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سایت آموزش مهندسی مکانیک

**Solution of Steady One-Dimensional Heat Conduction Problems**

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**2-52C** Yes, this claim is reasonable since in the absence of any heat generation the rate of heat transfer through a plain wall in steady operation must be constant. But the value of this constant must be zero since one side of the wall is perfectly insulated. Therefore, there can be no temperature difference between different parts of the wall; that is, the temperature in a plane wall must be uniform in steady operation.

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**2-53C** Yes, the temperature in a plane wall with constant thermal conductivity and no heat generation will vary linearly during steady one-dimensional heat conduction even when the wall loses heat by radiation from its surfaces. This is because the steady heat conduction equation in a plane wall is  $d^2T/dx^2 = 0$  whose solution is  $T(x) = C_1x + C_2$  regardless of the boundary conditions. The solution function represents a straight line whose slope is  $C_1$ .

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**2-54C** Yes, in the case of constant thermal conductivity and no heat generation, the temperature in a solid cylindrical rod whose ends are maintained at constant but different temperatures while the side surface is perfectly insulated will vary linearly during steady one-dimensional heat conduction. This is because the steady heat conduction equation in this case is  $d^2T/dx^2 = 0$  whose solution is  $T(x) = C_1x + C_2$  which represents a straight line whose slope is  $C_1$ .

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**2-55C** Yes, this claim is reasonable since no heat is entering the cylinder and thus there can be no heat transfer from the cylinder in steady operation. This condition will be satisfied only when there are no temperature differences within the cylinder and the outer surface temperature of the cylinder is the equal to the temperature of the surrounding medium.

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**2-56** A large plane wall is subjected to specified temperature on the left surface and convection on the right surface. The mathematical formulation, the variation of temperature, and the rate of heat transfer are to be determined for steady one-dimensional heat transfer.

**Assumptions** 1 Heat conduction is steady and one-dimensional. 2 Thermal conductivity is constant. 3 There is no heat generation.

**Properties** The thermal conductivity is given to be  $k = 2.3 \text{ W/m}\cdot\text{°C}$ .

**Analysis** (a) Taking the direction normal to the surface of the wall to be the  $x$  direction with  $x = 0$  at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and

$$T(0) = T_1 = 80^\circ\text{C}$$

$$-k \frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

(b) Integrating the differential equation twice with respect to  $x$  yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad T(0) = C_1 \times 0 + C_2 \rightarrow C_2 = T_1$$

$$x = L: \quad -kC_1 = h[(C_1L + C_2) - T_\infty] \rightarrow C_1 = -\frac{h(C_2 - T_\infty)}{k + hL} \rightarrow C_1 = -\frac{h(T_1 - T_\infty)}{k + hL}$$

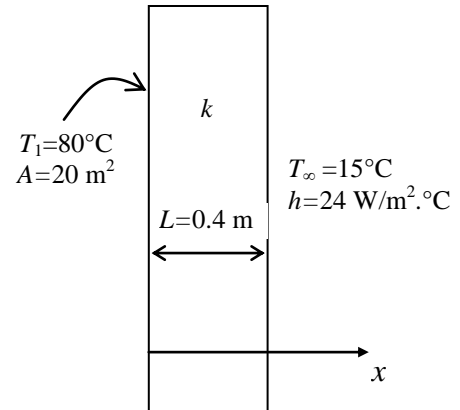
Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= -\frac{h(T_1 - T_\infty)}{k + hL}x + T_1 \\ &= -\frac{(24 \text{ W/m}^2\cdot\text{°C})(80 - 15)^\circ\text{C}}{(2.3 \text{ W/m}\cdot\text{°C}) + (24 \text{ W/m}^2\cdot\text{°C})(0.4 \text{ m})}x + 80^\circ\text{C} \\ &= 80 - 131.1x \end{aligned}$$

(c) The rate of heat conduction through the wall is

$$\begin{aligned} \dot{Q}_{\text{wall}} &= -kA \frac{dT}{dx} = -kAC_1 = kA \frac{h(T_1 - T_\infty)}{k + hL} \\ &= (2.3 \text{ W/m}\cdot\text{°C})(20 \text{ m}^2) \frac{(24 \text{ W/m}^2\cdot\text{°C})(80 - 15)^\circ\text{C}}{(2.3 \text{ W/m}\cdot\text{°C}) + (24 \text{ W/m}^2\cdot\text{°C})(0.4 \text{ m})} \\ &= \mathbf{6030 \text{ W}} \end{aligned}$$

Note that under steady conditions the rate of heat conduction through a plain wall is constant.



**2-57** The top and bottom surfaces of a solid cylindrical rod are maintained at constant temperatures of 20°C and 95°C while the side surface is perfectly insulated. The rate of heat transfer through the rod is to be determined for the cases of copper, steel, and granite rod.

**Assumptions** **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

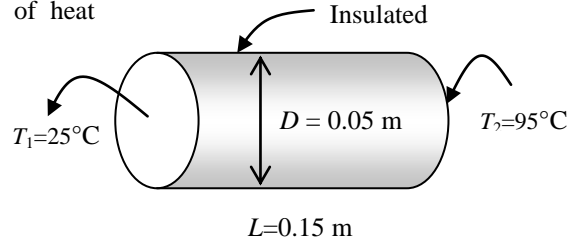
**Properties** The thermal conductivities are given to be  $k = 380 \text{ W/m}\cdot\text{°C}$  for copper,  $k = 18 \text{ W/m}\cdot\text{°C}$  for steel, and  $k = 1.2 \text{ W/m}\cdot\text{°C}$  for granite.

**Analysis** Noting that the heat transfer area (the area normal to the direction of heat transfer) is constant, the rate of heat transfer along the rod is determined from

$$\dot{Q} = kA \frac{T_1 - T_2}{L}$$

where  $L = 0.15 \text{ m}$  and the heat transfer area  $A$  is

$$A = \pi D^2 / 4 = \pi(0.05 \text{ m})^2 / 4 = 1.964 \times 10^{-3} \text{ m}^2$$



Then the heat transfer rate for each case is determined as follows:

(a) Copper: 
$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (380 \text{ W/m}\cdot\text{°C})(1.964 \times 10^{-3} \text{ m}^2) \frac{(95 - 20)^\circ\text{C}}{0.15 \text{ m}} = \mathbf{373.1 \text{ W}}$$

(b) Steel: 
$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (18 \text{ W/m}\cdot\text{°C})(1.964 \times 10^{-3} \text{ m}^2) \frac{(95 - 20)^\circ\text{C}}{0.15 \text{ m}} = \mathbf{17.7 \text{ W}}$$

(c) Granite: 
$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (1.2 \text{ W/m}\cdot\text{°C})(1.964 \times 10^{-3} \text{ m}^2) \frac{(95 - 20)^\circ\text{C}}{0.15 \text{ m}} = \mathbf{1.2 \text{ W}}$$

**Discussion:** The steady rate of heat conduction can differ by orders of magnitude, depending on the thermal conductivity of the material.

2-58

"GIVEN"

L=0.15 "[m]"

D=0.05 "[m]"

T<sub>1</sub>=20 "[C]"T<sub>2</sub>=95 "[C]"

"k=1.2 [W/m.C], parameter to be varied"

"ANALYSIS"

A=π\*D<sup>2</sup>/4Q<sub>dot</sub>=k\*A\*(T<sub>2</sub>-T<sub>1</sub>)/L

k [W/m.C]	Q [W]
1	0.9817
22	21.6
43	42.22
64	62.83
85	83.45
106	104.1
127	124.7
148	145.3
169	165.9
190	186.5
211	207.1
232	227.8
253	248.4
274	269
295	289.6
316	310.2
337	330.8
358	351.5
379	372.1
400	392.7

**2-59** The base plate of a household iron is subjected to specified heat flux on the left surface and to specified temperature on the right surface. The mathematical formulation, the variation of temperature in the plate, and the inner surface temperature are to be determined for steady one-dimensional heat transfer.

**Assumptions 1** Heat conduction is steady and one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** Heat loss through the upper part of the iron is negligible.

**Properties** The thermal conductivity is given to be  $k = 20 \text{ W/m}\cdot\text{C}$ .

**Analysis (a)** Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be

$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{800 \text{ W}}{160 \times 10^{-4} \text{ m}^2} = 50,000 \text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the  $x$  direction with  $x = 0$  at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and 
$$-k \frac{dT(0)}{dx} = \dot{q}_0 = 50,000 \text{ W/m}^2$$

$$T(L) = T_2 = 85^\circ\text{C}$$

(b) Integrating the differential equation twice with respect to  $x$  yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \quad T(L) = C_1L + C_2 = T_2 \rightarrow C_2 = T_2 - C_1L \rightarrow C_2 = T_2 + \frac{\dot{q}_0L}{k}$$

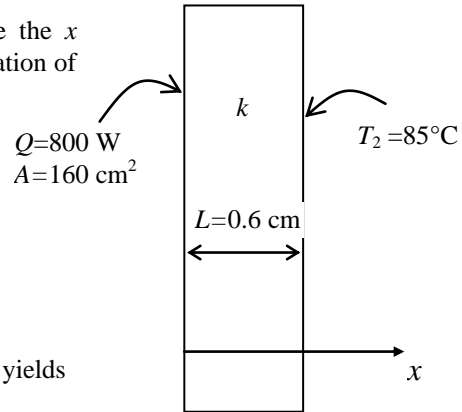
Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= -\frac{\dot{q}_0}{k}x + T_2 + \frac{\dot{q}_0L}{k} = \frac{\dot{q}_0(L-x)}{k} + T_2 \\ &= \frac{(50,000 \text{ W/m}^2)(0.006 - x)\text{m}}{20 \text{ W/m}\cdot\text{C}} + 85^\circ\text{C} \\ &= 2500(0.006 - x) + 85 \end{aligned}$$

(c) The temperature at  $x = 0$  (the inner surface of the plate) is

$$T(0) = 2500(0.006 - 0) + 85 = \mathbf{100^\circ\text{C}}$$

Note that the inner surface temperature is higher than the exposed surface temperature, as expected.



**2-60** The base plate of a household iron is subjected to specified heat flux on the left surface and to specified temperature on the right surface. The mathematical formulation, the variation of temperature in the plate, and the inner surface temperature are to be determined for steady one-dimensional heat transfer.

**Assumptions 1** Heat conduction is steady and one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** Heat loss through the upper part of the iron is negligible.

**Properties** The thermal conductivity is given to be  $k = 20 \text{ W/m}\cdot\text{°C}$ .

**Analysis (a)** Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be

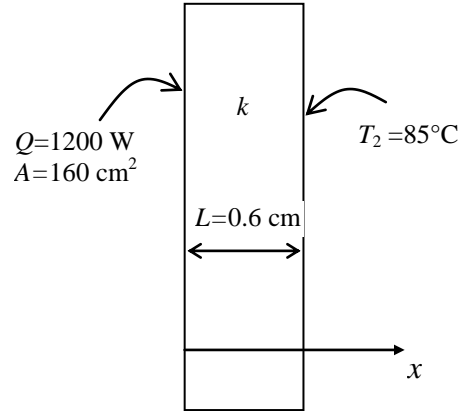
$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{1200 \text{ W}}{160 \times 10^{-4} \text{ m}^2} = 75,000 \text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the  $x$  direction with  $x = 0$  at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2 T}{dx^2} = 0$$

and 
$$-k \frac{dT(0)}{dx} = \dot{q}_0 = 75,000 \text{ W/m}^2$$

$$T(L) = T_2 = 85^\circ\text{C}$$



(b) Integrating the differential equation twice with respect to  $x$  yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \quad T(L) = C_1 L + C_2 = T_2 \rightarrow C_2 = T_2 - C_1 L \rightarrow C_2 = T_2 + \frac{\dot{q}_0 L}{k}$$

Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= -\frac{\dot{q}_0}{k} x + T_2 + \frac{\dot{q}_0 L}{k} = \frac{\dot{q}_0 (L-x)}{k} + T_2 \\ &= \frac{(75,000 \text{ W/m}^2)(0.006 - x) \text{ m}}{20 \text{ W/m}\cdot\text{°C}} + 85^\circ\text{C} \\ &= 3750(0.006 - x) + 85 \end{aligned}$$

(c) The temperature at  $x = 0$  (the inner surface of the plate) is

$$T(0) = 3750(0.006 - 0) + 85 = \mathbf{107.5^\circ\text{C}}$$

Note that the inner surface temperature is higher than the exposed surface temperature, as expected.

2-61

"GIVEN"

 $Q_{\dot{}}=800$  "[W]" $L=0.006$  "[m]" $A_{\text{base}}=160E-4$  "[m<sup>2</sup>]" $k=20$  "[W/m-C]" $T_2=85$  "[C]"

"ANALYSIS"

 $q_{\dot{}}_0=Q_{\dot{}}/A_{\text{base}}$  $T=q_{\dot{}}_0(L-x)/k+T_2$  "Variation of temperature"

"x is the parameter to be varied"

0	100
0.0006667	98.33
0.001333	96.67
0.002	95
0.002667	93.33
0.003333	91.67
0.004	90
0.004667	88.33
0.005333	86.67
0.006	85

**2-62E** A steam pipe is subjected to convection on the inner surface and to specified temperature on the outer surface. The mathematical formulation, the variation of temperature in the pipe, and the rate of heat loss are to be determined for steady one-dimensional heat transfer.

**Assumptions 1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

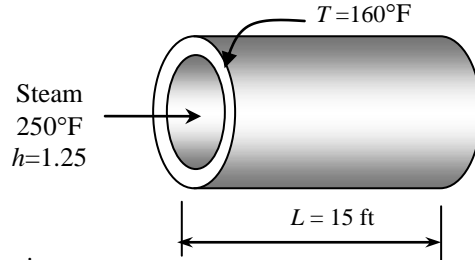
**Properties** The thermal conductivity is given to be  $k = 7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$ .

**Analysis (a)** Noting that heat transfer is one-dimensional in the radial  $r$  direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0$$

and 
$$-k \frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$T(r_2) = T_2 = 160^\circ\text{F}$$



(b) Integrating the differential equation once with respect to  $r$  gives

$$r \frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by  $r$  to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad -k \frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)]$$

$$r = r_2: \quad T(r_2) = C_1 \ln r_2 + C_2 = T_2$$

Solving for  $C_1$  and  $C_2$  simultaneously gives

$$C_1 = \frac{T_2 - T_\infty}{\ln \frac{r_2}{r_1} + \frac{k}{hr_1}} \quad \text{and} \quad C_2 = T_2 - C_1 \ln r_2 = T_2 - \frac{T_2 - T_\infty}{\ln \frac{r_2}{r_1} + \frac{k}{hr_1}} \ln r_2$$

Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(r) &= C_1 \ln r + T_2 - C_1 \ln r_2 = C_1 (\ln r - \ln r_2) + T_2 = \frac{T_2 - T_\infty}{\ln \frac{r_2}{r_1} + \frac{k}{hr_1}} \ln \frac{r}{r_2} + T_2 \\ &= \frac{(160 - 250)^\circ\text{F}}{\ln \frac{2.4}{2} + \frac{7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}}{(12.5 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(2/12 \text{ ft})}} \ln \frac{r}{2.4 \text{ in}} + 160^\circ\text{F} = -24.74 \ln \frac{r}{2.4 \text{ in}} + 160^\circ\text{F} \end{aligned}$$

(c) The rate of heat conduction through the pipe is

$$\begin{aligned} \dot{Q} &= -kA \frac{dT}{dr} = -k(2\pi rL) \frac{C_1}{r} = -2\pi Lk \frac{T_2 - T_\infty}{\ln \frac{r_2}{r_1} + \frac{k}{hr_1}} \\ &= -2\pi(15 \text{ ft})(7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}) \frac{(160 - 250)^\circ\text{F}}{\ln \frac{2.4}{2} + \frac{7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}}{(12.5 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(2/12 \text{ ft})}} = \mathbf{16,800 \text{ Btu/h}} \end{aligned}$$



**2-63** A spherical container is subjected to specified temperature on the inner surface and convection on the outer surface. The mathematical formulation, the variation of temperature, and the rate of heat transfer are to be determined for steady one-dimensional heat transfer.

**Assumptions 1** Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the midpoint. **2** Thermal conductivity is constant. **3** There is no heat generation.

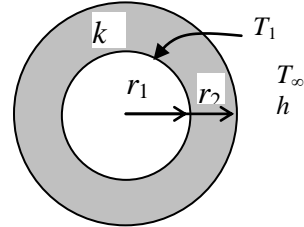
**Properties** The thermal conductivity is given to be  $k = 30 \text{ W/m}\cdot\text{°C}$ .

**Analysis** (a) Noting that heat transfer is one-dimensional in the radial  $r$  direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0$$

and  $T(r_1) = T_1 = 0^\circ\text{C}$

$$-k \frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty]$$



(b) Integrating the differential equation once with respect to  $r$  gives

$$r^2 \frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by  $r$  to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad T(r_1) = -\frac{C_1}{r_1} + C_2 = T_1$$

$$r = r_2: \quad -k \frac{C_1}{r_2^2} = h \left( -\frac{C_1}{r_2} + C_2 - T_\infty \right)$$

Solving for  $C_1$  and  $C_2$  simultaneously gives

$$C_1 = \frac{r_2(T_1 - T_\infty)}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \quad \text{and} \quad C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{T_1 - T_\infty}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \frac{r_2}{r_1}$$

Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(r) &= -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = C_1 \left( \frac{1}{r_1} - \frac{1}{r} \right) + T_1 = \frac{T_1 - T_\infty}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \left( \frac{r_2}{r_1} - \frac{r_2}{r} \right) + T_1 \\ &= \frac{(0 - 25)^\circ\text{C}}{1 - \frac{2.1}{2} - \frac{30 \text{ W/m}\cdot\text{°C}}{(18 \text{ W/m}^2 \cdot \text{°C})(2.1 \text{ m})}} \left( \frac{2.1}{2} - \frac{2.1}{r} \right) + 0^\circ\text{C} = 29.63(1.05 - 2.1/r) \end{aligned}$$

(c) The rate of heat conduction through the wall is

$$\begin{aligned}\dot{Q} &= -kA \frac{dT}{dx} = -k(4\pi r^2) \frac{C_1}{r^2} = -4\pi k C_1 = -4\pi k \frac{r_2(T_1 - T_\infty)}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \\ &= -4\pi(30 \text{ W/m}\cdot\text{°C}) \frac{(2.1 \text{ m})(0 - 25)\text{°C}}{1 - \frac{2.1}{2} - \frac{30 \text{ W/m}\cdot\text{°C}}{(18 \text{ W/m}^2\cdot\text{°C})(2.1 \text{ m})}} = \mathbf{23,460 \text{ W}}\end{aligned}$$

**2-64** A large plane wall is subjected to specified heat flux and temperature on the left surface and no conditions on the right surface. The mathematical formulation, the variation of temperature in the plate, and the right surface temperature are to be determined for steady one-dimensional heat transfer.

**Assumptions** **1** Heat conduction is steady and one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides of the wall are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall.

**Properties** The thermal conductivity is given to be  $k = 2.5 \text{ W/m}\cdot^\circ\text{C}$ .

**Analysis** (a) Taking the direction normal to the surface of the wall to be the  $x$  direction with  $x = 0$  at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and 
$$-k \frac{dT(0)}{dx} = \dot{q}_0 = 700 \text{ W/m}^2$$

$$T(0) = T_1 = 80^\circ\text{C}$$

(b) Integrating the differential equation twice with respect to  $x$  yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

Heat flux at  $x = 0$ : 
$$-kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

Temperature at  $x = 0$ : 
$$T(0) = C_1 \times 0 + C_2 = T_1 \rightarrow C_2 = T_1$$

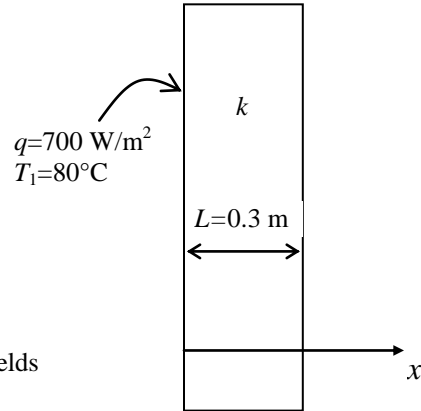
Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_1 = -\frac{700 \text{ W/m}^2}{2.5 \text{ W/m}\cdot^\circ\text{C}}x + 80^\circ\text{C} = -280x + 80$$

(c) The temperature at  $x = L$  (the right surface of the wall) is

$$T(L) = -280 \times (0.3 \text{ m}) + 80 = -4^\circ\text{C}$$

Note that the right surface temperature is lower as expected.



**2-65** A large plane wall is subjected to specified heat flux and temperature on the left surface and no conditions on the right surface. The mathematical formulation, the variation of temperature in the plate, and the right surface temperature are to be determined for steady one-dimensional heat transfer.

**Assumptions** **1** Heat conduction is steady and one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides of the wall are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall.

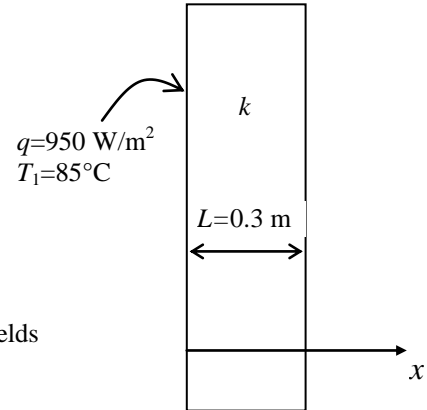
**Properties** The thermal conductivity is given to be  $k = 2.5 \text{ W/m}\cdot\text{C}$ .

**Analysis** (a) Taking the direction normal to the surface of the wall to be the  $x$  direction with  $x = 0$  at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and  $-k \frac{dT(0)}{dx} = \dot{q}_0 = 950 \text{ W/m}^2$

$$T(0) = T_1 = 85^\circ\text{C}$$



(b) Integrating the differential equation twice with respect to  $x$  yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

Heat flux at  $x = 0$ :  $-kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$

Temperature at  $x = 0$ :  $T(0) = C_1 \times 0 + C_2 = T_1 \rightarrow C_2 = T_1$

Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_1 = -\frac{950 \text{ W/m}^2}{2.5 \text{ W/m}\cdot\text{C}}x + 85^\circ\text{C} = -380x + 85$$

(c) The temperature at  $x = L$  (the right surface of the wall) is

$$T(L) = -380 \times (0.3 \text{ m}) + 85 = -29^\circ\text{C}$$

Note that the right surface temperature is lower as expected.

**2-66E** A large plate is subjected to convection, radiation, and specified temperature on the top surface and no conditions on the bottom surface. The mathematical formulation, the variation of temperature in the plate, and the bottom surface temperature are to be determined for steady one-dimensional heat transfer.

**Assumptions** **1** Heat conduction is steady and one-dimensional since the plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate.

**Properties** The thermal conductivity and emissivity are given to be  $k = 7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$  and  $\varepsilon = 0.6$ .

**Analysis** (a) Taking the direction normal to the surface of the plate to be the  $x$  direction with  $x = 0$  at the bottom surface, and the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

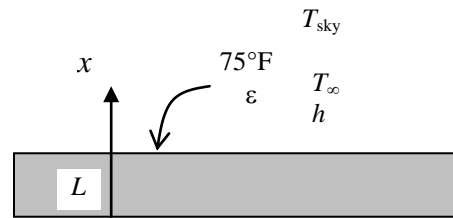
$$\text{and } -k \frac{dT(L)}{dx} = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{\text{sky}}^4] = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4]$$

$$T(L) = T_2 = 75^\circ\text{F}$$

(b) Integrating the differential equation twice with respect to  $x$  yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$



where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

$$\begin{aligned} \text{Convection at } x = L: \quad & -kC_1 = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4] \\ \rightarrow C_1 = & -\{h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4]\} / k \end{aligned}$$

$$\text{Temperature at } x = L: \quad T(L) = C_1 \times L + C_2 = T_2 \rightarrow C_2 = T_2 - C_1L$$

Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= C_1x + (T_2 - C_1L) = T_2 - (L - x)C_1 = T_2 + \frac{h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{sky}}^4]}{k}(L - x) \\ &= 75^\circ\text{F} + \frac{(12 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(75 - 90)^\circ\text{F} + 0.6(0.1714 \times 10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(535 \text{ R})^4 - (510 \text{ R})^4]}{7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}}(4/12 - x) \text{ ft} \\ &= 75 - 23.0(1/3 - x) \end{aligned}$$

(c) The temperature at  $x = 0$  (the bottom surface of the plate) is

$$T(0) = 75 - 23.0 \times (1/3 - 0) = \mathbf{67.3^\circ\text{F}}$$

**2-67E** A large plate is subjected to convection and specified temperature on the top surface and no conditions on the bottom surface. The mathematical formulation, the variation of temperature in the plate, and the bottom surface temperature are to be determined for steady one-dimensional heat transfer.

**Assumptions 1** Heat conduction is steady and one-dimensional since the plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate.

**Properties** The thermal conductivity is given to be  $k = 7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$ .

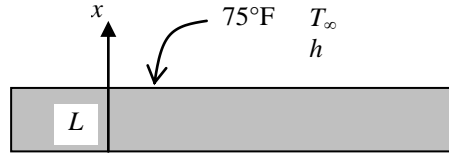
**Analysis (a)** Taking the direction normal to the surface of the plate to be the  $x$  direction with  $x = 0$  at the bottom surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and

$$-k \frac{dT(L)}{dx} = h[T(L) - T_\infty] = h(T_2 - T_\infty)$$

$$T(L) = T_2 = 75^\circ\text{F}$$



(b) Integrating the differential equation twice with respect to  $x$  yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where  $C_1$  and  $C_2$  are arbitrary constants. Applying the boundary conditions give

Convection at  $x = L$ :  $-kC_1 = h(T_2 - T_\infty) \rightarrow C_1 = -h(T_2 - T_\infty) / k$

Temperature at  $x = L$ :  $T(L) = C_1 \times L + C_2 = T_2 \rightarrow C_2 = T_2 - C_1L$

Substituting  $C_1$  and  $C_2$  into the general solution, the variation of temperature is determined to be

$$T(x) = C_1x + (T_2 - C_1L) = T_2 - (L - x)C_1 = T_2 + \frac{h(T_2 - T_\infty)}{k}(L - x)$$

$$= 75^\circ\text{F} + \frac{(12 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(75 - 90)^\circ\text{F}}{7.2 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}}(4/12 - x) \text{ ft}$$

$$= 75 - 25(1/3 - x)$$

(c) The temperature at  $x = 0$  (the bottom surface of the plate) is

$$T(0) = 75 - 25 \times (1/3 - 0) = \mathbf{66.7^\circ\text{F}}$$