}^3)(8\ \text{m/s})^2}{2} = 1.34\times10^{-4}$$



which is sufficiently close to the value calculated before. Therefore, the terminal velocity of the raindrop is ***V*** **= 8 m/s.** The Schmidt number is

$$\text{Sc} = \frac{\nu}{D_{\text{AB}}} = \frac{1.488\times10^{-5}\ \text{m}^2/\text{s}}{2.37\times10^{-5}\ \text{m}^2/\text{s}} = 0.628$$

Then the Sherwood number can be determined from the forced heat convection relation for a sphere by replacing Pr by the Sc number to be

$$\text{Sh} = \frac{h_{mass}D}{D_{AB}} = 2 + \left[0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}\right]\text{Sc}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(1536)^{1/2} + 0.06(1536)^{2/3}\right](0.628)^{0.4}\left(\frac{1.849\times10^{-5}}{1.759\times10^{-5}}\right)^{1/4} = 21.9$$

Then the mass transfer coefficient becomes

$$h_{\text{mass}} = \frac{\text{Sh}\,D_{AB}}{D} = \frac{(21.9)(2.37\times10^{-5}\ \text{m}^2/\text{s})}{0.003\text{m}} = \mathbf{0.173\ m/s}$$

14-108 Wet steel plates are to be dried by blowing air parallel to their surfaces. The rate of evaporation from both sides of a plate is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** The plates are far enough from each other so that they can be treated as flat plates. **4** The air is dry so that the amount of moisture in the air is negligible.

Properties The molar masses of air and water are M = 29 and M = 18 kg/kmol, respectively (Table A-1). Because of low mass flux conditions, we can use dry air properties for the mixture. The properties of the air at 1 atm and at the film temperature of (20 + 25) = 22.5ºC are (Table A-15)

$\nu = 1.539\times10^{-5}$ m^2/s C_p = 1007 J / kg K

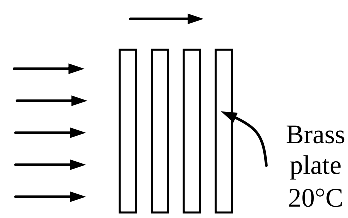
ρ = 1.194 kg /m^3 Pr = 0.7303

The saturation pressure of water at 20ºC is 2.339 kPa (Table A-9). The mass diffusivity of water vapor in air at 22.5ºC = 295.5 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(295.5\text{K})^{2.072}}{1\text{atm}} = 2.46\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis The Reynolds number for flow over the flat plate is

$$\text{Re} = \frac{VL}{\nu} = \frac{(4\ \text{m/s})(0.4\ \text{m})}{1.539\times10^{-5}\ \text{m}^2/\text{s}} = 103{,}964$$



which is less than 500,000, and thus the air flow is laminar over the entire plate. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.539\times10^{-5}\ \text{m}^2/\text{s}}{2.46\times10^{-5}\ \text{m}^2/\text{s}} = 0.626$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}_L^{0.5}\text{Sc}^{1/3} = 0.664(103{,}964)^{0.5}(0.626)^{1/3} = 183.1$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(183.1)(2.46\times10^{-5}\ \text{m}^2/\text{s})}{0.4\text{m}} = 0.0113\text{m/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 20ºC is 2.339 kPa, the mass fraction of water vapor in the air at the surface of the plate is, from Eq. 14-10,

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(2.339\text{kPa})}{101.325\text{kPa}}\left(\frac{18\text{kg/kmol}}{29\text{kg/kmol}}\right) = 0.01433$$

and $w_{A,\infty} = 0$

Then the rate of mass transfer to the air becomes

$$\begin{aligned}\dot{m}_{\text{evap.}} &= h_{\text{mass}}\rho A(w_{A,s} - w_{A,\infty})\\ &= (0.0113\text{m/s})(1.194\text{kg/m}^3)(2\times0.4\text{m}\times0.4\text{m})(0.01433-0)\\ &= \mathbf{6.19\times10^{-5}kg/s}\end{aligned}$$

Discussion This is the upper limit for the evaporation rate since the air is assumed to be completely dry.

14-109E Air is blown over a square pan filled with water. The rate of evaporation of water and the rate of heat transfer to the pan to maintain the water temperature constant are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as the air.

Properties The molar masses of air and water are M = 29 and M = 18 lbm/lbmol, respectively (Table A-1E). Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 80°F and 1 atm, for which $\nu = 0.17 \times 10^{-3}$ ft^2/s, and $\rho = 0.074$ lbm/ft^3 (Table A-15E). The saturation pressure of water at 80ºF is 0.5073 psia, and the heat of vaporization is 1048 Btu/lbm. The mass diffusivity of water vapor in air at 80ºF = 540 R = 300 K is determined from Eq. 14-15 to be

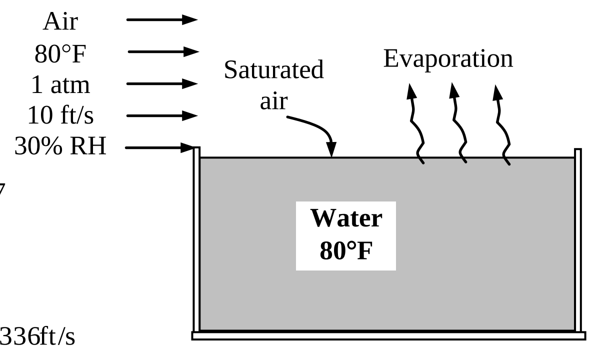
$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(300\text{K})^{2.072}}{1\text{atm}} = 2.54\times10^{-5}\ \text{m}^2/\text{s} = 2.74\times10^{-4}\ \text{ft}^2/\text{s}$$

Analysis The Reynolds number for flow over the free surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(10\text{ft/s})(15/12\text{ ft})}{0.17\times10^{-3}\ \text{ft}^2/\text{s}} = 73{,}530$$

which is less than 500,000, and thus the flow is laminar over the entire surface. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{0.17\times10^{-3}\ \text{ft}^2/\text{s}}{2.734\times10^{-4}\ \text{ft}^2/\text{s}} = 0.622$$



Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}_L^{0.5}\text{Sc}^{1/3} = 0.664(73{,}530)^{0.5}(0.622)^{1/3} = 153.7$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}\,D_{AB}}{L} = \frac{(153.7)(2.734\times10^{-4}\ \text{ft}^2/\text{s})}{15/12\text{ft}} = 0.0336\text{ft/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 80ºF is 0.5073 psia (= 0.0345 atm), the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(0.3)(0.5073\text{psia})}{14.7\text{psia}}\left(\frac{18\text{lbm/lbmol}}{29\text{lbm/lbmol}}\right) = 0.00643$$

$$w_{A,\infty} = y_{A,\infty}\frac{M_A}{M_{air}} = \frac{\phi P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(1.0)(0.5073\text{psia})}{14.7\text{psia}}\left(\frac{18\text{lbm/lbmol}}{29\text{lbm/lbmol}}\right) = 0.02142$$

Then the rate of mass transfer to the air becomes

$$\dot{m}_{\text{evap}} = h_{\text{mass}}\rho A_s(w_{A,s} - w_{A,\infty}) = (0.0336\text{ft/s})(0.074\text{lbm/ft}^3)(15/12\text{ft}^2)(0.02142 - 0.00642) = \mathbf{5.83\times10^{-5}\ lbm/s}$$

Noting that the latent heat of vaporization of water at 80ºF is h_{fg} = 1048 Btu/ lbm, the required rate of heat supply to the water to maintain its temperature constant is

$$\dot{Q} = \dot{m}_{evap.}\ h_{fg} = (5.83\times10^{-5}\ \text{lbm/s})(1048\ \text{Btu/lbm}) = \mathbf{0.0611\ Btu/s = 220\ Btu/h}$$

Discussion If no heat is supplied to the pan, the heat of vaporization of water will come from the water, and thus the water temperature will have to drop below the air temperature.

14-110E Air is blown over a square pan filled with water. The rate of evaporation of water and the rate of heat transfer to the pan to maintain the water temperature constant are to be determined.

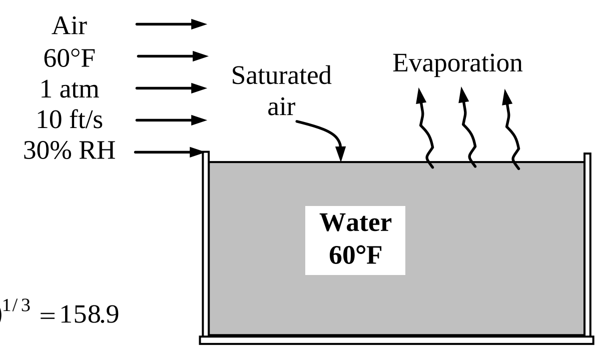
Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 60°F). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as air.

Properties The molar masses of air and water are M = 29 and M = 18 lbm/lbmol, respectively (Table A-1E). Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 60°F and 1 atm, for which $\nu = 0.159\times10^{-3}$ ft^2/s, and $\rho = 0.076$ lbm / ft^3 (Table A-15E). The saturation pressure of water at 60ºF is 0.2563 psia, and the heat of vaporization is 1060 Btu/lbm. The mass diffusivity of water vapor in air at 60ºF = 520 R = 288.9 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(288.9\,\text{K})^{2.072}}{1\,\text{atm}} = 2.35\times10^{-5}\ \text{m}^2/\text{s} = 2.53\times10^{-4}\ \text{ft}^2/\text{s}$$

Analysis The Reynolds number for flow over the free surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(10\,\text{ft/s})(15/12\,\text{ft})}{0.159\times10^{-3}\ \text{ft}^2/\text{s}} = 78{,}620$$



which is less than 500,000, and thus the flow is laminar over the entire surface. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{0.159\times10^{-3}\ \text{ft}^2/\text{s}}{2.53\times10^{-4}\ \text{ft}^2/\text{s}} = 0.628$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}_L^{\ 0.5}\,\text{Sc}^{\ 1/3} = 0.664(78{,}620)^{0.5}(0.622)^{1/3} = 158.9$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}\,D_{AB}}{L} = \frac{(158.9)(2.53\times10^{-4}\ \text{ft}^2/\text{s})}{15/12\,\text{ft}} = 0.0322\,\text{ft/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 60ºF is 0.2563 psia, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(0.3)(0.2563\,\text{psia})}{14.7\,\text{psia}}\left(\frac{18\,\text{lbm/lbmol}}{29\,\text{lbm/lbmol}}\right) = 0.00325$$

$$w_{A,\infty} = y_{A,\infty}\frac{M_A}{M_{air}} = \frac{\phi P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(1.0)(0.2565\,\text{psia})}{14.7\,\text{psia}}\left(\frac{18\,\text{lbm/lbmol}}{29\,\text{lbm/lbmol}}\right) = 0.01082$$

Then the rate of mass transfer to the air becomes

$$\dot{m}_{\text{evap}} = h_{\text{mass}}\rho A(w_{A,s} - w_{A,\infty}) = (0.0322\,\text{ft/s})(0.076\,\text{lbm/ft}^3)(15/12\,\text{ft}^3)(0.01082 - 0.00325) = \mathbf{2.35\times10^{-5}\ lbm/s}$$

Noting that the latent heat of vaporization of water at 60ºF is h_{fg} = 1060 Btu/ lbm, the required rate of heat supply to the water to maintain its temperature constant is

$$\dot{Q} = \dot{m}_{\text{evap}}\,h_{fg} = (2.35\times10^{-5}\ \text{lbm/s})(1060\ \text{Btu/lbm}) = \mathbf{0.0249\ Btu/s = 89.5\ Btu/h}$$

Discussion If no heat is supplied to the pan, the heat of vaporization of water will come from the water, and thus the water temperature will have to drop below the air temperature.

Simultaneous Heat and Mass Transfer

14-111C In steady operation, the mass transfer process does not have to involve heat transfer. However, a mass transfer process that involves phase change (evaporation, sublimation, condensation, melting etc.) must involve heat transfer. For example, the evaporation of water from a lake into air (mass transfer) requires the transfer of latent heat of water at a specified temperature to the liquid water at the surface (heat transfer).

14-112C It is possible for a shallow body of water to freeze during a cool and dry night even when the ambient air and surrounding surface temperatures never drop to 0°C. This is because when the air is not saturated (ϕ < 100 percent), there will be a difference between the concentration of water vapor at the water-air interface (which is always saturated) and some distance above it. Concentration difference is the driving force for mass transfer, and thus this concentration difference will drive the water into the air. But the water must vaporize first, and it must absorb the latent heat of vaporization from the water. The temperature of water near the surface must drop as a result of the sensible heat loss, possibly below the freezing point.

14-113C During evaporation from a water body to air, the latent heat of vaporization will be equal to *convection* heat transfer from the air when *conduction* from the lower parts of the water body to the surface is negligible, and temperature of the surrounding surfaces is at about the temperature of the water surface so that the *radiation* heat transfer is negligible.

14-114 Air is blown over a jug made of porous clay to cool it by simultaneous heat and mass transfer. The temperature of the water in the jug when steady conditions are reached is to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** Radiation effects are negligible.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ which cannot be determined at this point because of the unknown surface temperature T_s. We know that $T_s < T_\infty$ and, for the purpose of property evaluation, we take T_s to be 20°C. Then, the properties of water at 20°C and the properties of dry air at the average temperature of 25°C and 1 atm are (Tables A-9 and A-15)

$$\text{Water at 20°C}: \; h_{fg} = 2454 \text{ kJ/kg}, \; P_v = 2.34 \text{ kPa. Also, at 30°C, } P_{sat\,@\,30°C} = 4.25 \text{ kPa}$$

$$\text{Dry air at 25°C}: \; C_p = 1.007 \text{ kJ/kg}\cdot°\text{C}, \; \alpha = 2.141\times10^{-5} \text{ m}^2/\text{s}$$

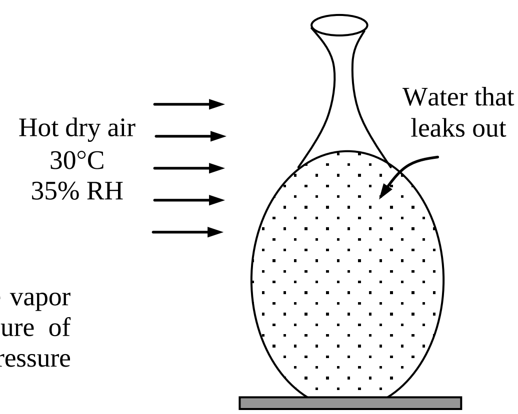
Also, the mass diffusivity of water vapor in air at 25°C is $D_{H_2O\text{-air}} = 2.50\times10^{-5} \text{ m}^2/\text{s}$ (Table 14-4), and the molar masses of water and air are 18 and 29 kg/kmol, respectively (Table A-1).

Analysis The surface temperature of the jug can be determined by rearranging Chilton-Colburn equation as

$$T_s = T_\infty - \frac{h_{fg}}{C_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P}$$

where the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.141\times10^{-5} \text{ m}^2/\text{s}}{2.50\times10^{-5} \text{ m}^2/\text{s}} = 0.856$$



Note that we could take the Lewis number to be 1 for simplicity, but we chose to incorporate it for better accuracy.

The air at the surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (2.34 kPa). The vapor pressure of air far from the surface is determined from

$$P_{v,\infty} = \phi P_{sat@T_\infty} = (0.35) P_{sat@30°C} = (0.35)(4.25 \text{ kPa}) = 1.488 \text{ kPa}$$

Noting that the atmospheric pressure is 1 atm = 101.3 Pa, substituting the known quantities gives

$$T_s = 30°\text{C} - \frac{2454 \text{ kJ/kg}}{(1.007 \text{ kJ/kg.°C})(0.856)^{2/3}} \frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \frac{(2.34 - 1.488) \text{ kPa}}{101.3 \text{ kPa}} = \mathbf{15.9°C}$$

Therefore, the temperature of the drink can be lowered to 15.9°C by this process.

Discussion The accuracy of this result can be improved by repeating the calculations with dry air properties evaluated at (30+16)/2 = 18°C and water properties at 16.0°C. But the improvement will be minor.

14-115 "!PROBLEM 14-115"

```
"GIVEN"
P=101.3 "[kPa]"
T_infinity=30 "[C]"
"phi=0.35 parameter to be varied"

"PROPERTIES"
Fluid$='steam_NBS'
h_f=enthalpy(Fluid$, T=T_s, x=0)
h_g=enthalpy(Fluid$, T=T_s, x=1)
h_fg=h_g-h_f
P_sat_s=Pressure(Fluid$, T=T_s, x=0)
P_sat_infinity=Pressure(Fluid$, T=T_infinity, x=0)
C_p_air=CP(air, T=T_ave)
T_ave=1/2*(T_infinity+T_s)
alpha=2.18E-5 "[m^2/s], from the tables in the text"
D_AB=2.50E-5 "[m^2/s], from the text"
MM_H2O=molarmass(H2O)
MM_air=molarmass(air)

"ANALYSIS"
Le=alpha/D_AB
P_v_infinity=phi*P_sat_infinity
P_v_s=P_sat_s
T_s=T_infinity-h_fg/(C_p_air*Le^(2/3))*MM_H2O/MM_air*(P_v_s-P_v_infinity)/P
```

ϕ	T_s [C]
0.1	12.72
0.15	14.05
0.2	15.32
0.25	16.53
0.3	17.68
0.35	18.79
0.4	19.85
0.45	20.87
0.5	21.85
0.55	22.8
0.6	23.71
0.65	24.58
0.7	25.43
0.75	26.25
0.8	27.05
0.85	27.82
0.9	28.57
0.95	29.29
1	30

14-116E In a hot summer day, a bottle of drink is to be cooled by wrapping it in a wet cloth, and blowing air to it. The temperature of the drink in the bottle when steady conditions are reached is to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** Radiation effects are negligible.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ which cannot be determined at this point because of the unknown surface temperature T_s. We know that $T_s < T_\infty$ and, for the purpose of property evaluation, we take T_s to be 60°F. Then the properties of water at 60°F and the properties of dry air at the average temperature of (60+80)/2 = 70°F and 1 atm are (Tables A-9E and A-15E)

$$\text{Water at } 60°\text{F}: \; h_{fg} = 1060 \text{ Btu/lbm}, \; P_v = 0.2563 \text{ psia. Also, at } 80°\text{F}, \; P_{sat\,@\,80°\text{F}} = 0.5073 \text{ psia}$$

$$\text{Dry air at } 70°\text{F}: \; C_p = 0.24 \text{ Btu/lbm}\cdot°\text{F}, \; \alpha = 0.8093 \text{ ft}^2/\text{h} = 2.25\times10^{-4} \text{ ft}^2/\text{s}$$

Also, the molar masses of water and air are 18 and 29 lbm/lbmol, respectively (Table A-1E), and the mass diffusivity of water vapor in air at 80°F (= 294.4 K) is

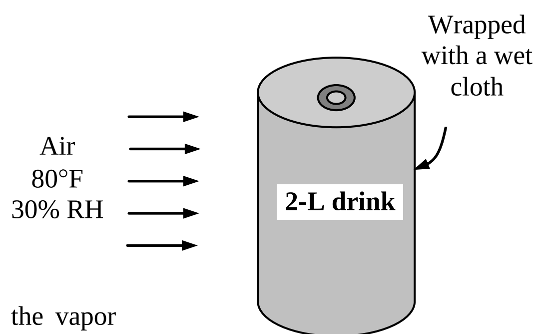
$$D_{AB} = D_{H_2O\text{-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(294.4\text{ K})^{2.072}}{1\text{ atm}} = 2.44\times10^{-5} \text{ m}^2/\text{s} = 2.63\times10^{-4} \text{ ft}^2/\text{s}$$

Analysis The surface temperature of the jug can be determined by rearranging Chilton-Colburn equation as

$$T_s = T_\infty - \frac{h_{fg}}{C_p \text{Le}^{2/3}}\frac{M_v}{M}\frac{P_{v,s} - P_{v,\infty}}{P}$$

where the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.25\times10^{-4} \text{ ft}^2/\text{s}}{2.63\times10^{-4} \text{ ft}^2/\text{s}} = 0.856$$



Note that we could take the Lewis number to be 1 for simplicity, but we chose to incorporate it for better accuracy.

The air at the surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (0.2563 psia). The vapor pressure of air far from the surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.3)P_{\text{sat@80°F}} = (0.3)(0.5073 \text{ psia}) = 0.152 \text{ psia}$$

Noting that the atmospheric pressure is 1 atm = 14.7 psia, substituting the known quantities gives

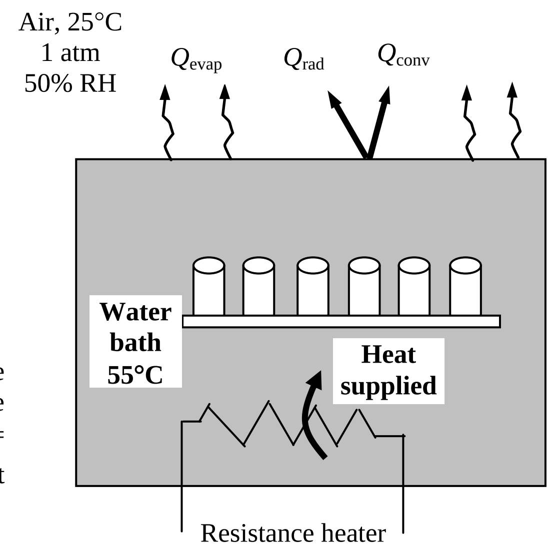
$$T_s = 80°\text{F} - \frac{1060 \text{ Btu/lbm}}{(0.24 \text{ Btu/lbm.°F})(0.856)^{2/3}}\left(\frac{18 \text{ lbm/lbmol}}{29 \text{ lbm/lbmol}}\right)\frac{(0.2563 - 0.152) \text{ psia}}{14.7 \text{ psia}} = \mathbf{58.4°F}$$

Therefore, the temperature of the drink can be lowered to 58.4°F by this process.

Discussion Note that the value obtained is very close to the assumed value of 60°F for the surface temperature. Therefore, there is no need to repeat the calculations with properties at the new surface temperature of 58.7°F

14-117 Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat loss from the top and side surfaces of the bath by radiation, natural convection, and evaporation as well as the rates of heat and water mass that need to be supplied to the water are to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body and the metal container are maintained at a uniform temperature of 55°C. **4** Heat losses from the bottom surface are negligible. **5** The air motion around the bath is negligible so that there are no forced convection effects.



Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2 =$ (25+55)/2 = 40°C = 313 K. The properties of dry air at 40°C and 1 atm are, from Table A-15,

$$k = 0.0266 \text{W/m}\cdot°\text{C}, \quad \Pr = 0.726$$

$$\alpha = 2.35\times10^{-5}\ \text{m}^2/\text{s} \quad \nu = 1.70\times10^{-5}\ \text{m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 313 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(313\text{K})^{2.072}}{1\text{atm}} = 2.77\times10^{-5}\ \text{m}^2/\text{s}$$

The saturation pressure of water at 25°C is $P_{\text{sat@25°C}} = 3.169$ kPa. Properties of water at 55°C are $h_{fg} = 2371$ kJ/kg and $P_v = 15.76$ kPa (Table A-9). The specific heat of water at the average temperature of (15+55)/2 = 35°C is C_p = 4.178 kJ/kg.°C.

The gas constants of dry air and water are R_{air} = 0.287 kPa.m^3/kg.K and R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1). Also, the emissivities of water and the sheet metal are given to be 0.61 and 0.95, respectively, and the specific heat of glass is given to be 1.0 kJ/kg.°C.

Analysis (*a*) The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot{m}_{\text{bottle}} = m_{\text{bottle}} \times \text{Bottle flow rate} = (0.150\ \text{kg/bottle})(800\ \text{bottles/min}) = 120\ \text{kg/min} = 2\ \text{kg/s}$$

Then the rate of heat removal by the bottles as they are heated from 25 to 55°C is

$$\dot{Q}_{\text{bottle}} = \dot{m}_{\text{bottle}} C_p \Delta T = (2\,\text{kg/s})(1\text{kJ/kg.°C})(55-25)°\text{C} = 60{,}000\text{W}$$

The amount of water removed by the bottles is

$$\dot{m}_{\text{water,out}} = (\text{Flow rate of bottles})(\text{Water removed per bottle})$$

$$= (800\text{bottles/min})(0.6\text{g/bottle}) = 480\text{g/min} = 8\times10^{-3}\ \text{kg/s}$$

Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot{Q}_{\text{water removed}} = \dot{m}_{\text{water removed}} C_p \Delta T = (8\times10^{-3}\ \text{kg/s})(4178\,\text{J/kg.°C})(55-15)°\text{C} = 1337\ \text{W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot{Q}_{\text{total, removed}} = \dot{Q}_{\text{glass removed}} + \dot{Q}_{\text{water removed}} = 60{,}000 + 1337 = \mathbf{61{,}337\ W}$$

(*b*) The rate of heat loss from the top surface of the water bath is the sum of the heat losses by radiation, natural convection, and evaporation. Then the radiation heat loss from the top surface of water to the surrounding surfaces is

$$\dot{Q}_{\text{rad,top}} = \varepsilon A\sigma(T_s^4 - T_{\text{surr}}^4) = (0.95)(8\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(55+273\text{ K})^4 - (15+273\text{ K})^4] = 2023\text{ W}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (15.76 kPa at 55°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.50)P_{\text{sat@25°C}} = (0.50)(3.169\text{ kPa}) = 1.585\text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

At the surface:

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{15.76\text{ kPa}}{(0.4615\text{ kPa.m}^3/\text{kg}\cdot\text{K})(55+273\text{ K})} = 0.1041\text{ kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325-15.76)\text{ kPa}}{(0.287\text{ kPa.m}^3/\text{kg}\cdot\text{K})(55+273\text{ K})} = 0.9090\text{ kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.1041 + 0.9090 = 1.0131\text{ kg/m}^3$$

and

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.585\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273\text{ K})} = 0.0115\text{ kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325-1.585)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273\text{ K})} = 1.1662\text{ kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0115 + 1.1662 = 1.1777\text{ kg/m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The area of the top surface of the water bath is A_s = 2 m × 4 m = 8 m^2 and its perimeter is p = 2(2 + 4) = 12 m. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{8\text{ m}^2}{12\text{ m}} = 0.667\text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81\text{ m/s}^2)(1.1777-1.0131\text{ kg/m}^3)(0.667\text{ m})^3}{[(1.1777+1.0131)/2\text{ kg/m}^3](1.70\times10^{-5}\text{ m}^2/\text{s})^2} = 1.51\times10^9$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.51\times10^9\times0.726)^{1/3} = 155$$

and $$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(155)(0.0266\text{ W/m}\cdot°\text{C})}{0.667\text{ m}} = 6.17\text{ W/m}^2\cdot°\text{C}$$

Then the natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (6.17\text{ W/m}^2\cdot°\text{C})(8\text{ m}^2)(55-25)°\text{C} = \mathbf{1480\text{ W}}$$

(*c*) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.70\times10^{-5}\text{ m}^2/\text{s}}{2.77\times10^{-5}\text{ m}^2/\text{s}} = 0.614$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{GrSc})^{1/3} = 0.15(1.51\times10^9\times0.614)^{1/3} = 146$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(146)(2.77\times10^{-5}\ \text{m}^2/\text{s})}{0.667\ \text{m}} = 0.00606\ \text{m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned}\dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty})\\ &= (0.00606\text{m/s})(8\,\text{m}^2)(0.1041 - 0.0116)\text{kg/m}^3\\ &= 0.00448\,\text{kg/s} = 16.1\,\text{kg/h}\end{aligned}$$

and $\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.00448\ \text{kg/s})(2371\ \text{kJ/kg}) = 10.6\ \text{kW} = 10{,}600\ \text{W}$

Then the total rate of heat loss from the open top surface of the bath to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 2023 + 1480 + 10{,}600 = \mathbf{14{,}103\,W}$$

Therefore, if the water bath is heated electrically, a 14 kW resistance heater will be needed just to make up for the heat loss from the top surface.

(*c*) The side surfaces are vertical plates, and treating the air as dry air for simplicity, heat transfer from them by natural convection is determined to be

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\text{m/s}^2)(1/313\text{K})(55-25)\,\text{K})(1\text{m})^3}{(1.70\times10^{-5}\,\text{m}^2/\text{s})^2} = 3.25\times10^9$$

$$\text{Nu} = 0.1(\text{Gr Pr})^{1/3} = 0.1(3.25\times10^9\times0.726)^{1/3} = 133$$

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(133)(0.0266\,\text{W/m}\cdot{}^\circ\text{C})}{1\,\text{m}} = 3.54\,\text{W/m}^2\cdot{}^\circ\text{C}$$

$$\dot{Q}_{\text{conv, side}} = h_{\text{conv}}A_s(T_s - T_\infty) = (3.54\,\text{W/m}^2\cdot{}^\circ\text{C})(12\times1\,\text{m}^2)(55-25)^\circ\text{C} = 1275\,\text{W}$$

The radiation heat loss from the side surfaces of the bath to the surrounding surfaces is

$$\dot{Q}_{\text{rad,side}} = \varepsilon A_s\sigma(T_s^4 - T_{\text{surr}}^4) = (0.61)(12\,\text{m}\times1\,\text{m})(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)[(55+273\text{K})^4 - (15+273\text{K})^4] = 2498\,\text{W}$$

and $\dot{Q}_{\text{total, side}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1275 + 2498 = \mathbf{3773\,W}$

(*d*) The rate at which water must be supplied to the maintain steady operation is equal to the rate of water removed by the bottles plus the rate evaporation,

$$\dot{m}_{\text{make-up}} = \dot{m}_{\text{removed}} + \dot{m}_{\text{evap}} = 0.00800 + 0.00448 = \mathbf{0.01248\ kg/s = 44.9\ kg/h}$$

Noting that the entire make-up water enters the bath 15°C, the rate of heat supply to preheat the make-up water to 55°C is

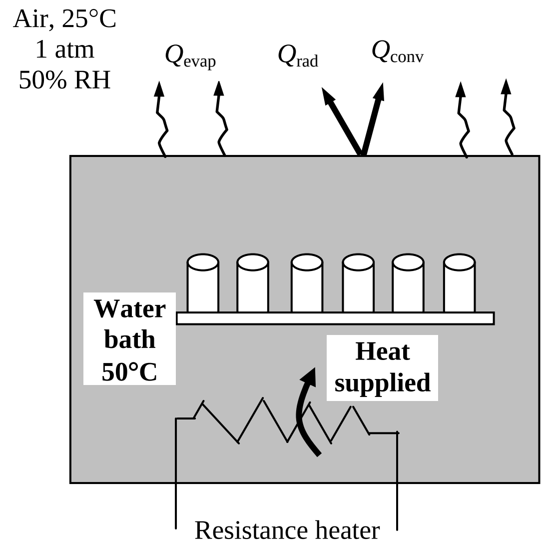
$$\dot{Q}_{\text{preheating water}} = \dot{m}_{\text{make-up water}}C_p\,\Delta T = (0.01248\ \text{kg/s})(4178\,\text{J/kg}\cdot{}^\circ\text{C})(55-15)^\circ\text{C} = 2086\ \text{W}$$

Then the rate of required heat supply for the bath becomes the sum of heat losses from the top and side surfaces, plus the heat needed for preheating the make-up water and the bottles,

$$\begin{aligned}\dot{Q}_{\text{total}} &= \dot{Q}_{\text{bottle}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}}\right)_{\text{top}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}}\right)_{\text{side}} + \dot{Q}_{\text{makeupwater}}\\ &= 60{,}000 + 14{,}103 + 3773 + 2086 = \mathbf{79{,}962W}\end{aligned}$$

Therefore, the heater must be able to supply heat at a rate of 80 kW to maintain steady operating conditions

14-118 Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat loss from the top and side surfaces of the bath by radiation, natural convection, and evaporation as well as the rates of heat and water mass that need to be supplied to the water are to be determined.



Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body and the metal container are maintained at a uniform temperature of 50°C. **4** Heat losses from the bottom surface are negligible. **5** The air motion around the bath is negligible so that there are no forced convection effects.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2 =$ (25+50)/2 = 37.5°C = 310.5 K. The properties of dry air at 310.5 K and 1 atm are, from Table A-15,

$$k = 0.0264 \text{ W/m}\cdot°\text{C}, \quad \text{Pr} = 0.726$$

$$\alpha = 2.31\times10^{-5} \text{ m}^2/\text{s} \quad \nu = 1.68\times10^{-5} \text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 310.5 K is, from Eq. 14-15,

$$D_{AB} = D_{H_2O\text{-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(310.5\text{K})^{2.072}}{1\text{atm}} = 2.72\times10^{-5} \text{ m²/s}$$

The saturation pressure of water at 25°C is $P_{sat@25°C} = 3.169$ kPa. Properties of water at 50°C are $h_{fg} = 2383$ kJ/kg and $P_v = 12.35$ kPa (Table A-9). The specific heat of water at the average temperature of (15+50)/2 = 32.5°C is C_p = 4.178 kJ/kg.°C.

The gas constants of dry air and water are R_{air} = 0.287 kPa.m^3/kg.K and R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1). Also, the emissivities of water and the sheet metal are given to be 0.61 and 0.95, respectively, and the specific heat of glass is given to be 1.0 kJ/kg.°C.

Analysis (*a*) The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot{m}_{bottle} = m_{bottle} \times \text{Bottle flow rate} = (0.150 \text{ kg/bottle})(800 \text{ bottles/min}) = 120 \text{ kg/min} = 2 \text{ kg/s}$$

Then the rate of heat removal by the bottles as they are heated from 25 to 55°C is

$$\dot{Q}_{bottle} = \dot{m}_{bottle} C_p \Delta T = (2\text{kg/s})(1\text{kJ/kg.°C})(55-25)°\text{C} = 60{,}000\text{W}$$

The amount of water removed by the bottles is

$$\dot{m}_{water,out} = (\text{Flow rate of bottles})(\text{Water removed per bottle})$$
$$= (800 \text{bottles/min})(0.6\text{g/bottle}) = 480\text{g/min} = 8\times10^{-3} \text{ kg/s}$$

Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot{Q}_{water\ removed} = \dot{m}_{water\ removed} C_p \Delta T = (8\times10^{-3} \text{ kg/s})(4178 \text{ J/kg.°C})(55-15)°\text{C} = 1337 \text{ W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot{Q}_{total,\ removed} = \dot{Q}_{glass\ removed} + \dot{Q}_{water\ removed} = 60{,}000 + 1337 = \mathbf{61{,}337\ W}$$

(*b*) The rate of heat loss from the top surface of the water bath is the sum of the heat losses by radiation, natural convection, and evaporation. Then the radiation heat loss from the top surface of water to the surrounding surfaces is

$$\dot{Q}_{rad,top} = \varepsilon A_s \sigma (T_s^4 - T_{surr}^4) = (0.95)(8 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(50+273\text{K})^4 - (15+273\text{K})^4] = 1726\text{W}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (12.35 kPa at 50°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.50)P_{\text{sat@25°C}} = (0.50)(3.169 \text{ kPa}) = 1.585 \text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

At the surface:

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{12.35 \text{ kPa}}{(0.4615 \text{ kPa.m}^3/\text{kg}\cdot\text{K})(50+273 \text{ K})} = 0.0829 \text{ kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325-12.35) \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg}\cdot\text{K})(50+273 \text{ K})} = 0.9598 \text{ kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.0829 + 0.9598 = 1.0427 \text{ kg/m}^3$$

and

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.585 \text{ kPa}}{(0.4615 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273 \text{ K})} = 0.0115 \text{ kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325-1.585) \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273 \text{ K})} = 1.1662 \text{ kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0115 + 1.1662 = 1.1777 \text{ kg/m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The area of the top surface of the water bath is A_s = 2 m × 4 m = 8 m^2 and its perimeter is p = 2(2 + 4) = 12 m. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{8 \text{ m}^2}{12 \text{ m}} = 0.667 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1777-1.0427 \text{ kg/m}^3)(0.667\text{m})^3}{[(1.1777+1.0427)/2 \text{ kg/m}^3](1.68\times10^{-5} \text{ m}^2/\text{s})^2} = 1.27\times10^9$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.27\times10^9 \times 0.726)^{1/3} = 146$$

and $$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(146)(0.0264 \text{ W/m}\cdot°\text{C})}{0.667 \text{ m}} = 5.78 \text{ W/m}^2\cdot°\text{C}$$

Then the natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_s - T_\infty) = (5.78 \text{ W/m}^2\cdot°\text{C})(8 \text{ m}^2)(50-25)°\text{C} = 1156 \text{ W}$$

(*c*) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.68\times10^{-5} \text{ m}^2/\text{s}}{2.72\times10^{-5} \text{ m}^2/\text{s}} = 0.618$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{GrSc})^{1/3} = 0.15(1.27\times10^9 \times 0.618)^{1/3} = 138$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(138)(2.72\times10^{-5} \text{ m}^2/\text{s})}{0.667 \text{ m}} = 0.00564 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\dot{m}_v = h_{\text{mass}} A_s(\rho_{v,s} - \rho_{v,\infty})$$
$$= (0.00567\,\text{m/s})(8\,\text{m}^2)(0.0829 - 0.0116)\text{kg/m}^3$$
$$= 0.00323\,\text{kg/s} = 11.6\,\text{kg/h}$$

and $\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.00323 \text{ kg/s})(2383 \text{ kJ/kg}) = 7.67 \text{ kW} = 7670 \text{ W}$

The total rate of heat loss from the open top surface of the bath to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 1726 + 1156 + 7670 = \mathbf{10{,}552\,W}$$

Therefore, if the water bath is heated electrically, a 10.55 kW resistance heater will be needed just to make up for the heat loss from the top surface.

(*c*) The side surfaces are vertical plates, and treating the air as dry air for simplicity, heat transfer from them by natural convection is determined to be

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\,\text{m/s}^2)(1/310.5\text{K})(50-25)\,\text{K})(1\,\text{m})^3}{(1.68\times10^{-5}\,\text{m}^2/\text{s})^2} = 2.83\times10^9$$

$$\text{Nu} = 0.1(\text{Gr Pr})^{1/3} = 0.1(2.83\times10^9 \times 0.726)^{1/3} = 127$$

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(127)(0.0264\,\text{W/m}\cdot{}^\circ\text{C})}{1\,\text{m}} = 3.36\,\text{W/m}^2\cdot{}^\circ\text{C}$$

$$\dot{Q}_{\text{conv, side}} = h_{\text{conv}} A_s(T_s - T_\infty) = (3.36\,\text{W/m}^2\cdot{}^\circ\text{C})(12\times1\,\text{m}^2)(50-25)^\circ\text{C} = 1007\,\text{W}$$

The radiation heat loss from the side surfaces of the bath to the surrounding surfaces is

$\dot{Q}_{\text{rad,side}} = \varepsilon A_s\sigma(T_s^4 - T_{\text{surr}}^4) = (0.61)(12\,\text{m}\times1\,\text{m})(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)[(50+273\text{K})^4 - (15+273\text{K})^4] = 1662\,\text{W}$ a

nd $\dot{Q}_{\text{total, side}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1007 + 1662 = \mathbf{2669\,W}$

(*d*) The rate at which water must be supplied to the maintain steady operation is equal to the rate of water removed by the bottles plus the rate evaporation,

$$\dot{m}_{\text{make-up}} = \dot{m}_{\text{removed}} + \dot{m}_{\text{evap}} = 0.00800 + 0.00323 = \mathbf{0.01123\ kg/s = 40.4\ kg/h}$$

Noting that the entire make-up water enters the bath 15°C, the rate of heat supply to preheat the make-up water to 50°C is

$$\dot{Q}_{\text{preheating water}} = \dot{m}_{\text{make-up water}} C_p\,\Delta T = (0.01123 \text{ kg/s})(4178\,\text{J/kg}.^\circ\text{C})(50-15)^\circ\text{C} = 1642 \text{ W}$$

Then the rate of required heat supply for the bath becomes the sum of heat losses from the top and side surfaces, plus the heat needed for preheating the make-up water and the bottles,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{bottle}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}}\right)_{\text{top}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}}\right)_{\text{side}} + \dot{Q}_{\text{makeupwater}}$$
$$= 60{,}000 + 10{,}552 + 2669 + 1642 = \mathbf{74{,}863\,W}$$

Therefore, the heater must be able to supply heat at a rate of 75 kW to maintain steady operating conditions