

<http://www.Drshokuhi.com>

سایت آموزش مهندسی مکانیک

**14-134** A circular pan filled with water is cooled naturally. The rate of evaporation of water, the rate of heat transfer by natural convection, and the rate of heat supply to the water needed to maintain its temperature constant are to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 25°C). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Radiation heat transfer is negligible. **4** Both air and water vapor are ideal gases.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (15+20)/2 = 17.5^\circ\text{C} = 290.5\text{ K}$ . The properties of dry air at 290.5 K and 1 atm are, from Table A-15,

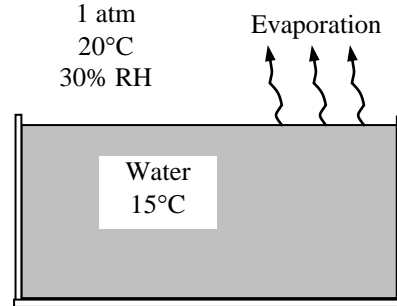
$$k = 0.0251\text{ W/m}\cdot^\circ\text{C}, \text{ Pr} = 0.731$$

$$\alpha = 2.04 \times 10^{-5}\text{ m}^2/\text{s} \quad \nu = 1.49 \times 10^{-5}\text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 290.5 K is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P}$$

$$= 1.87 \times 10^{-10} \frac{(290.5\text{ K})^{2.072}}{1\text{ atm}} = 2.37 \times 10^{-5}\text{ m}^2/\text{s}$$



The saturation pressure of water at 20°C is  $P_{\text{sat}@20^\circ\text{C}} = 2.339\text{ kPa}$ . Properties of water at 15°C are  $h_{fg} = 2466\text{ kJ/kg}$  and  $P_v = 1.7051\text{ kPa}$  (Table A-9). The specific heat of water at the average temperature of  $(15+20)/2 = 17.5^\circ\text{C}$  is  $C_p = 4.184\text{ kJ/kg}\cdot^\circ\text{C}$ . The gas constants of dry air and water are  $R_{\text{air}} = 0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  and  $R_{\text{water}} = 0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  (Table A-1).

**Analysis (a)** The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (1.7051 kPa at 15°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.30)P_{\text{sat}@20^\circ\text{C}} = (0.30)(2.339\text{ kPa}) = 0.7017\text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{1.7051\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(15+273)\text{ K}} = 0.01283\text{ kg/m}^3$$

At the surface:

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 1.7051)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(15+273)\text{ K}} = 1.2052\text{ kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.01283 + 1.2052 = 1.21803\text{ kg/m}^3$$

and

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.7017\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\text{ K}} = 0.00520\text{ kg/m}^3$$

Away from the surface:

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 0.7017)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\text{ K}} = 1.1966\text{ kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0052 + 1.1966 = 1.2018\text{ kg/m}^3$$

Note that  $\rho_\infty < \rho_s$ , and thus this corresponds to hot surface facing down. The area of the top surface of the water  $A_s = \pi r_o^2$  and its perimeter is  $p = 2\pi r_o$ . Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{\pi r_o^2}{2\pi r_o} = \frac{r_o}{2} = \frac{0.15 \text{ m}}{2} = 0.075 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.2180 - 1.2018 \text{ kg/m}^3)(0.075 \text{ m})^3}{[(1.2180 + 1.2018)/2 \text{ kg/m}^3](1.49 \times 10^{-5} \text{ m}^2/\text{s})^2} = 2.53 \times 10^5$$

Recognizing that this is a natural convection problem with cold horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be (Eq. 14-13)

$$\text{Nu} = 0.27(\text{Gr Pr})^{1/4} = 0.27(2.53 \times 10^5 \times 0.731)^{1/4} = 5.60$$

and

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(5.60)(0.0250 \text{ W/m} \cdot \text{°C})}{0.075 \text{ m}} = 1.87 \text{ W/m}^2 \cdot \text{°C}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_\infty - T_s) = (1.87 \text{ W/m}^2 \cdot \text{°C})[\pi(0.15 \text{ m})^2](20 - 15) \text{°C} = \mathbf{0.66 \text{ W}} \quad (\text{to water})$$

(b) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.49 \times 10^{-5} \text{ m}^2/\text{s}}{2.37 \times 10^{-5} \text{ m}^2/\text{s}} = 0.629$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.27(\text{GrSc})^{1/4} = 0.27(2.53 \times 10^5 \times 0.629)^{1/4} = 5.39$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(5.39)(2.37 \times 10^{-5} \text{ m}^2/\text{s})}{0.075 \text{ m}} = 0.00170 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.00170 \text{ m/s})[\pi(0.15 \text{ m})^2](0.01283 - 0.00520) \text{ kg/m}^3 \\ &= 9.17 \times 10^{-7} \text{ kg/s} = \mathbf{0.0033 \text{ kg/h}} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (9.17 \times 10^{-7} \text{ kg/s})(2466 \text{ kJ/kg}) = 0.00226 \text{ kW} = 2.26 \text{ W}$$

(c) The net rate of heat transfer to the water needed to maintain its temperature constant at 15°C is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{evap}} + \dot{Q}_{\text{conv}} = 2.26 + (-0.66) = \mathbf{1.6 \text{ W}}$$

**Discussion** Note that if no heat is supplied to the water (by a resistance heater, for example), the temperature of the water in the pan would drop until the heat gain by convection equals the heat loss by evaporation.

**14-135** Air is blown over a circular pan filled with water. The rate of evaporation of water, the rate of heat transfer by convection, and the rate of energy supply to the water to maintain its temperature constant are to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 25°C). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Radiation heat transfer is negligible. **4** Both air and water vapor are ideal gases.

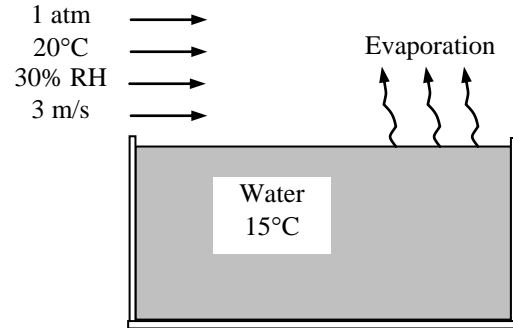
**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (15+20)/2 = 17.5^\circ\text{C} = 290.5\text{ K}$ . The properties of dry air at 290.5 K and 1 atm are, from Table A-15,

$$k = 0.0251\text{ W/m}\cdot^\circ\text{C}, \text{ Pr} = 0.731$$

$$\alpha = 2.04 \times 10^{-5}\text{ m}^2/\text{s} \quad \nu = 1.49 \times 10^{-5}\text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 290.5 K is, from Eq. 14-15,

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(290.5\text{ K})^{2.072}}{1\text{ atm}} = 2.37 \times 10^{-5}\text{ m}^2/\text{s} \end{aligned}$$



The saturation pressure of water at 20°C is  $P_{\text{sat}@20^\circ\text{C}} = 2.339\text{ kPa}$ . Properties of water at 15°C are  $h_{fg} = 2466\text{ kJ/kg}$  and  $P_v = 1.7051\text{ kPa}$  (Table A-9). Also, the gas constants of water is  $R_{\text{water}} = 0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  (Table A-1).

**Analysis (a)** Taking the radius of the pan  $r_0 = 0.15\text{ m}$  to be the characteristic length, the Reynolds number for flow over the pan is

$$\text{Re} = \frac{VL}{\nu} = \frac{(3\text{ m/s})(0.15\text{ m})}{1.49 \times 10^{-5}\text{ m}^2/\text{s}} = 30,201$$

which is less than 500,000, and thus the flow is laminar over the entire surface. The Nusselt number and the heat transfer coefficient are

$$\text{Nu} = 0.664\text{Re}_L^{0.5}\text{Pr}^{1/3} = 0.664(30,201)^{0.5}(0.731)^{1/3} = 103.9$$

$$h_{\text{heat}} = \frac{\text{Nu}k}{L} = \frac{(103.9)(0.0250\text{ W/m}\cdot^\circ\text{C})}{0.15\text{ m}} = 17.3\text{ W/m}^2\cdot^\circ\text{C}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_\infty - T_s) = (17.3\text{ W/m}^2\cdot^\circ\text{C})[\pi(0.15\text{ m})^2](20 - 15)^\circ\text{C} = \mathbf{6.1\text{ W}} \quad (\text{to water})$$

**(b)** Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.49 \times 10^{-5}\text{ m}^2/\text{s}}{2.37 \times 10^{-5}\text{ m}^2/\text{s}} = 0.629$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\text{Re}_L^{0.5}\text{Sc}^{1/3} = 0.664(30,201)^{0.5}(0.629)^{1/3} = 98.9$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(98.9)(2.37 \times 10^{-5}\text{ m}^2/\text{s})}{0.15\text{ m}} = 0.0156\text{ m/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (1.7051 kPa at 15°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.30)P_{\text{sat}@20^\circ\text{C}} = (0.30)(2.339 \text{ kPa}) = 0.7017 \text{ kPa}$$

Treating the water vapor and the air as ideal gases, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface: } \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{1.7051 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(15 + 273) \text{ K}} = 0.01283 \text{ kg} / \text{m}^3$$

$$\text{Away from the surface: } \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.7017 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 0.00520 \text{ kg} / \text{m}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.0156 \text{ m/s})[\pi(0.15 \text{ m})^2](0.01283 - 0.00520) \text{ kg/m}^3 \\ &= 8.41 \times 10^{-6} \text{ kg/s} = \mathbf{0.0303 \text{ kg/h}} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (8.41 \times 10^{-6} \text{ kg/s})(2466 \text{ kJ/kg}) = 0.0207 \text{ kW} = 20.7 \text{ W}$$

(c) The net rate of heat transfer to the water needed to maintain its temperature constant at 15°C is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{evap}} + \dot{Q}_{\text{conv}} = 20.7 + (-6.1) = \mathbf{14.6 \text{ W}}$$

**Discussion** Note that if no heat is supplied to the water (by a resistance heater, for example), the temperature of the water in the pan would drop until the heat gain by convection equals the heat loss by evaporation.

Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation  $\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty})$ .

**14-136** A spherical naphthalene ball is hanged in a closet. The time it takes for the naphthalene to sublimate completely is to be determined.

**Assumptions 1** The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable (will be verified). **2** Both air and naphthalene vapor are ideal gases. **3** The naphthalene and the surrounding air are at the same temperature. **4** The radiation effects are negligible.

**Properties** The molar mass of naphthalene is 128.2 kg/kmol. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 298 K and 1 atm, at which  $\rho = 1.18 \text{ kg/m}^3$ ,  $C_p = 1007 \text{ J/kg} \cdot \text{K}$ , and  $\alpha = 2.14 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15).

**Analysis** The incoming air is free of naphthalene, and thus the mass fraction of naphthalene at free stream conditions is zero,  $w_{A,\infty} = 0$ . Noting that the vapor pressure of naphthalene at the surface is 11 Pa, the mass fraction of naphthalene on the air side of the surface is

$$w_{A,s} = \frac{P_{A,s}}{P} \left( \frac{M_A}{M_{air}} \right) = \frac{11 \text{ Pa}}{101,325 \text{ Pa}} \left( \frac{128.2 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 4.8 \times 10^{-4}$$

Normally we would expect natural convection currents to develop around the naphthalene ball because the amount of naphthalene near the surface is much larger, and determine the Nusselt number (and its counterpart in mass transfer, the Sherwood number) from Eq. 14-16,

$$\text{Nu} = 2 + \frac{0.589 \text{ Ra}^{1/4}}{[1 + (0.469 / \text{Pr})^{9/16}]^{4/9}}$$

But the mass fraction value determined above indicates that the amount of naphthalene in the air is so low that it will not cause any significant difference in the density of air. With no density gradient, there will be no natural convection and thus the Rayleigh number can be taken to be zero. Then the Nusselt number relation above will reduce to  $\text{Nu} = 2$  or its equivalent  $\text{Sh} = 2$ . Then using the definition of Sherwood number, the mass transfer coefficient can be expressed as

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{2 D_{AB}}{D}$$

The mass of naphthalene ball can be expressed as  $m = \rho_{\text{naph}} V = \frac{1}{6} \rho_{\text{naph}} (\pi D^3)$ . The rate of change of the mass of naphthalene is equal to the rate of mass transfer from naphthalene to the air, and is expressed as

$$\begin{aligned} \frac{dm}{dt} &= -h_{\text{mass}} \rho_{\text{air}} A (w_{A,s} - w_{A,\infty}) \\ \frac{d}{dt} \left( \frac{1}{6} \rho_{\text{naph}} (\pi D^3) \right) &= -\frac{2 D_{AB}}{D} \rho_{\text{air}} (\pi D^2) (w_{A,s} - w_{A,\infty}) \\ \frac{3}{6} \pi \rho_{\text{naph}} D^2 \frac{dD}{dt} &= -2 D_{AB} (\pi D) \rho_{\text{air}} (w_{A,s} - w_{A,\infty}) \end{aligned}$$

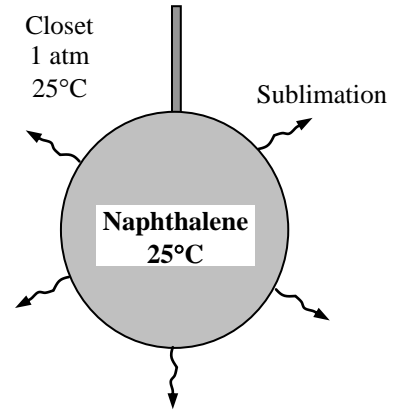
Simplifying and rearranging,  $D dD = -\frac{4 \rho_{\text{air}} D_{AB}}{\rho_{\text{naph}}} (w_{A,s} - w_{A,\infty}) dt$

Integrating from  $D = D_i = 0.03 \text{ m}$  at time  $t = 0$  to  $D = 0$  (complete sublimation) at time  $t = t$  gives

$$t = \frac{\rho_{\text{naph}} D_i^2}{8 \rho_{\text{air}} D_{AB} (w_{A,s} - w_{A,\infty})}$$

Substituting, the time it takes for the naphthalene to sublimate completely is determined to be

$$t = \frac{\rho_{\text{naph}} D_i^2}{8 \rho_{\text{air}} D_{AB} (w_{A,s} - w_{A,\infty})} = \frac{(1100 \text{ kg/m}^3)(0.01 \text{ m})^2}{8(1.19 \text{ kg/m}^3)(4.80 \times 10^{-4} - 0) \text{ m}^2/\text{s}} = 3.95 \times 10^6 \text{ s} = \mathbf{45.7 \text{ days}}$$



**14-137E** A swimmer extends his wet arms into the windy air outside. The rate at which water evaporates from both arms and the corresponding rate of heat transfer by evaporation are to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 60°F). 2 The arm can be modeled as a long cylinder.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of  $(40 + 80)/2 = 60^\circ\text{F}$  and 1 atm, for which  $\nu = 0.159 \times 10^{-3} \text{ ft}^2/\text{s}$ , and  $\rho = 0.077 \text{ lbm}/\text{ft}^3$  (Table A-15E). The saturation pressure of water at 40°F is 0.1217 psia. Also, at 80°F, the saturation pressure is 0.5073 psia and the heat of vaporization is 1048 Btu/lbm (Table A-9E). The molar mass of water is  $R = 0.5956 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R}$  (Table A-1E). The mass diffusivity of water vapor in air at 60°F = 520 R = 288.9 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288.9 \text{ K})^{2.072}}{1 \text{ atm}} = 2.35 \times 10^{-5} \text{ m}^2/\text{s} = 2.53 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** The Reynolds number for flow over a cylinder is

$$\text{Re} = \frac{VD}{\nu} = \frac{(20 \times 5280/3600 \text{ ft/s})(3/12 \text{ ft})}{0.159 \times 10^{-3} \text{ ft}^2/\text{s}} = 46,120$$

The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{0.159 \times 10^{-3} \text{ ft}^2/\text{s}}{2.53 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.628$$

Then utilizing the analogy between heat and mass convection, the Sherwood number is determined from Eq. 10-32 by replacing Pr number by the Schmidt number to be

$$\text{Sh} = 0.3 + \frac{0.62 \text{Re}^{0.5} \text{Sc}^{1/3}}{\left[1 + (0.4/\text{Sc})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\text{Re}}{28200}\right)^{5/8}\right]^{4/5} = 0.3 + \frac{0.62(46,120)^{0.5} (0.628)^{1/3}}{\left[1 + (0.4/0.628)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{46,120}{28200}\right)^{5/8}\right]^{4/5} = 198$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(198)(2.53 \times 10^{-4} \text{ ft}^2/\text{s})}{3/12 \text{ ft}} = 0.2004 \text{ ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (0.5073 psia at 80°F). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.50)P_{\text{sat}@40^\circ\text{F}} = (0.50)(0.1217 \text{ psia}) = 0.0609 \text{ psia}$$

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface: } \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.5073 \text{ psia}}{(0.5956 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(80 + 460) \text{ R}} = 0.00158 \text{ lbm}/\text{ft}^3$$

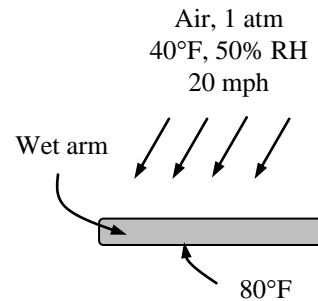
$$\text{Away from the surface: } \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.0609 \text{ psia}}{(0.5956 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(40 + 460) \text{ R}} = 0.000205 \text{ lbm}/\text{ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.2004 \text{ ft/s}) [2 \times \pi (3/12 \text{ ft})(2 \text{ ft})] (0.00158 - 0.000205) \text{ lbm}/\text{ft}^3 \\ &= 8.66 \times 10^{-4} \text{ lbm/s} = \mathbf{3.12 \text{ lbm/h}} \end{aligned}$$

$$\text{and } \dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (8.66 \times 10^{-4} \text{ lbm/s})(1048 \text{ Btu/lbm}) = \mathbf{0.907 \text{ Btu/s}}$$

**Discussion** The rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation  $\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty})$ .



**14-138** A nickel part is put into a room filled with hydrogen. The ratio of hydrogen concentrations at the surface of the part and at a depth of 2-mm from the surface after 24 h is to be determined.

**Assumptions 1** Hydrogen penetrates into a thin layer beneath the surface of the nickel component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial hydrogen concentration in the nickel part is zero.

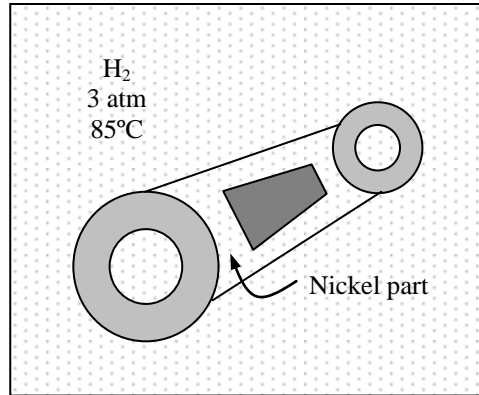
**Properties** The molar mass of hydrogen  $H_2$  is  $M = 2$  kg/kmol (Table A-1). The solubility of hydrogen in nickel at 358 K (=85°C) is  $0.00901$  kmol/m<sup>3</sup>.bar (Table 14-7). The mass diffusivity of hydrogen in nickel at 358 K is  $D_{AB} = 1.2 \times 10^{-12}$  m<sup>2</sup>/s (Table A-3b). Also, 1 atm = 1.01325 bar.

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x,t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The molar density of hydrogen in the nickel at the interface is determined from Eq. 14-20 to be

$$\begin{aligned} C_{H_2, \text{solid side}}(0) &= S \times P_{H_2, \text{gas side}} \\ &= (0.00901 \text{ kmol} / \text{m}^3 \cdot \text{bar})(3 \times 1.01325 \text{ bar}) \\ &= 0.0274 \text{ kmol} / \text{m}^3 \end{aligned}$$



The argument of the complementary error function is

$$\xi = \frac{x}{2\sqrt{D_{AB}t}} = \frac{2 \times 10^{-3} \text{ m}}{2\sqrt{(1.2 \times 10^{-12} \text{ m}^2 / \text{s})(24 \times 3600 \text{ s})}} = 3.105$$

The corresponding value of the complementary error function is determined from Table 4-3 to be

$$\operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right) = \operatorname{erfc}(3.105) = 0.000015$$

Substituting the known quantities,

$$\frac{C_A(x,t) - 0}{0.0274 - 0} = 0.000015 \rightarrow C_A(x,t) = 4.1 \times 10^{-7} \text{ kmol} / \text{m}^3$$

Therefore, the hydrogen concentration in the steel component at a depth of 2 mm in 24 h is very small.

**14-139** A 0.1-mm thick soft rubber membrane separates pure O<sub>2</sub> from air. The mass flow rate of O<sub>2</sub> through the membrane per unit area and the direction of flow are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Mass transfer through the membrane is one-dimensional. 3 The permeability of the membrane is constant.

**Properties** The mass diffusivity of oxygen in rubber at 298 K is  $D_{AB} = 2.1 \times 10^{-10} \text{ m}^2/\text{s}$  (Table 11-3). The solubility of oxygen in rubber at 298 K is  $0.00312 \text{ kmol} / \text{m}^3 \cdot \text{bar}$  (Table 14-7). The molar mass of oxygen is  $32 \text{ kg} / \text{kmol}$  (Table A-1).

**Analysis** The molar fraction of oxygen in air is 0.21. Therefore, the partial pressure of oxygen in the air is

$$y_{\text{O}_2} = \frac{P_{\text{O}_2,2}}{P} \rightarrow P_{\text{O}_2,2} = y_{\text{O}_2} P = 0.21 \times (1.2 \text{ atm}) = 0.252 \text{ atm}$$

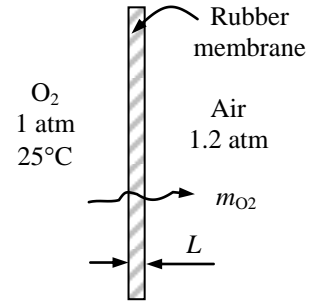
The partial pressure of oxygen on the other side is simply  $P_{\text{O}_2,1} = 1 \text{ atm}$ . Then the molar flow rate of oxygen through the membrane by diffusion can readily be determined to be

$$\begin{aligned} \dot{N}_{\text{diff,A,wall}} &= D_{AB} S \frac{P_{A,1} - P_{A,2}}{L} \\ &= (2.1 \times 10^{-10} \text{ m}^2/\text{s})(0.00312 \text{ kmol}/\text{m}^3 \cdot \text{bar}) \frac{(1 - 0.252) \text{ atm}}{0.1 \times 10^{-3} \text{ m}} \left( \frac{1.01325 \text{ bar}}{1 \text{ atm}} \right) \\ &= 4.97 \times 10^{-9} \text{ kmol}/\text{m}^2 \cdot \text{s} \end{aligned}$$

Then the mass flow rate of oxygen gas through the membrane becomes

$$\dot{m}_{\text{diff}} = M \dot{N}_{\text{diff}} = (32 \text{ kg} / \text{kmol})(4.97 \times 10^{-9} \text{ kmol} / \text{m}^2 \cdot \text{s}) = \mathbf{1.59 \times 10^{-7} \text{ kg} / \text{m}^2 \cdot \text{s}}$$

The direction of the flow will be from the pure oxygen inside to the air outside since the partial pressure of oxygen is higher inside.



**14-140E** The top section of a solar pond is maintained at a constant temperature. The rates of heat loss from the top surface of the pond by radiation, natural convection, and evaporation are to be determined.

**Assumptions 1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The water in the pool is maintained at a uniform temperature of 80°F. **4** The critical Reynolds number for flow over a flat surface is 500,000.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (70+80)/2 = 75^\circ\text{F}$ . The properties of dry air at 75°F and 1 atm are, from Table A-15E,

$$k = 0.0147 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$$

$$\text{Pr} = 0.73$$

$$\alpha = 0.824 \text{ ft}^2/\text{h}$$

$$\nu = 0.167 \times 10^{-3} \text{ ft}^2/\text{s}$$

The saturation pressure of water at 70°F is  $P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia}$ . Properties of water at 80°F are  $h_{fg} = 1048 \text{ Btu/lbm}$  and  $P_v = 0.5073 \text{ psia}$  (Table A-9). The gas constant of water is  $R_{\text{water}} = 0.5956 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R}$  (Table A-1E). The emissivity of water is 0.95 (Table A-15). The mass diffusivity of water vapor in air at the average temperature of 75°F = 535 R = 297.2 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(297.2\text{K})^{2.072}}{1 \text{ atm}} = 2.49 \times 10^{-5} \text{ m}^2/\text{s} = 2.68 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** (a) The pond surface can be treated as a flat surface. The Reynolds number for flow over a flat surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(40 \times 5280 / 3600 \text{ ft/s})(100 \text{ ft})}{0.167 \times 10^{-3} \text{ ft}^2/\text{s}} = 3.51 \times 10^7$$

which is much larger than the critical Reynolds number of 500,000. Therefore, the air flow over the pond surface is turbulent, and the Nusselt number and the heat transfer coefficient are determined to be

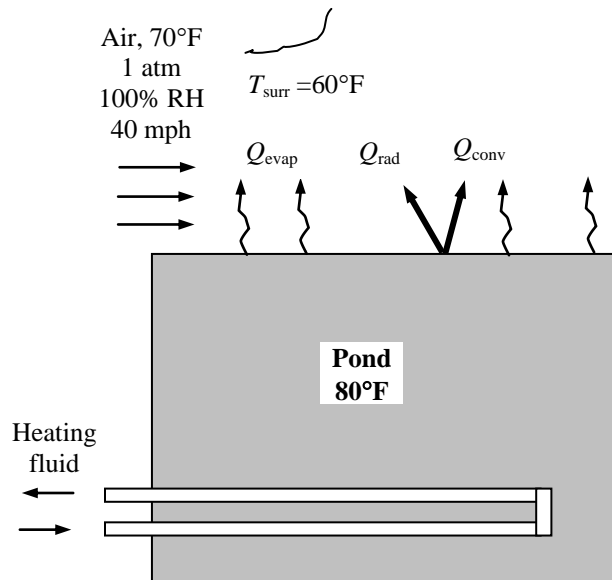
$$\text{Nu} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{1/3} = 0.037(3.51 \times 10^7)^{0.8} (0.73)^{1/3} = 36,215$$

$$h_{\text{heat}} = \frac{\text{Nu}k}{L} = \frac{(36,215)(0.0147 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F})}{100 \text{ ft}} = 5.32 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_\infty - T_s) = (5.32 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(10,000 \text{ ft}^2)(80 - 70)^\circ\text{F} = \mathbf{532,000 \text{ Btu/h}} \quad (\text{to water})$$

(b) Noting that the emissivity of water is 0.95 and the surface area of the pool is  $A_s = (100 \text{ ft})(100 \text{ ft}) = 10,000 \text{ ft}^2$ , heat transfer from the top surface of the pool by radiation is



$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(10,000 \text{ ft}^2)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)[(540 \text{ R})^4 - (520 \text{ R})^4] = \mathbf{194,000 \text{ Btu/h}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{0.167 \times 10^{-3} \text{ ft}^2/\text{s}}{2.68 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.623$$

Then utilizing the analogy between heat and mass convection, the Sherwood number is determined by replacing Pr number by the Schmidt number to be

$$\text{Sh} = 0.037 \text{Re}_L^{0.8} \text{Sc}^{1/3} = 0.037(3.51 \times 10^7)^{0.8} (0.623)^{1/3} = 34,350$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(34,350)(2.68 \times 10^{-4} \text{ ft}^2/\text{s})}{100 \text{ ft}} = 0.092 \text{ lft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature ( $P_{v,s} = 0.5073 \text{ psia}$  at  $80^\circ\text{F}$ ). The humidity of air is given to be 100%, and thus the air far from the water surface is also saturated. Therefore,  $P_{v,\infty} = P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia}$ .

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface:} \quad \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.5073 \text{ psia}}{(0.5956 \text{ psia} \cdot \text{ft}^3 / \text{lbm} \cdot \text{R})(80 + 460) \text{ R}} = 0.00158 \text{ lbm} / \text{ft}^3$$

$$\text{Away from the surface:} \quad \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.3632 \text{ psia}}{(0.5956 \text{ psia} \cdot \text{ft}^3 / \text{lbm} \cdot \text{R})(70 + 460) \text{ R}} = 0.00115 \text{ lbm} / \text{ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.0921 \text{ lft/s})(10,000 \text{ ft}^2)(0.00158 - 0.00115) \text{ lbm/ft}^3 \\ &= 0.396 \text{ lbm/s} = \mathbf{1426 \text{ lbm/h}} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (1425 \text{ lbm/h})(1048 \text{ Btu/lbm}) = \mathbf{1,493,000 \text{ Btu/h}}$$

**Discussion** All of the quantities calculated above represent heat loss for the pond, and the total rate of heat loss from the open top surface of the pond to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 194,000 + 532,000 + 1,493,000 = 2,219,000 \text{ Btu/h}$$

This heat loss will come from the deeper parts of the pond, and thus the pond will start cooling unless it gains heat from the sun or another heat source. Note that the evaporative heat losses dominate. Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation  $\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A_s (w_{A,s} - w_{A,\infty})$ .

**14-141E** The top section of a solar pond is maintained at a constant temperature. The rates of heat loss from the top surface of the pond by radiation, natural convection, and evaporation are to be determined.

**Assumptions 1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The water in the pool is maintained at a uniform temperature of 90°F. **4** The critical Reynolds number for flow over a flat surface is 500,000.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (70+90)/2 = 80^\circ\text{F}$ . The properties of dry air at 80°F and 1 atm are, from Table A-15E,

$$k = 0.0148 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$$

$$\text{Pr} = 0.73$$

$$\alpha = 0.838 \text{ ft}^2/\text{h}$$

$$\nu = 0.170 \times 10^{-3} \text{ ft}^2/\text{s}$$

The saturation pressure of water at 70°F is  $P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia}$ . Properties of water at 90°F are  $h_{fg} = 1043 \text{ Btu/lbm}$  and  $P_v = 0.6988 \text{ psia}$  (Table A-9). The gas constant of water is  $R_{\text{water}} = 0.5956 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R}$  (Table A-1E). The emissivity of water is 0.95 (Table A-15). The mass diffusivity of water vapor in air at the average temperature of 80°F = 540 R = 300 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(300\text{K})^{2.072}}{1 \text{ atm}} = 2.54 \times 10^{-5} \text{ m}^2/\text{s} = 2.72 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** (a) The pond surface can be treated as a flat surface. The Reynolds number for flow over a flat surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(40 \times 5280 / 3600 \text{ ft/s})(100 \text{ ft})}{0.170 \times 10^{-3} \text{ ft}^2/\text{s}} = 3.45 \times 10^7$$

which is much larger than the critical Reynolds number of 500,000. Therefore, the air flow over the pond surface is turbulent, and the Nusselt number and the heat transfer coefficient are determined to be

$$\text{Nu} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{1/3} = 0.037(3.45 \times 10^7)^{0.8} (0.73)^{1/3} = 35,720$$

$$h_{\text{heat}} = \frac{\text{Nu}k}{L} = \frac{(35,720)(0.0148 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F})}{100 \text{ ft}} = 5.29 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

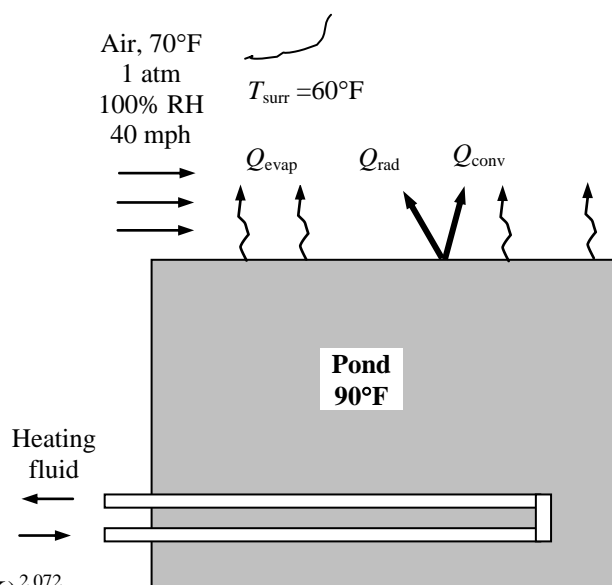
Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_\infty - T_s) = (5.29 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(10,000 \text{ ft}^2)(90 - 70)^\circ\text{F} = \mathbf{1,057,000 \text{ Btu/h}} \quad (\text{to water})$$

(b) Noting that the emissivity of water is 0.95 and the surface area of the pool is  $A_s = (100 \text{ ft})(100 \text{ ft}) = 10,000 \text{ ft}^2$ , heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(10,000 \text{ ft}^2)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)[(550 \text{ R})^4 - (520 \text{ R})^4] = \mathbf{299,400 \text{ Btu/h}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be



$$Sc = \frac{\nu}{D_{AB}} = \frac{0.170 \times 10^{-3} \text{ ft}^2/\text{s}}{2.72 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.625$$

Then utilizing the analogy between heat and mass convection, the Sherwood number is determined by replacing Pr number by the Schmidt number to be

$$Sh = 0.037 Re_L^{0.8} Sc^{1/3} = 0.037(3.45 \times 10^7)^{0.8} (0.625)^{1/3} = 33,920$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{Sh D_{AB}}{D} = \frac{(33,920)(2.72 \times 10^{-4} \text{ ft}^2/\text{s})}{100 \text{ ft}} = 0.0923 \text{ ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature ( $P_{v,s} = 0.6988 \text{ psia}$  at  $90^\circ\text{F}$ ). The humidity of air is given to be 100%, and thus the air far from the water surface is also saturated. Therefore,  $P_{v,\infty} = P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia}$ .

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface:} \quad \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.6988 \text{ psia}}{(0.5956 \text{ psia} \cdot \text{ft}^3 / \text{lbm} \cdot \text{R})(90 + 460) \text{ R}} = 0.00213 \text{ lbm/ft}^3$$

$$\text{Away from the surface:} \quad \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.3632 \text{ psia}}{(0.5956 \text{ psia} \cdot \text{ft}^3 / \text{lbm} \cdot \text{R})(70 + 460) \text{ R}} = 0.00115 \text{ lbm/ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.0923 \text{ ft/s})(10,000 \text{ ft}^2)(0.00213 - 0.00115) \text{ lbm/ft}^3 \\ &= 0.905 \text{ lbm/s} = \mathbf{3256 \text{ lbm/h}} \end{aligned}$$

$$\text{and} \quad \dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (3256 \text{ lbm/h})(1043 \text{ Btu/lbm}) = \mathbf{3,396,000 \text{ Btu/h}}$$

**Discussion** All of the quantities calculated above represent heat loss for the pond, and the total rate of heat loss from the open top surface of the pond to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 299,400 + 1,057,000 + 3,396,000 = 4,752,400 \text{ Btu/h}$$

This heat loss will come from the deeper parts of the pond, and thus the pond will start cooling unless it gains heat from the sun or another heat source. Note that the evaporative heat losses dominate. Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation  $\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty})$ .

#### 14-142 .... 14-146 Design and Essay Problems

