2-68 A compressed air pipe is subjected to uniform heat flux on the outer surface and convection on the inner surface. The mathematical formulation, the variation of temperature in the pipe, and the surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

Properties The thermal conductivity is given to be k = 14 W/m·°C.

Analysis (*a*) Noting that the 85% of the 300 W generated by the strip heater is transferred to the pipe, the heat flux through the outer surface is determined to be

$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{0.85\times 300\,\text{W}}{2\pi(0.04\,\text{m})(6\,\text{m})} = 169.1\,\text{W/m}^2$$

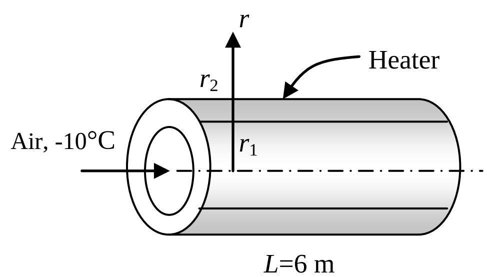
Noting that heat transfer is one-dimensional in the radial r direction and heat flux is in the negative r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and

$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$



(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $$k\frac{C_1}{r_2} = \dot{q}_s \quad\rightarrow\quad C_1 = \frac{\dot{q}_s r_2}{k}$$

$r = r_1$: $$-k\frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)] \;\rightarrow\; C_2 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln r - \ln r_1 + \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln\frac{r}{r_1} + \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$$

$$= -10°\text{C} + \left(\ln\frac{r}{r_1} + \frac{14\,\text{W/m}\cdot°\text{C}}{(30\,\text{W/m}^2\cdot°\text{C})(0.037\,\text{m})}\right)\frac{(169.1\,\text{W/m}^2)(0.04\,\text{m})}{14\,\text{W/m}\cdot°\text{C}} = -10 + 0.483\left(\ln\frac{r}{r_1} + 12.61\right)$$

(*c*) The inner and outer surface temperatures are determined by direct substitution to be

Inner surface ($r = r_1$): $T(r_1) = -10 + 0.483\left(\ln\frac{r_1}{r_1} + 12.61\right) = -10 + 0.483(0 + 12.61) = \mathbf{-3.91°C}$

Outer surface ($r = r_2$): $T(r_1) = -10 + 0.483\left(\ln\frac{r_2}{r_1} + 12.61\right) = -10 + 0.483\left(\ln\frac{0.04}{0.037} + 12.61\right) = \mathbf{-3.87°C}$

Note that the pipe is essentially isothermal at a temperature of about -3.9°C.

2-69

```
"GIVEN"
L=6 "[m]"
r_1=0.037 "[m]"
r_2=0.04 "[m]"
k=14 "[W/m-C]"
Q_dot=300 "[W]"
T_infinity=-10 "[C]"
h=30 "[W/m^2-C]"
f_loss=0.15

"ANALYSIS"
q_dot_s=((1-f_loss)*Q_dot)/A
A=2*pi*r_2*L
T=T_infinity+(ln(r/r_1)+k/(h*r_1))*(q_dot_s*r_2)/k "Variation of temperature"
"r is the parameter to be varied"
```

r [m]	T [C]
0.037	3.906
0.03733	3.902
0.03767	3.898
0.038	3.893
0.03833	3.889
0.03867	3.885
0.039	3.881
0.03933	3.877
0.03967	3.873
0.04	3.869

2-70 A spherical container is subjected to uniform heat flux on the outer surface and specified temperature on the inner surface. The mathematical formulation, the variation of temperature in the pipe, and the outer surface temperature, and the maximum rate of hot water supply are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the mid point. **2** Thermal conductivity is constant. **3** There is no heat generation in the container.

Properties The thermal conductivity is given to be k = 1.5 W/m·°C. The specific heat of water at the average temperature of (100+20)/2 = 60°C is 4.185 kJ/kg·°C (Table A-9).

Analysis (*a*) Noting that the 90% of the 500 W generated by the strip heater is transferred to the container, the heat flux through the outer surface is determined to be

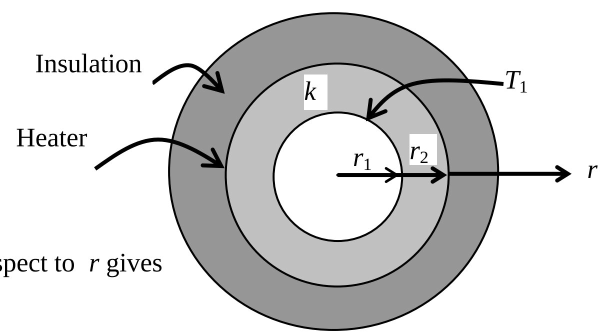
$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{4\pi r_2^2} = \frac{0.90\times 500\,\text{W}}{4\pi(0.41\,\text{m})^2} = 213.0\ \text{W/m}^2$$

Noting that heat transfer is one-dimensional in the radial r direction and heat flux is in the negative r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

and $$T(r_1) = T_1 = 100°\text{C}$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$



(*b*) Integrating the differential equation once with respect to r gives

$$r^2\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r^2 and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $$k\frac{C_1}{r_2^2} = \dot{q}_s \quad\rightarrow\quad C_1 = \frac{\dot{q}_s r_2^2}{k}$$

$r = r_1$: $$T(r_1) = T_1 = -\frac{C_1}{r_1} + C_2 \quad\rightarrow\quad C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{\dot{q}_s r_2^2}{k r_1}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{C_1}{r} + C_2 = -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)C_1 = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)\frac{\dot{q}_s r_2^2}{k}$$

$$= 100°\text{C} + \left(\frac{1}{0.40\,\text{m}} - \frac{1}{r}\right)\frac{(213\,\text{W/m}^2)(0.41\,\text{m})^2}{1.5\,\text{W/m}\cdot°\text{C}} = 100 + 23.87\left(2.5 - \frac{1}{r}\right)$$

(*c*) The outer surface temperature is determined by direct substitution to be

Outer surface ($r = r_2$): $$T(r_2) = 100 + 23.87\left(2.5 - \frac{1}{r_2}\right) = 100 + 23.87\left(2.5 - \frac{1}{0.41}\right) = \mathbf{101.5°C}$$

Noting that the maximum rate of heat supply to the water is $0.9\times 500\ \text{W} = 450\ \text{W}$, water can be heated from 20 to 100°C at a rate of

$$\dot{Q} = \dot{m}C_p\Delta T \quad \rightarrow \quad \dot{m} = \frac{\dot{Q}}{C_p\Delta T} = \frac{0.450 \text{ kJ/s}}{(4.185 \text{ kJ/kg}\cdot°\text{C})(100-20)°\text{C}} = 0.00134 \text{ kg/s} = \mathbf{4.84 \text{ kg/h}}$$

2-71

```
"GIVEN"
r_1=0.40 "[m]"
r_2=0.41 "[m]"
k=1.5 "[W/m-C]"
T_1=100 "[C]"
Q_dot=500 "[W]"
f_loss=0.10

"ANALYSIS"
q_dot_s=((1-f_loss)*Q_dot)/A
A=4*pi*r_2^2
T=T_1+(1/r_1-1/r)*(q_dot_s*r_2^2)/k "Variation of temperature"
"r is the parameter to be varied"
```

r [m]	T [C]
0.4	100
0.4011	100.2
0.4022	100.3
0.4033	100.5
0.4044	100.7
0.4056	100.8
0.4067	101
0.4078	101.1
0.4089	101.3
0.41	101.5

Heat Generation in Solids

2-72C No. Heat generation in a solid is simply the conversion of some form of energy into sensible heat energy. For example resistance heating in wires is conversion of electrical energy to heat.

2-73C Heat generation in a solid is simply conversion of some form of energy into sensible heat energy. Some examples of heat generations are resistance heating in wires, exothermic chemical reactions in a solid, and nuclear reactions in nuclear fuel rods.

2-74C The rate of heat generation inside an iron becomes equal to the rate of heat loss from the iron when steady operating conditions are reached and the temperature of the iron stabilizes.

2-75C No, it is not possible since the highest temperature in the plate will occur at its center, and heat cannot flow "uphill."

2-76C The cylinder will have a higher center temperature since the cylinder has less surface area to lose heat from per unit volume than the sphere.

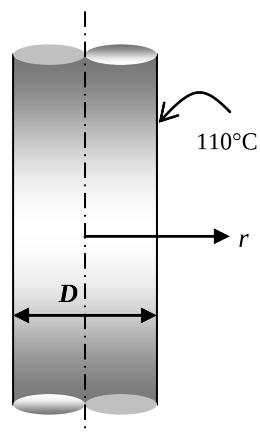
2-77 A 2-kW resistance heater wire with a specified surface temperature is used to boil water. The center temperature of the wire is to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be k = 20 W/m·°C.

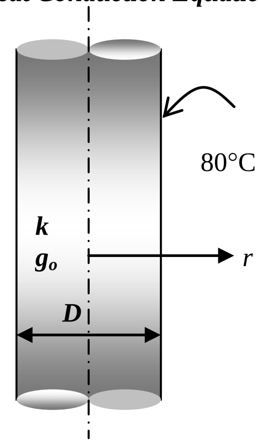
Analysis The resistance heater converts electric energy into heat at a rate of 2 kW. The rate of heat generation per unit volume of the wire is

$$\dot{g} = \frac{\dot{Q}_{gen}}{V_{\text{wire}}} = \frac{\dot{Q}_{gen}}{\pi r_o^2 L} = \frac{2000\,\text{W}}{\pi(0.0025\,\text{m})^2(0.7\,\text{m})} = 1.455\times10^8 \text{ W/m}^3$$



The center temperature of the wire is then determined from Eq. 2-71 to be

$$T_o = T_s + \frac{\dot{g} r_o^2}{4k} = 110°\text{C} + \frac{(1.455\times10^8 \text{ W/m}^3)(0.0025\,\text{m})^2}{4(20\,\text{W/m.°C})} = \mathbf{121.4°C}$$



2-78 Heat is generated in a long solid cylinder with a specified surface temperature. The variation of temperature in the cylinder is given by

$$T(r) = \frac{\dot{g}r_0^2}{k}\left[1-\left(\frac{r}{r_0}\right)^2\right]+T_s$$

(*a*) Heat conduction is steady since there is no time *t* variable involved.

(*b*) Heat conduction is a one-dimensional.

(*c*) Using Eq. (1), the heat flux on the surface of the cylinder at $r = r_0$ is determined from its definition to be

$$\dot{q}_s = -k\frac{dT(r_0)}{dr} = -k\left[\frac{\dot{g}r_0^2}{k}\left(-\frac{2r}{r_0^2}\right)\right]_{r=r_0} = -k\left[\frac{\dot{g}r_0^2}{k}\left(-\frac{2r_0}{r_0^2}\right)\right] = 2\dot{g}r_0 = 2(35\text{ W/cm}^3)(4\text{ cm}) = \mathbf{280\ W/cm^2}$$

2-79

```
"GIVEN"
r_0=0.04 "[m]"
k=25 "[W/m-C]"
g_dot_0=35E+6 "[W/m^3]"
T_s=80 "[C]"

"ANALYSIS"
T=(g_dot_0*r_0^2)/k*(1-(r/r_0)^2)+T_s "Variation of temperature"
"r is the parameter to be varied"
```

r [m]	T [C]
0	2320
0.004444	2292
0.008889	2209
0.01333	2071
0.01778	1878
0.02222	1629
0.02667	1324
0.03111	964.9
0.03556	550.1
0.04	80

2-80E A long homogeneous resistance heater wire with specified convection conditions at the surface is used to boil water. The mathematical formulation, the variation of temperature in the wire, and the temperature at the centerline of the wire are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 8.6 Btu/h·ft·°F.

Analysis Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

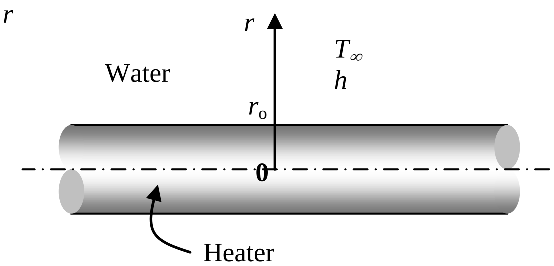
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0$$

and $-k\frac{dT(r_0)}{dr}=h[T(r_0)-T_\infty]$ (convection at the outer surface)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and rearranging gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{g}}{k}r$$



Integrating with respect to r gives

$$r\frac{dT}{dr}=-\frac{\dot{g}}{k}\frac{r^2}{2}+C_1 \qquad (a)$$

It is convenient at this point to apply the second boundary condition since it is related to the first derivative of the temperature by replacing all occurrences of r and dT/dr in the equation above by zero. It yields

B.C. at $r = 0$: $\quad 0\times\frac{dT(0)}{dr}=-\frac{\dot{g}}{2k}\times 0+C_1 \quad\rightarrow\quad C_1=0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{g}}{2k}r$$

and $\quad T(r)=-\frac{\dot{g}}{4k}r^2+C_2 \qquad (b)$

Applying the second boundary condition at $r=r_0$,

B. C. at $r=r_0$: $\quad -k\frac{\dot{g}r_0}{2k}=h\left(-\frac{\dot{g}}{4k}r_0^2+C_2-T_\infty\right) \quad\rightarrow\quad C_2=T_\infty+\frac{\dot{g}r_0}{2h}+\frac{\dot{g}}{4k}r_0^2$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r)=T_\infty+\frac{\dot{g}}{4k}(r_0^2-r^2)+\frac{\dot{g}r_0}{2h}$$

which is the desired solution for the temperature distribution in the wire as a function of r. Then the temperature at the center line (r = 0) is determined by substituting the known quantities to be

$$T(0) = T_\infty + \frac{\dot{g}}{4k} r_0^2 + \frac{\dot{g} r_0}{2h}$$

$$= 212°\text{F} + \frac{(1800\,\text{Btu/h.in}^3)(0.25\,\text{in})^2}{4 \times (8.6\,\text{Btu/h.ft.°F})}\left(\frac{12\,\text{in}}{1\,\text{ft}}\right) + \frac{(1800\,\text{Btu/h.in}^3)(0.25\,\text{in})}{2 \times (820\,\text{Btu/h}\cdot\text{ft}^2\cdot\text{°F})}\left(\frac{12\,\text{in}}{1\,\text{ft}}\right)^2 = \mathbf{290.8°F}$$

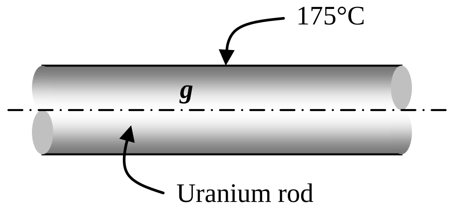
Thus the centerline temperature will be about 80°F above the temperature of the surface of the wire.

2-81E

```
"GIVEN"
r_0=0.25/12 "[ft]"
k=8.6 "[Btu/h-ft-F]"
"g_dot=1800 [Btu/h-in^3], parameter to be varied"
T_infinity=212 "[F]"
h=820 "[Btu/h-ft^2-F]"

"ANALYSIS"
T_0=T_infinity+(g_dot/Convert(in^3, ft^3))/(4*k)*(r_0^2-r^2)+((g_dot/Convert(in^3, ft^3))*r_0)/(2*h) "Variation of temperature"
r=0 "for centerline temperature"
```

g [Btu/h.in^3]	T_0 [F]
400	229.5
600	238.3
800	247
1000	255.8
1200	264.5
1400	273.3
1600	282
1800	290.8
2000	299.5
2200	308.3
2400	317

2-82 A nuclear fuel rod with a specified surface temperature is used as the fuel in a nuclear reactor. The center temperature of the rod is to be determined.



Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the rod is uniform.

Properties The thermal conductivity is given to be k = 29.5 W/m·°C.

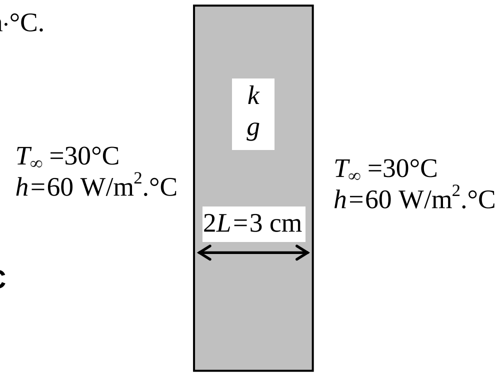
Analysis The center temperature of the rod is determined from

$$T_o = T_s + \frac{\dot{g}{r_o}^2}{4k} = 175°\text{C} + \frac{(7\times10^7\ \text{W/m}^3)(0.025\,\text{m})^2}{4(29.5\,\text{W/m.°C})} = \mathbf{545.8\,°C}$$

2-83 Both sides of a large stainless steel plate in which heat is generated uniformly are exposed to convection with the environment. The location and values of the highest and the lowest temperatures in the plate are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness, and there is thermal symmetry about the center plane **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k =15.1 W/m·°C.



Analysis The lowest temperature will occur at surfaces of plate while the highest temperature will occur at the midplane. Their values are determined directly from

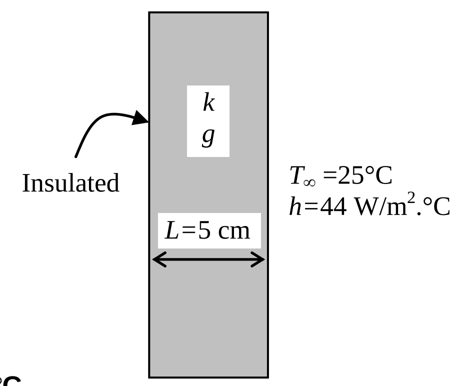
$$T_s = T_\infty + \frac{\dot{g}L}{h} = 30°\text{C} + \frac{(5\times10^5\ \text{W/m}^3)(0.015\,\text{m})}{60\,\text{W/m}^2.°\text{C}} = \mathbf{155°C}$$

$$T_o = T_s + \frac{\dot{g}L^2}{2k} = 155°\text{C} + \frac{(5\times10^5\ \text{W/m}^3)(0.015\,\text{m})^2}{2(15.1\,\text{W/m.°C})} = \mathbf{158.7°C}$$

2-84 Heat is generated uniformly in a large brass plate. One side of the plate is insulated while the other side is subjected to convection. The location and values of the highest and the lowest temperatures in the plate are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness, and there is thermal symmetry about the center plane **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k =111 W/m·°C.



Analysis This insulated plate whose thickness is L is equivalent to one-half of an uninsulated plate whose thickness is $2L$ since the midplane of the uninsulated plate can be treated as insulated surface. The highest temperature will occur at the insulated surface while the lowest temperature will occur at the surface which is exposed to the environment. Note that L in the following relations is the full thickness of the given plate since the insulated side represents the center surface of a plate whose thickness is doubled. The desired values are determined directly from

$$T_s = T_\infty + \frac{\dot{g}L}{h} = 25°\text{C} + \frac{(2\times10^5\ \text{W/m}^3)(0.05\,\text{m})}{44\,\text{W/m}^2.°\text{C}} = \mathbf{252.3\ °C}$$

$$T_o = T_s + \frac{\dot{g}L^2}{2k} = 252.3°\text{C} + \frac{(2\times10^5\ \text{W/m}^3)(0.05\,\text{m})^2}{2(111\,\text{W/m.°C})} = \mathbf{254.5\ °C}$$

2-85

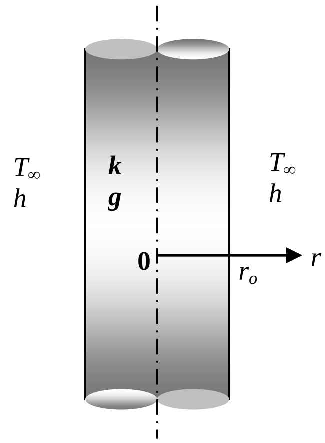
```
"GIVEN"
L=0.05 "[m]"
k=111 "[W/m-C]"
g_dot=2E5 "[W/m^3]"
T_infinity=25 "[C]"
"h=44 [W/m^2-C], parameter to be varied"

"ANALYSIS"
T_min=T_infinity+(g_dot*L)/h
T_max=T_min+(g_dot*L^2)/(2*k)
```

h [W/m^2.C]	T_{min} [C]	T_{max} [C]
20	525	527.3
25	425	427.3
30	358.3	360.6
35	310.7	313
40	275	277.3
45	247.2	249.5
50	225	227.3
55	206.8	209.1
60	191.7	193.9
65	178.8	181.1
70	167.9	170.1
75	158.3	160.6
80	150	152.3
85	142.6	144.9
90	136.1	138.4
95	130.3	132.5
100	125	127.3

2-86 A long resistance heater wire is subjected to convection at its outer surface. The surface temperature of the wire is to be determined using the applicable relations directly and by solving the applicable differential equation.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 15.1 W/m·°C.



Analysis (a) The heat generation per unit volume of the wire is

$$\dot{g} = \frac{\dot{Q}_{gen}}{V_{\text{wire}}} = \frac{\dot{Q}_{gen}}{\pi r_o^2 L} = \frac{2000 \text{ W}}{\pi(0.001 \text{ m})^2 (6 \text{ m})} = 1.061\times10^8 \text{ W/m}^3$$

The surface temperature of the wire is then (Eq. 2-68)

$$T_s = T_\infty + \frac{\dot{g}r_o}{2h} = 30^\circ\text{C} + \frac{(1.061\times10^8 \text{ W/m}^3)(0.001 \text{ m})}{2(140 \text{ W/m}^2.^\circ\text{C})} = \mathbf{409^\circ C}$$

(b) The mathematical formulation of this problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

and $-k\dfrac{dT(r_0)}{dr} = h[T(r_0) - T_\infty]$ (convection at the outer surface)

$$\frac{dT(0)}{dr} = 0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and integrating gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{g}}{k}r \quad \rightarrow \quad r\frac{dT}{dr} = -\frac{\dot{g}}{k}\frac{r^2}{2} + C_1 \qquad (a)$$

Applying the boundary condition at the center line,

B.C. at r = 0: $\quad 0\times\dfrac{dT(0)}{dr} = -\dfrac{\dot{g}}{2k}\times 0 + C_1 \quad \rightarrow \quad C_1 = 0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr} = -\frac{\dot{g}}{2k}r \quad \rightarrow \quad T(r) = -\frac{\dot{g}}{4k}r^2 + C_2 \qquad (b)$$

Applying the boundary condition at $r = r_0$,

B. C. at $r = r_0$: $\quad -k\dfrac{\dot{g}r_0}{2k} = h\left(-\dfrac{\dot{g}}{4k}r_0^2 + C_2 - T_\infty\right) \quad \rightarrow \quad C_2 = T_\infty + \dfrac{\dot{g}r_0}{2h} + \dfrac{\dot{g}}{4k}r_0^2$

Substituting this C_2 relation into Eq. (b) and rearranging give

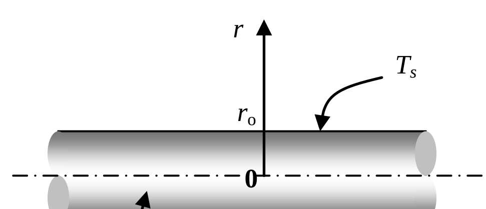
$$T(r) = T_\infty + \frac{\dot{g}}{4k}(r_0^2 - r^2) + \frac{\dot{g}r_0}{2h}$$

which is the temperature distribution in the wire as a function of r. Then the temperature of the wire at the surface ($r = r_0$) is determined by substituting the known quantities to be

$$T(r_0) = T_\infty + \frac{\dot{g}}{4k}(r_0^2 - r_0^2) + \frac{\dot{g}r_0}{2h} = T_\infty + \frac{\dot{g}r_o}{2h} = 30^\circ\text{C} + \frac{(1.061\times10^8 \text{ W/m}^3)(0.001 \text{ m})}{2(140 \text{ W/m}^2.^\circ\text{C})} = \mathbf{409^\circ C}$$

Note that both approaches give the same result.

2-87E Heat is generated uniformly in a resistance heater wire. The temperature difference between the center and the surface of the wire is to be determined.



Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be k = 5.8 Btu/h·ft·°F.

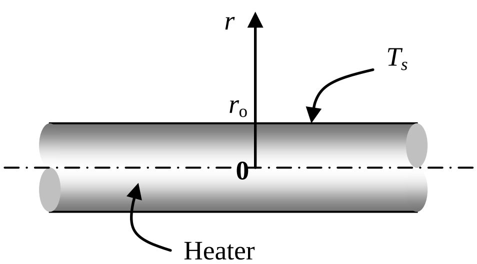
Analysis The resistance heater converts electric energy into heat at a rate of 3 kW. The rate of heat generation per unit length of the wire is

$$\dot{g} = \frac{\dot{Q}_{gen}}{V_{\text{wire}}} = \frac{\dot{Q}_{gen}}{\pi r_o^2 L} = \frac{(3\times 3412.14\,\text{Btu/h})}{\pi (0.04/12\,\text{ft})^2 (1\,\text{ft})} = 2.933\times 10^8\ \text{Btu/h.ft}^3$$

Then the temperature difference between the centerline and the surface becomes

$$\Delta T_{\max} = \frac{\dot{g} r_o^2}{4k} = \frac{(2.933\times 10^8\ \text{Btu/h.ft}^3)(0.04/12\,\text{ft})^2}{4(5.8\,\text{Btu/h.ft.°F})} = \mathbf{140.4°F}$$

2-88E Heat is generated uniformly in a resistance heater wire. The temperature difference between the center and the surface of the wire is to be determined.



Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be k = 4.5 Btu/h·ft·°F.

Analysis The resistance heater converts electric energy into heat at a rate of 3 kW. The rate of heat generation per unit volume of the wire is

$$\dot{g} = \frac{\dot{Q}_{gen}}{V_{\text{wire}}} = \frac{\dot{Q}_{gen}}{\pi r_o^2 L} = \frac{(3\times 3412.14\,\text{Btu/h})}{\pi (0.04/12\,\text{ft})^2 (1\,\text{ft})} = 2.933\times 10^8\ \text{Btu/h.ft}^3$$

Then the temperature difference between the centerline and the surface becomes

$$\Delta T_{\max} = \frac{\dot{g} r_o^2}{4k} = \frac{(2.933\times 10^8\ \text{Btu/h.ft}^3)(0.04/12\,\text{ft})^2}{4(4.5\,\text{Btu/h.ft.°F})} = \mathbf{181.0°F}$$

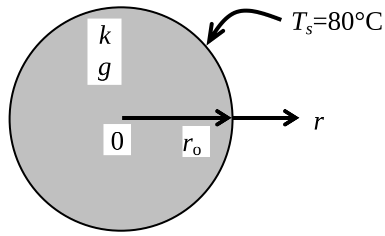
2-89 Heat is generated uniformly in a spherical radioactive material with specified surface temperature. The mathematical formulation, the variation of temperature in the sphere, and the center temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady since there is no indication of any changes with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the mid point. **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k = 15 W/m·°C.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0 \quad \text{with} \quad \dot{g}=\text{constant}$$



and $T(r_0)=T_s=80°\text{C}$ (specified surface temperature)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the mid point)}$$

(*b*) Multiplying both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{g}}{k}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr}=-\frac{\dot{g}}{k}\frac{r^3}{3}+C_1 \qquad (a)$$

Applying the boundary condition at the mid point,

B.C. at r = 0: $0\times\frac{dT(0)}{dr}=-\frac{\dot{g}}{3k}\times 0+C_1 \quad \rightarrow \quad C_1=0$

Dividing both sides of Eq. (*a*) by r^2 to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{g}}{3k}r$$

and $T(r)=-\frac{\dot{g}}{6k}r^2+C_2 \qquad (b)$

Applying the other boundary condition at $r=r_0$,

B. C. at $r=r_0$: $T_s=-\frac{\dot{g}}{6k}r_0^2+C_2 \quad \rightarrow \quad C_2=T_s+\frac{\dot{g}}{6k}r_0^2$

Substituting this C_2 relation into Eq. (*b*) and rearranging give

$$T(r)=T_s+\frac{\dot{g}}{6k}(r_0^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r.

(*c*) The temperature at the center of the sphere (r = 0) is determined by substituting the known quantities to be

$$T(0)=T_s+\frac{\dot{g}}{6k}(r_0^2-0^2)=T_s+\frac{\dot{g}r_0^2}{6k}=80°\text{C}+\frac{(4\times10^7\ \text{W/m}^3)(0.04\,\text{m})^2}{6\times(15\,\text{W/m.°C})}=\mathbf{791°C}$$

Thus the temperature at center will be about 711°C above the temperature of the outer surface of the sphere.

2-90

```
"GIVEN"
r_0=0.04 "[m]"
g_dot=4E7 "[W/m^3]"
T_s=80 "[C]"
k=15 "[W/m-C], Parameter to be varied"

"ANALYSIS"
T=T_s+g_dot/(6*k)*(r_0^2-r^2) "Temperature distribution as a function of r"
"r is the parameter to be varied"
T_0=T_s+g_dot/(6*k)*r_0^2 "Temperature at the center (r=0)"
```

r [m]	T [C]
0	791.1
0.002105	789.1
0.004211	783.2
0.006316	773.4
0.008421	759.6
0.01053	741.9
0.01263	720.2
0.01474	694.6
0.01684	665
0.01895	631.6
0.02105	594.1
0.02316	552.8
0.02526	507.5
0.02737	458.2
0.02947	405
0.03158	347.9
0.03368	286.8
0.03579	221.8
0.03789	152.9
0.04	80

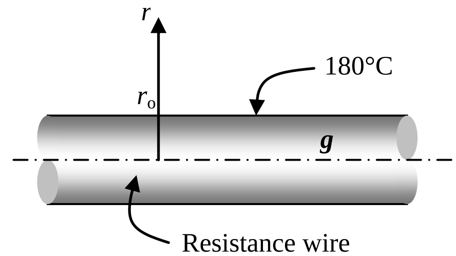
k [W/m.C]	T_0 [C]
10	1147
30.53	429.4
51.05	288.9
71.58	229
92.11	195.8
112.6	174.7
133.2	160.1
153.7	149.4
174.2	141.2
194.7	134.8
215.3	129.6
235.8	125.2
256.3	121.6
276.8	118.5
297.4	115.9
317.9	113.6
338.4	111.5
358.9	109.7
379.5	108.1
400	106.7

2-91 A long homogeneous resistance heater wire with specified surface temperature is used to boil water. The temperature of the wire 2 mm from the center is to be determined in steady operation.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 8 W/m·°C.

Analysis Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0$$

and $$T(r_0)=T_s=180°\text{C} \quad \text{(specified surface temperature)}$$

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and rearranging gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{g}}{k}r$$

Integrating with respect to r gives

$$r\frac{dT}{dr}=-\frac{\dot{g}}{k}\frac{r^2}{2}+C_1 \qquad (a)$$

It is convenient at this point to apply the boundary condition at the center since it is related to the first derivative of the temperature. It yields

B.C. at r = 0: $$0\times\frac{dT(0)}{dr}=-\frac{\dot{g}}{2k}\times 0+C_1 \quad \rightarrow \quad C_1=0$$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{g}}{2k}r$$

and $$T(r)=-\frac{\dot{g}}{4k}r^2+C_2 \qquad (b)$$

Applying the other boundary condition at $r=r_0$,

B. C. at $r=r_0$: $$T_s=-\frac{\dot{g}}{4k}r_0^2+C_2 \quad \rightarrow \quad C_2=T_s+\frac{\dot{g}}{4k}r_0^2$$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r)=T_s+\frac{\dot{g}}{4k}(r_0^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r. The temperature 2 mm from the center line (r = 0.002 m) is determined by substituting the known quantities to be

$$T(0.002\text{ m})=T_s+\frac{\dot{g}}{4k}(r_0^2-r^2)=180°\text{C}+\frac{5\times10^7\text{ W/m}^3}{4\times(8\text{ W/ m.°C})}[(0.005\text{ m})^2-(0.002\text{ m})^2]=\mathbf{212.8°C}$$

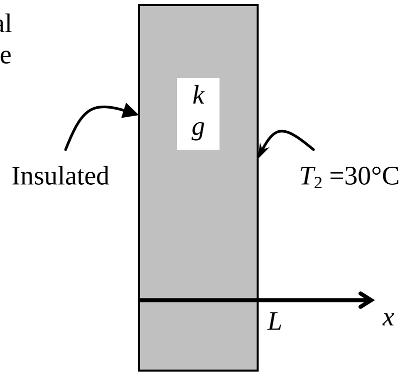
Thus the temperature at that location will be about 33°C above the temperature of the outer surface of the wire.

2-92 Heat is generated in a large plane wall whose one side is insulated while the other side is maintained at a specified temperature. The mathematical formulation, the variation of temperature in the wall, and the temperature of the insulated surface are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness, and there is thermal symmetry about the center plane. **3** Thermal conductivity is constant. **4** Heat generation varies with location in the x direction.

Properties The thermal conductivity is given to be k = 30 W/m·°C.

Analysis (a) Noting that heat transfer is steady and one-dimensional in x direction, the mathematical formulation of this problem can be expressed as



$$\frac{d^2T}{dx^2}+\frac{\dot{g}(x)}{k}=0$$

where $\dot{g}=\dot{g}_0 e^{-0.5x/L}$ and $\dot{g}_0 = 8\times10^6\ \text{W/m}^3$

and $\dfrac{dT(0)}{dx}=0$ (insulated surface at x = 0)

$$T(L)=T_2=30°\text{C} \text{ (specified surface temperature)}$$

(b) Rearranging the differential equation and integrating,

$$\frac{d^2T}{dx^2}=-\frac{\dot{g}_0}{k}e^{-0.5x/L} \quad\to\quad \frac{dT}{dx}=-\frac{\dot{g}_0}{k}\frac{e^{-0.5x/L}}{-0.5/L}+C_1 \quad\to\quad \frac{dT}{dx}=\frac{2\dot{g}_0L}{k}e^{-0.5x/L}+C_1$$

Integrating one more time,

$$T(x)=\frac{2\dot{g}_0L}{k}\frac{e^{-0.5x/L}}{-0.5/L}+C_1x+C_2 \quad\to\quad T(x)=-\frac{4\dot{g}_0L^2}{k}e^{-0.5x/L}+C_1x+C_2 \qquad (1)$$

Applying the boundary conditions:

B.C. at x = 0: $\quad \dfrac{dT(0)}{dx}=\dfrac{2\dot{g}_0L}{k}e^{-0.5\times0/L}+C_1 \quad\to\quad 0=\dfrac{2\dot{g}_0L}{k}+C_1 \quad\to\quad C_1=-\dfrac{2\dot{g}_0L}{k}$

B. C. at $x = L$: $\quad T(L)=T_2=-\dfrac{4\dot{g}_0L^2}{k}e^{-0.5L/L}+C_1L+C_2 \quad\to\quad C_2=T_2+\dfrac{4\dot{g}_0L^2}{k}e^{-0.5}+\dfrac{2\dot{g}_0L^2}{k}$

Substituting the C_1 and C_2 relations into Eq. (1) and rearranging give

$$T(x)=T_2+\frac{\dot{g}_0L^2}{k}[4(e^{-0.5}-e^{-0.5x/L})+(2-x/L)]$$

which is the desired solution for the temperature distribution in the wall as a function of x.

(c) The temperature at the insulate surface (x = 0) is determined by substituting the known quantities to be

$$T(0)=T_2+\frac{\dot{g}_0L^2}{k}[4(e^{-0.5}-e^{0})+(2-0/L)]$$

$$=30°\text{C}+\frac{(8\times10^6\ \text{W/m}^3)(0.05\,\text{m})^2}{(30\,\text{W/m}\cdot°\text{C})}[4(e^{-0.5}-1)+(2-0)]=\mathbf{314.1°C}$$

Therefore, there is a temperature difference of almost 300°C between the two sides of the plate.

2-93

```
"GIVEN"
L=0.05 "[m]"
T_s=30 "[C]"
k=30 "[W/m-C]"
g_dot_0=8E6 "[W/m^3]"

"ANALYSIS"
g_dot=g_dot_0*exp((-0.5*x)/L) "Heat generation as a function of x"
"x is the parameter to be varied"
```

x [m]	**g [W/m^3]**
0	8.000E+06
0.005	7.610E+06
0.01	7.239E+06
0.015	6.886E+06
0.02	6.550E+06
0.025	6.230E+06
0.03	5.927E+06
0.035	5.638E+06
0.04	5.363E+06
0.045	5.101E+06
0.05	4.852E+06