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14-119 A person is standing outdoors in windy weather. The rates of heat loss from the head by radiation, forced convection, and evaporation are to be determined for the cases of the head being wet and dry.

Assumptions 1 The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The head can be approximated as a sphere of 30 cm diameter maintained at a uniform temperature of 30°C. **4** The surrounding surfaces are at the same temperature as the ambient air.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture. The properties of air at the free stream temperature of 25°C and 1 atm are, from Table A-15,

$$k = 0.0255 \text{ W/m} \cdot \text{C}, \quad \text{Pr} = 0.73$$

$$\mu = 1.85 \times 10^{-5} \text{ kg/m} \cdot \text{s} \quad \nu = 1.56 \times 10^{-5} \text{ m}^2/\text{s}$$

Also, $\mu_s = \mu_{@30^\circ\text{C}} = 1.87 \times 10^{-5} \text{ kg/m} \cdot \text{s}$. The mass diffusivity of water vapor in air at the average temperature of $(25 + 30)/2 = 27.5^\circ\text{C} = 300.5 \text{ K}$ is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(300.5 \text{ K})^{2.072}}{1 \text{ atm}} = 2.55 \times 10^{-5} \text{ m}^2/\text{s}$$

The saturation pressure of water at 25°C is $P_{\text{sat}@25^\circ\text{C}} = 3.169 \text{ kPa}$. Properties of water at 30°C are $h_{fg} = 2431 \text{ kJ/kg}$ and $P_v = 4.246 \text{ kPa}$ (Table A-9).

The gas constants of dry air and water are $R_{\text{air}} = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$ and $R_{\text{water}} = 0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$ (Table A-1). Also, the emissivity of the head is given to be 0.95.

Analysis (a) When the head is dry, heat transfer from the head is by forced convection and radiation only. The radiation heat transfer is

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)[\pi(0.3 \text{ m})^2](5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(30 + 273 \text{ K})^4 - (25 + 273 \text{ K})^4] = 8.3 \text{ W}$$

The Reynolds number for flow over the head is

$$\text{Re} = \frac{V_\infty D}{\nu} = \frac{(25/3.6 \text{ m/s})(0.3 \text{ m})}{1.56 \times 10^{-5} \text{ m}^2/\text{s}} = 133,550$$

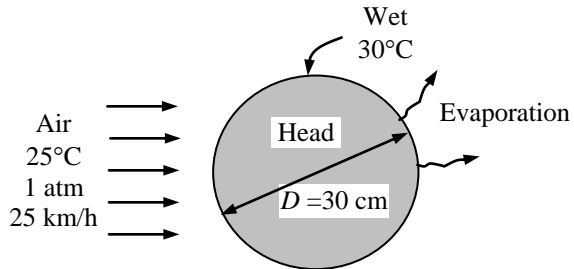
Then the Nusselt number and the heat transfer coefficient become

$$\begin{aligned} \text{Nu} &= 2 + \left[0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3} \right] \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s} \right)^{1/4} \\ &= 2 + \left[0.4(133,550)^{1/2} + 0.06(133,550)^{2/3} \right] (0.73)^{0.4} \left(\frac{1.85 \times 10^{-5}}{1.87 \times 10^{-5}} \right)^{1/4} = 269 \\ h &= \frac{k}{D} \text{Nu} = \frac{0.0255 \text{ W/m} \cdot \text{C}}{0.3 \text{ m}} (269) = 22.9 \text{ W/m}^2 \cdot \text{C} \end{aligned}$$

Then the rate of convection heat transfer from the head becomes

$$\dot{Q}_{\text{conv}} = h A_s (T_s - T_\infty) = (22.9 \text{ W/m}^2 \cdot \text{C}) [\pi(0.3 \text{ m})^2] (30 - 25)^\circ\text{C} = 32.3 \text{ W}$$

Therefore,



$$\dot{Q}_{\text{total,dry}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 32.3 + 8.3 = \mathbf{40.6W}$$

(b) When the head is wet, there is additional heat transfer mechanism by evaporation. The Schmidt number is

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.56 \times 10^{-5} \text{ m}^2/\text{s}}{2.55 \times 10^{-5} \text{ m}^2/\text{s}} = 0.612$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\begin{aligned} Sh &= 2 + \left[0.4 Re^{1/2} + 0.06 Re^{2/3} \right] Sc^{0.4} \left(\frac{\mu_{\infty}}{\mu_s} \right)^{1/4} \\ &= 2 + \left[0.4(133,550)^{1/2} + 0.06(133,550)^{2/3} \right] (0.612)^{0.4} \left(\frac{1.85 \times 10^{-5}}{1.87 \times 10^{-5}} \right)^{1/4} = 251 \\ h_{\text{mass}} &= \frac{Sh D_{AB}}{L} = \frac{(251)(2.55 \times 10^{-5} \text{ m}^2/\text{s})}{0.3 \text{ m}} = 0.0213 \text{ m/s} \end{aligned}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (4.246 kPa at 30°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_{\infty}} = (0.40) P_{\text{sat}@25^{\circ}\text{C}} = (0.40)(3.169 \text{ kPa}) = 1.268 \text{ kPa}$$

Treating the water vapor and the air as ideal gases, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface:} \quad \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{4.246 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(30 + 273) \text{ K}} = 0.0304 \text{ kg} / \text{m}^3$$

$$\text{Away from the surface:} \quad \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_s} = \frac{1.268 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(25 + 273) \text{ K}} = 0.0092 \text{ kg} / \text{m}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.0213 \text{ m/s}) [\pi (0.3 \text{ m})^2] (0.0304 - 0.0092) \text{ kg} / \text{m}^3 \\ &= 0.000128 \text{ kg/s} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.000128 \text{ kg} / \text{s})(2431 \text{ kJ} / \text{kg}) = 0.311 \text{ kW} = 311 \text{ W}$$

Then the total rate of heat loss from the wet head to the surrounding air and surfaces becomes

$$\dot{Q}_{\text{total,wet}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} + \dot{Q}_{\text{evap}} = 32.3 + 8.3 + 311 = \mathbf{351.6W}$$

Discussion Note that the heat loss from the head can be increased by more than 8 times in this case by wetting the head and allowing heat transfer by evaporation.

14-120 The heating system of a heated swimming pool is being designed. The rates of heat loss from the top surface of the pool by radiation, natural convection, and evaporation are to be determined.

Assumptions 1 The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body in the pool is maintained at a uniform temperature of 30°C. **4** The air motion around the pool is negligible so that there are no forced convection effects.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2 = (20+30)/2 = 25^\circ\text{C} = 298\text{ K}$. The properties of dry air at 298 K and 1 atm are, from Table A-15,

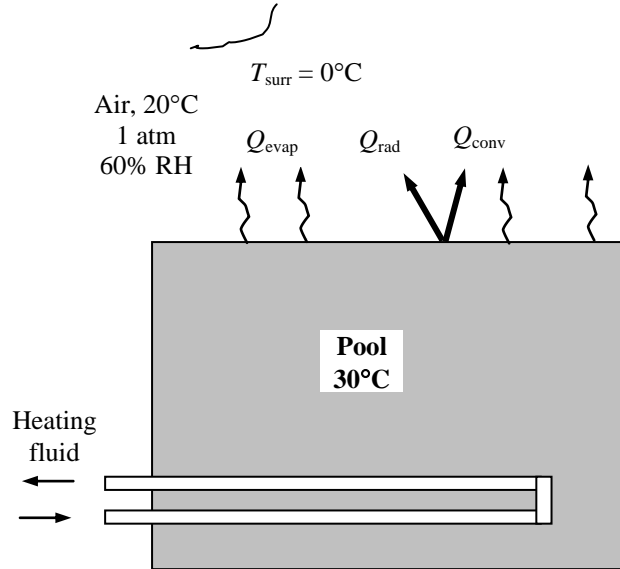
$$k = 0.0255\text{ W/m}\cdot^\circ\text{C}, \text{ Pr} = 0.73$$

$$\alpha = 2.14 \times 10^{-5}\text{ m}^2/\text{s} \quad \nu = 1.56 \times 10^{-5}\text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 298 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P}$$

$$= 1.87 \times 10^{-10} \frac{(298\text{ K})^{2.072}}{1\text{ atm}} = 2.50 \times 10^{-5}\text{ m}^2/\text{s}$$



The saturation pressure of water at 20°C is $P_{\text{sat}@20^\circ\text{C}} = 2.339\text{ kPa}$. Properties of water at 30°C are $h_{fg} = 2431\text{ kJ/kg}$ and $P_v = 4.246\text{ kPa}$ (Table A-9). The gas constants of dry air and water are $R_{\text{air}} = 0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ and $R_{\text{water}} = 0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ (Table A-1). The emissivity of water is 0.95 (Table A-15).

Analysis (a) Noting that the emissivity of water is 0.95 and the surface area of the pool is $A = (20\text{ m})(20\text{ m}) = 400\text{ m}^2$, heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \epsilon A \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(400\text{ m}^2)(5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4)[(30 + 273\text{ K})^4 - (0 + 273\text{ K})^4] = \mathbf{61,930\text{ W}}$$

(b) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (4.246 kPa at 30°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.60)P_{\text{sat}@20^\circ\text{C}} = (0.60)(2.339\text{ kPa}) = 1.40\text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{4.246\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(30 + 273)\text{ K}} = 0.0304\text{ kg/m}^3$$

At the surface:

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 4.246)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(30 + 273)\text{ K}} = 1.1164\text{ kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.0304 + 1.1164 = 1.1168\text{ kg/m}^3$$

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.40 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 0.0104 \text{ kg} / \text{m}^3$$

Away from the surface:

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 1.40) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 1.1883 \text{ kg} / \text{m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0104 + 1.1883 = 1.1987 \text{ kg} / \text{m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The perimeter of the top surface of the pool is $p = 2(20 + 20) = 80 \text{ m}$. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{400 \text{ m}^2}{80 \text{ m}} = 5 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1987 - 1.1468 \text{ kg/m}^3)(5 \text{ m})^3}{[(1.1968 + 1.1468)/2 \text{ kg/m}^3](1.56 \times 10^{-5} \text{ m}^2/\text{s})^2} = 2.26 \times 10^{11}$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(2.26 \times 10^{11} \times 0.73)^{1/3} = 823$$

and
$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(823)(0.0255 \text{ W/m} \cdot \text{°C})}{5 \text{ m}} = 4.20 \text{ W/m}^2 \cdot \text{°C}$$

Then natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (4.20 \text{ W/m}^2 \cdot \text{°C})(400 \text{ m}^2)(30 - 20) \text{°C} = \mathbf{16,780 \text{ W}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.56 \times 10^{-5} \text{ m}^2/\text{s}}{2.50 \times 10^{-5} \text{ m}^2/\text{s}} = 0.624$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{Gr Sc})^{1/3} = 0.15(2.26 \times 10^{11} \times 0.624)^{1/3} = 781$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(781)(2.50 \times 10^{-5} \text{ m}^2/\text{s})}{5 \text{ m}} = 0.00390 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.00390 \text{ m/s})(400 \text{ m}^2)(0.0304 - 0.0104) \text{ kg/m}^3 \\ &= 0.0312 \text{ kg/s} = 112 \text{ kg/h} \end{aligned}$$

and
$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.00312 \text{ kg/s})(2,431,000 \text{ J/kg}) = \mathbf{75,850 \text{ W}}$$

Then the total rate of heat loss from the open top surface of the pool to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 61,930 + 16,780 + 75,850 = \mathbf{154,560 \text{ W}}$$

Therefore, if the pool is heated electrically, a 155 kW resistance heater will be needed to make up for the heat losses from the top surface.

14-121 The heating system of a heated swimming pool is being designed. The rates of heat loss from the top surface of the pool by radiation, natural convection, and evaporation are to be determined.

Assumptions 1 The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body in the pool is maintained at a uniform temperature of 25°C. **4** The air motion around the pool is negligible so that there are no forced convection effects.

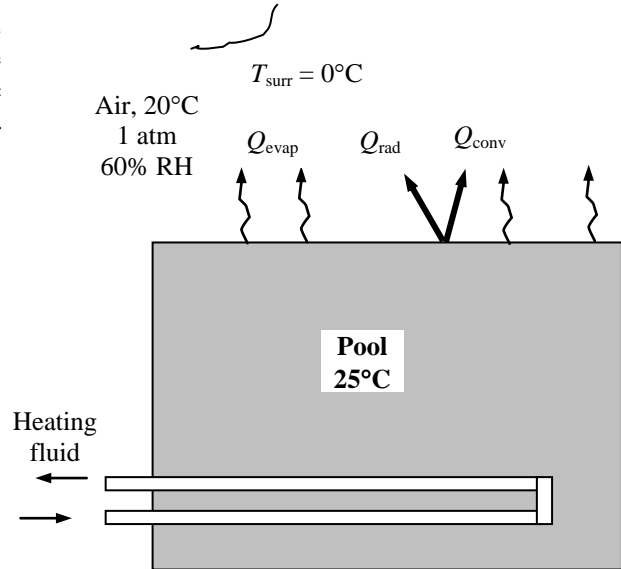
Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2 = (20+25)/2 = 22.5^\circ\text{C} = 295.5\text{ K}$. The properties of dry air at 295.5 K and 1 atm are, from Table A-15,

$$k = 0.0253\text{ W/m}\cdot^\circ\text{C}, \text{ Pr} = 0.73$$

$$\alpha = 2.1 \times 10^{-5} \text{ m}^2/\text{s} \quad \nu = 1.54 \times 10^{-5} \text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 295.5 K is, from Eq. 14-15,

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(295.5\text{ K})^{2.072}}{1\text{ atm}} \\ &= 2.46 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$



The saturation pressure of water at 20°C is $P_{\text{sat}@20^\circ\text{C}} = 2.339\text{ kPa}$. Properties of water at 25°C are $h_{fg} = 2442\text{ kJ/kg}$ and $P_v = 3.169\text{ kPa}$ (Table A-9). The gas constants of dry air and water are $R_{\text{air}} = 0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ and $R_{\text{water}} = 0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ (Table A-1). The emissivity of water is 0.95 (Table A-15).

Analysis (a) Noting that the emissivity of water is 0.95 and the surface area of the pool is $A_s = (20\text{ m})(20\text{ m}) = 400\text{ m}^2$, heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon A \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(400\text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4) [(25 + 273\text{ K})^4 - (0 + 273\text{ K})^4] = \mathbf{50,236\text{ W}}$$

(b) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (3.169 kPa at 25°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.60) P_{\text{sat}@20^\circ\text{C}} = (0.60)(2.339\text{ kPa}) = 1.40\text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{3.169\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25 + 273)\text{ K}} = 0.0230\text{ kg/m}^3$$

At the surface:

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 3.169)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25 + 273)\text{ K}} = 1.1477\text{ kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.0230 + 1.1477 = 1.1707\text{ kg/m}^3$$

and

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.40 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 0.0104 \text{ kg} / \text{m}^3$$

Away from the surface:

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 1.40) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 1.1883 \text{ kg} / \text{m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0104 + 1.1883 = 1.1987 \text{ kg} / \text{m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The perimeter of the top surface of the pool is $p = 2(20 + 20) = 80 \text{ m}$. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{400 \text{ m}^2}{80 \text{ m}} = 5 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1987 - 1.1707 \text{ kg/m}^3)(5 \text{ m})^3}{[(1.1968 + 1.1707) / 2 \text{ kg/m}^3](1.54 \times 10^{-5} \text{ m}^2 / \text{s})^2} = 1.24 \times 10^{11}$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.24 \times 10^{11} \times 0.73)^{1/3} = 674$$

and
$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(674)(0.0253 \text{ W/m} \cdot \text{°C})}{5 \text{ m}} = 3.41 \text{ W/m}^2 \cdot \text{°C}$$

Then natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (3.41 \text{ W/m}^2 \cdot \text{°C})(400 \text{ m}^2)(25 - 20) \text{°C} = \mathbf{6820 \text{ W}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.54 \times 10^{-5} \text{ m}^2 / \text{s}}{2.45 \times 10^{-5} \text{ m}^2 / \text{s}} = 0.629$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{Gr Sc})^{1/3} = 0.15(1.24 \times 10^{11} \times 0.629)^{1/3} = 641$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(641)(2.45 \times 10^{-5} \text{ m}^2 / \text{s})}{5 \text{ m}} = 0.00314 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.00314 \text{ m/s})(400 \text{ m}^2)(0.0230 - 0.0104) \text{ kg/m}^3 \\ &= 0.0158 \text{ kg/s} = 57.0 \text{ kg/h} \end{aligned}$$

and
$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.0158 \text{ kg/s})(2,441,000 \text{ J/kg}) = \mathbf{38,570 \text{ W}}$$

Then the total rate of heat loss from the open top surface of the pool to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 50,236 + 6820 + 38,570 = \mathbf{95,626 \text{ W}}$$

Therefore, if the pool is heated electrically, a 96 kW resistance heater will be needed to make up for the heat losses from the top surface.

Review Problems

14-122C (a) T, (b) F, (c) F, (d) T

14-123 Henry's law is expressed as

$$y_{i, \text{liquid side}}(0) = \frac{P_{i, \text{gas side}}(0)}{H}$$

Henry's constant H increases with temperature, and thus the fraction of gas i in the liquid $y_{i, \text{liquid side}}$ decreases. Therefore, heating a liquid will drive off the dissolved gases in a liquid.

14-124 The ideal gas relation can be expressed as $PV = NR_uT = mRT$ where R_u is the universal gas constant, whose value is the same for all gases, and R is the gas constant whose value is different for different gases. The molar and mass densities of an ideal gas mixture can be expressed as

$$PV = NR_uT \rightarrow C = \frac{N}{V} = \frac{P}{R_uT} = \text{constant}$$

and $PV = mRT \rightarrow \rho = \frac{m}{V} = \frac{P}{RT} \neq \text{constant}$

Therefore, for an ideal gas mixture maintained at a constant temperature and pressure, the molar concentration C of the mixture remains constant but this is not necessarily the case for the density ρ of mixture.

14-125E The masses of the constituents of a gas mixture at a specified temperature and pressure are given. The partial pressure of each gas and the volume of the mixture are to be determined.

Assumptions The gas mixture and its constituents are ideal gases.

Properties The molar masses of CO_2 and CH_4 are 44 and 16 kg/kmol, respectively (Table A-1)

Analysis The mole numbers of each gas and of the mixture are

$$\text{CO}_2: \quad N_{\text{CO}_2} = \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}} = \frac{1 \text{ lbmol}}{44 \text{ lbmol}} = 0.0227 \text{ lbmol}$$

$$\text{CH}_4: \quad N_{\text{CH}_4} = \frac{m_{\text{CH}_4}}{M_{\text{CH}_4}} = \frac{3 \text{ lbmol}}{16 \text{ lbmol}} = 0.1875 \text{ lbmol}$$

$$N_{\text{total}} = N_{\text{CO}_2} + N_{\text{CH}_4} = 0.0227 + 0.1875 = 0.2102$$

1 lbm CO_2 3 lbm CH_4 600 R 20 psia
--

Using the ideal gas relation for the mixture and for the constituents, the volume of the mixture and the partial pressures of the constituents are determined to be

$$V = \frac{NR_u T}{P} = \frac{(0.2102 \text{ lbmol})(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})}{20 \text{ psia}} = \mathbf{67.66 \text{ ft}^3}$$

$$P_{\text{CO}_2} = \frac{N_{\text{CO}_2} R_u T}{V} = \frac{(0.0227 \text{ lbmol})(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})(600 \text{ R})}{67.66 \text{ ft}^3} = \mathbf{2.16 \text{ psia}}$$

$$P_{\text{CH}_4} = \frac{N_{\text{CH}_4} R_u T}{V} = \frac{(0.1875 \text{ lbmol})(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})(600 \text{ R})}{67.66 \text{ ft}^3} = \mathbf{17.84 \text{ psia}}$$

Discussion Note that each constituent of a gas mixture occupies the same volume (the volume of the container), and that the total pressure of a gas mixture is equal to the sum of the partial pressures of its constituents. That is, $P_{\text{total}} = P_{\text{CO}_2} + P_{\text{CH}_4} = 2.16 + 17.84 = 20 \text{ psia}$.

14-126 Dry air flows over a water body at constant pressure and temperature until it is saturated. The molar analysis of the saturated air and the density of air before and after the process are to be determined.

Assumptions The air and the water vapor are ideal gases.

Properties The molar masses of N₂, O₂, Ar, and H₂O are 28.0, 32.0, 39.9 and 18 kg / kmol, respectively (Table A-1). The molar analysis of dry air is given to be 78.1 percent N₂, 20.9 percent O₂, and 1 percent Ar. The saturation pressure of water at 25°C is 3.169 kPa (Table A-9). Also, 1 atm = 101.325 kPa.

Analysis (a) Noting that the total pressure remains constant at 101.32 kPa during this process, the partial pressure of air becomes

$$P = P_{\text{air}} + P_{\text{vapor}}$$

$$P_{\text{air}} = P - P_{\text{vapor}}$$

$$= 101.325 - 3.169 = 98.156 \text{ kPa}$$

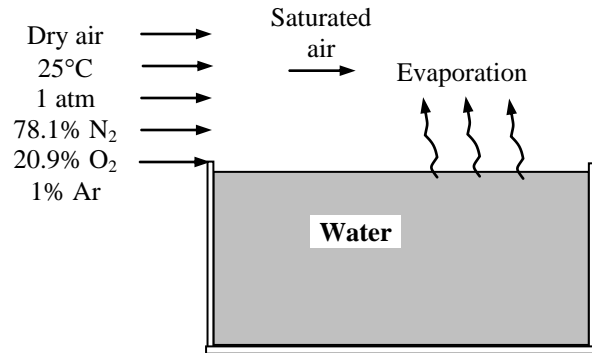
Then the molar analysis of the saturated air becomes

$$y_{\text{H}_2\text{O}} = \frac{P_{\text{H}_2\text{O}}}{P} = \frac{3.169}{101.325} = \mathbf{0.0313}$$

$$y_{\text{N}_2} = \frac{P_{\text{N}_2}}{P} = \frac{y_{\text{N}_2, \text{dry}} P_{\text{dry air}}}{P} = \frac{0.781(98.156 \text{ kPa})}{101.325} = \mathbf{0.7566}$$

$$y_{\text{O}_2} = \frac{P_{\text{O}_2}}{P} = \frac{y_{\text{O}_2, \text{dry}} P_{\text{dry air}}}{P} = \frac{0.209(98.156 \text{ kPa})}{101.325} = \mathbf{0.2025}$$

$$y_{\text{Ar}} = \frac{P_{\text{Ar}}}{P} = \frac{y_{\text{Ar, dry}} P_{\text{dry air}}}{P} = \frac{0.01(98.156 \text{ kPa})}{101.325} = \mathbf{0.0097}$$



(b) The molar masses of dry and saturated air are

$$M_{\text{dry air}} = \sum y_i M_i = 0.781 \times 28.0 + 0.209 \times 32.0 + 0.01 \times 39.9 = 29.0 \text{ kg / kmol}$$

$$M_{\text{sat air}} = \sum y_i M_i = 0.7565 \times 28.0 + 0.2025 \times 32.0 + 0.0097 \times 39.9 + 0.0313 \times 18 = 28.62 \text{ kg / kmol}$$

Then the densities of dry and saturated air are determined from the ideal gas relation to be

$$\rho_{\text{dry air}} = \frac{P}{(R_u / M_{\text{dry air}}) T} = \frac{101.325 \text{ kPa}}{[(8.314 \text{ kPa} \cdot \text{m}^3 / \text{kmol} \cdot \text{K}) / 29.0 \text{ kg / kmol}](25 + 273) \text{ K}} = \mathbf{1.186 \text{ kg / m}^3}$$

$$\rho_{\text{sat air}} = \frac{P}{(R_u / M_{\text{sat air}}) T} = \frac{101.325 \text{ kPa}}{[(8.314 \text{ kPa} \cdot \text{m}^3 / \text{kmol} \cdot \text{K}) / 28.62 \text{ kg / kmol}](25 + 273) \text{ K}} = \mathbf{1.170 \text{ kg / m}^3}$$

Discussion We conclude that the density of saturated air is less than that of the dry air, as expected. This is due to the molar mass of water being less than that of dry air.

14-127 A glass of water is left in a room. The mole fraction of the water vapor in the air at the water surface and far from the surface as well as the mole fraction of air in the water near the surface are to be determined when the water and the air are at the same temperature.

Assumptions 1 Both the air and water vapor are ideal gases. 2 Air is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 25°C is 3.169 kPa (Table A-9). Henry's constant for air dissolved in water at 25°C (298 K) is given in Table 14-6 to be $H = 71,200$ bar. Molar masses of dry air and water are 29 and 18 kg/kmol, respectively (Table A-1).

Analysis (a) Noting that the relative humidity of air is 70%, the partial pressure of water vapor in the air far from the water surface will be

$$P_{v, \text{room air}} = \phi P_{\text{sat}@25^\circ\text{C}} = (0.7)(3.169 \text{ kPa}) = 2.218 \text{ kPa}$$

Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the room air is

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{2.218 \text{ kPa}}{100 \text{ kPa}} = \mathbf{0.0222} \quad (\text{or } \mathbf{2.22\%})$$

(b) Noting that air at the water surface is saturated, the partial pressure of water vapor in the air near the surface will simply be the saturation pressure of water at 20°C, $P_{v, \text{interface}} = P_{\text{sat}@25^\circ\text{C}} = 3.169 \text{ kPa}$. Then the mole fraction of water vapor in the air at the interface becomes

$$y_{v, \text{surface}} = \frac{P_{v, \text{surface}}}{P} = \frac{3.169 \text{ kPa}}{100 \text{ kPa}} = \mathbf{0.0317} \quad (\text{or } \mathbf{3.17\%})$$

(c) Noting that the total pressure is 100 kPa, the partial pressure of dry air at the water surface is

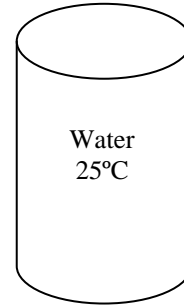
$$P_{\text{air, surface}} = P - P_{v, \text{surface}} = 100 - 3.169 = 96.831 \text{ kPa}$$

From Henry's law, the mole fraction of air in the water is determined to be

$$y_{\text{dry air, liquid side}} = \frac{P_{\text{dry air, gas side}}}{H} = \frac{(96.831/101.325) \text{ bar}}{71,200 \text{ bar}} = \mathbf{1.34 \times 10^{-5}}$$

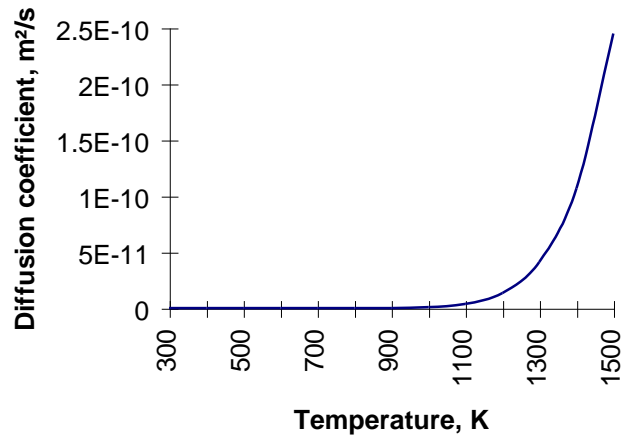
Discussion The water cannot remain at the room temperature when the air is not saturated. Therefore, some water will evaporate and the water temperature will drop until a balance is reached between the rate of heat transfer to the water and the rate of evaporation.

Air
25°C
100 kPa
70% RH



14-128 Using the relation $D_{AB} = 2.67 \times 10^{-5} \exp(-17,400/T)$ the diffusion coefficient of carbon in steel is determined to be

T, K	$D_{AB}, m^2/s$
300	1.728×10^{-30}
400	3.426×10^{-24}
500	2.056×10^{-20}
600	6.792×10^{-18}
700	4.277×10^{-16}
800	9.563×10^{-15}
900	1.071×10^{-13}
1000	7.409×10^{-13}
1100	3.604×10^{-12}
1200	1.347×10^{-11}
1300	4.108×10^{-11}
1400	1.068×10^{-10}
1500	2.440×10^{-10}



14-129 A 2-L bottle is filled with carbonated drink that is fully charged (saturated) with CO₂ gas. The volume that the CO₂ gas would occupy if it is released and stored in a container at room conditions is to be determined.

Assumptions 1 The liquid drink can be treated as water. 2 Both the CO₂ gas and the water vapor are ideal gases. 3 The CO₂ gas is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 17°C is 1.96 kPa (Table A-9). Henry's constant for CO₂ dissolved in water at 17°C (290 K) is $H = 1280$ bar (Table 14-6). Molar masses of CO₂ and water are 44.01 and 18.015 kg/kmol, respectively (Table A-1). The gas constant of CO₂ is 0.1889 kPa·m³/kg·K. Also, 1 bar = 100 kPa.

Analysis (a) In the charging station, the CO₂ gas and water vapor mixture above the liquid will form a saturated mixture. Noting that the saturation pressure of water at 17°C is 1.96 kPa, the partial pressure of the CO₂ gas is

$$P_{\text{CO}_2, \text{gas side}} = P - P_{\text{vapor}} = P - P_{\text{sat}@17^\circ\text{C}} = 600 - 1.96 = 598.04 \text{ kPa} = 5.9804 \text{ bar}$$

From Henry's law, the mole fraction of CO₂ in the liquid drink is determined to be

$$y_{\text{CO}_2, \text{liquid side}} = \frac{P_{\text{CO}_2, \text{gas side}}}{H} = \frac{5.9804 \text{ bar}}{1280 \text{ bar}} = 0.00467$$

Then the mole fraction of water in the drink becomes

$$y_{\text{water, liquid side}} = 1 - y_{\text{CO}_2, \text{liquid side}} = 1 - 0.00467 = 0.99533$$

The mass and mole fractions of a mixture are related to each other by

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

where the apparent molar mass of the drink (liquid water - CO₂ mixture) is

$$M_m = \sum y_i M_i = y_{\text{liquid water}} M_{\text{water}} + y_{\text{CO}_2} M_{\text{CO}_2} = 0.99533 \times 18.015 + 0.00467 \times 44.01 = 18.14 \text{ kg/kmol}$$

Then the mass fraction of dissolved CO₂ in liquid drink becomes

$$w_{\text{CO}_2, \text{liquid side}} = y_{\text{CO}_2, \text{liquid side}} \frac{M_{\text{CO}_2}}{M_m} = 0.00467 \frac{44.01}{18.14} = 0.0113$$

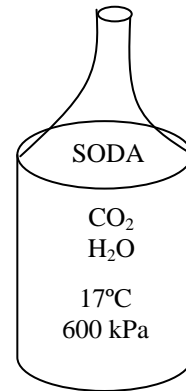
Therefore, the mass of dissolved CO₂ in a 2 L ≈ 2 kg drink is

$$m_{\text{CO}_2} = w_{\text{CO}_2} m_m = 0.0113(2 \text{ kg}) = 0.0226 \text{ kg}$$

Then the volume occupied by this CO₂ at the room conditions of 25°C and 100 kPa becomes

$$V = \frac{mRT}{P} = \frac{(0.0226 \text{ kg})(0.1889 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(298 \text{ K})}{100 \text{ kPa}} = \mathbf{0.0127 \text{ m}^3 = 12.7 \text{ L}}$$

Discussion Note that the amount of dissolved CO₂ in a 2-L pressurized drink is large enough to fill 6 such bottles at room temperature and pressure. Also, we could simplify the calculations by assuming the molar mass of carbonated drink to be the same as that of water, and take it to be 18 kg/kmol because of the very low mole fraction of CO₂ in the drink.



14-130 The walls of a house are made of 20-cm thick bricks. The maximum amount of water vapor that will diffuse through a 4 m × 7 m section of the wall in 24-h is to be determined.

Assumptions 1 Steady operating conditions exist. 2 Mass transfer through the wall is one-dimensional. 3 The vapor permeability of the wall is constant. 4 The vapor pressure at the outer side of the wall is zero.

Properties The permeance of the brick wall is given to be 23×10^{-12} kg/s.m².Pa. The saturation pressure of water at 20°C is 2339 Pa (Table 14-9).

Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given by (Eq. 14-31)

$$\dot{m}_v = PA \frac{P_{v,1} - P_{v,2}}{L} = PA \frac{\phi_1 P_{sat,1} - \phi_2 P_{sat,2}}{L} = MA(\phi_1 P_{sat,1} - \phi_2 P_{sat,2})$$

where P is the vapor permeability and $M = P/L$ is the permeance of the material, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall.

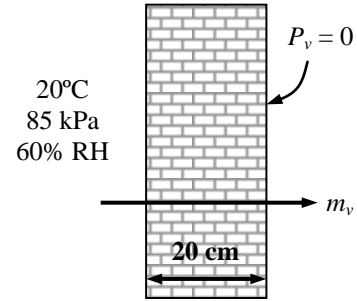
Noting that the vapor pressure at the outer side of the wallboard is zero ($\phi_2 = 0$) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (23 \times 10^{-12} \text{ kg/s.m}^2.\text{Pa})(4 \times 7 \text{ m}^2)[0.60(2339 \text{ Pa}) - 0] = 9.038 \times 10^{-7} \text{ kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v \Delta t = (9.038 \times 10^{-7} \text{ kg/s})(24 \times 3600 \text{ s}) = \mathbf{0.0781 \text{ kg} = 78.1 \text{ g}}$$

Discussion This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero.



14-131E The thermal and vapor resistances of different layers of a wall are given. The rates of heat and moisture transfer through the wall under steady conditions are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Heat transfer through the wall is one-dimensional. 3 Thermal and vapor resistances of different layers of the wall and the heat transfer coefficients are constant. 4 Condensation does not occur inside the wall.

Properties The thermal and vapor resistances are as given in the problem statement. The saturation pressures of water at 70°F and 32°F are 0.3632 and 0.0887 psia, respectively (Table 14-9E).

Analysis Noting that all the layers of the wall are in series, the total thermal resistance of the wall for a 1-ft² section is determined by simply adding the R-values of all layers

$$R_{\text{total}} = \sum R\text{-value} = 0.17 + 0.43 + 0.10 + 4.20 + 1.02 + 0.45 + 0.68 = 7.05 \text{ h} \cdot \text{ft}^2 \cdot ^\circ\text{F}/\text{Btu}$$

Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{wall}} = A \frac{T_i - T_o}{R_{\text{total}}} = (9 \times 25 \text{ ft}^2) \frac{(70 - 32)^\circ\text{F}}{7.05 \text{ h} \cdot \text{ft}^2 \cdot ^\circ\text{F}/\text{Btu}} = \mathbf{1436 \text{ Btu/h}}$$

The vapor pressures at the indoors and the outdoors is

$$P_{v,1} = \phi_1 P_{\text{sat},1} = 0.65 \times (0.3632 \text{ psia}) = 0.2361 \text{ psia}$$

$$P_{v,2} = \phi_2 P_{\text{sat},2} = 0.40 \times (0.0887 \text{ psia}) = 0.0355 \text{ psia}$$

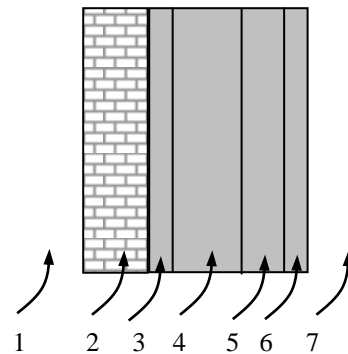
The total vapor resistance of the wall for a 1-ft² section is determined by simply adding the R_v-values of all layers,

$$R_{v,\text{total}} = \sum R_v\text{-value} = 15,000 + 1930 + 23,000 + 77.6 + 332 = 40,340 \text{ s} \cdot \text{ft}^2 \cdot \text{psia}/\text{lbm}$$

Then the rate of moisture flow through the interior and exterior parts of the wall becomes

$$\dot{m}_{v,\text{wall}} = A \frac{P_{v,1} - P_{v,2}}{R_{v,\text{total}}} = (9 \times 25 \text{ ft}^2) \frac{(0.2361 - 0.0355) \text{ psia}}{40,340 \text{ s} \cdot \text{ft}^2 \cdot \text{psia}/\text{lbm}} = 0.00112 \text{ lbm/s} = \mathbf{4.03 \text{ lbm/h}}$$

Construction	R-value, h.ft ² .°F/Btu	R _v -value, s.ft ² .psi/lbm
1. Outside surface, 15 mph wind	0.17	-
2. Face brick, 4 in.	0.43	15,000
3. Cement mortar, 0.5 in.	0.10	1930
4. Concrete block, 6 in.	4.20	23,000
5. Air space, ¾ in.	1.02	77.6
6. Gypsum wallboard, 0.5 in.	0.45	332
7. Inside surface, still air	0.68	-



14-132 An aquarium is oxygenated by forcing air to the bottom of it. The mole fraction of water vapor is to be determined at the center of the air bubbles when they reach the free surface of water.

Assumptions **1** The air bubbles are initially completely dry. **2** The bubbles are spherical and possess symmetry about the midpoint. **3** Air is weakly soluble in water and thus Henry's law is applicable. **4** Convection effects in the bubble are negligible. **5** The pressure and temperature of the air bubbles remain constant at 1 atm and 25°C. **6** Both the air and the vapor are ideal gases.

Properties Henry's constant for oxygen dissolved in water at 300 K ($\cong 25^\circ\text{C}$) is given in Table 14-6 to be $H = 43,600$ bar. The saturation pressure of water at 25°C is 3.169 kPa (Table A-9). The mass diffusivity of water vapor in air at 298 K is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(298 \text{ K})^{2.072}}{1 \text{ atm}} = 2.50 \times 10^{-5} \text{ m}^2 / \text{s}$$

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a sphere with specified surface temperature, and thus can be solved accordingly. Noting that the air in the bubble at the air-water interface will be saturated, the vapor pressure at the interface will be

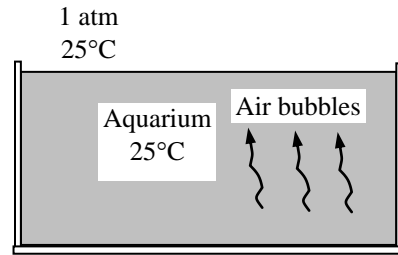
$$P_{v,\text{surface}} = P_{\text{sat}@25^\circ\text{C}} = 3.169 \text{ kPa}$$

Then the mole fraction of vapor at the bubble interface becomes

$$y_{v,\text{surface}} = \frac{P_{v,\text{surface}}}{P} = \frac{3.169 \text{ kPa}}{101.325 \text{ kPa}} = 0.0313$$

The mass transfer Fourier number for $t = 2$ s is

$$\tau = \frac{D_{AB}t}{r_0^2} = \frac{(2.50 \times 10^{-5} \text{ m}^2 / \text{s})(2 \text{ s})}{(2 \times 10^{-3} \text{ m})^2} = 12.5$$



Then the mole fraction of water vapor at the center of the bubble in 2 s can be determined from

$$\frac{y_{v,\text{center}} - y_{v,\text{surface}}}{y_{v,\text{initial}} - y_{v,\text{surface}}} = A_1 e^{-\lambda_1^2 \tau}$$

The Biot number $\text{Bi} = hr_0/k$ in this case is infinity since a specified surface concentration corresponds to an infinitely large mass transfer coefficient ($h \rightarrow \infty$). Then the two constants in the equation above are determined from Table 4-1 to be $\lambda_1 = 3.1416$ and $A_1 = 2$. Also, $y_{v,\text{initial}} = 0$ since the air is initially dry.

Substituting, the mole fraction of water vapor at the center of the bubble in 2 s is determined to be

$$\frac{y_{v,\text{center}} - 0.0313}{0 - 0.0313} = 2 e^{-(3.1416)^2 (12.5)} = 5.27 \times 10^{-54} \cong 0 \rightarrow y_{v,\text{center}} = y_{v,\text{surface}} = \mathbf{0.0313}$$

That is, the air bubbles become saturated when they leave the aquarium.

14-133 An aquarium is oxygenated by forcing oxygen to the bottom of it, and letting the oxygen bubbles rise. The penetration depth of oxygen in the water during the rising time is to be determined.

Assumptions **1** Convection effects in the water are negligible. **2** The pressure and temperature of the oxygen bubbles remain constant.

Properties The mass diffusivity of oxygen in liquid water at 298 K is $D_{AB} = 2.5 \times 10^{-9} \text{ m}^2/\text{s}$ (Table 14-3b).

Analysis The penetration depth can be determined directly from its definition (Eq. 14-38) to be

$$\begin{aligned} \delta_{\text{diff}} &= \sqrt{\pi D_{AB} t} = \sqrt{\pi (2.5 \times 10^{-9} \text{ m}^2/\text{s})(2 \text{ s})} \\ &= 1.25 \times 10^{-4} \text{ m} = \mathbf{0.125 \text{ mm}} \end{aligned}$$

Therefore, oxygen will penetrate the water only a fraction of a millimeter.

1 atm
25°C

