

<http://www.Drshokuhi.com>

سایت آموزش مهندسی مکانیک

2-126 A spherical liquid nitrogen container is subjected to specified temperature on the inner surface and convection on the outer surface. The mathematical formulation, the variation of temperature, and the rate of evaporation of nitrogen are to be determined for steady one-dimensional heat transfer.

Assumptions 1 Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the midpoint. **2** Thermal conductivity is constant. **3** There is no heat generation.

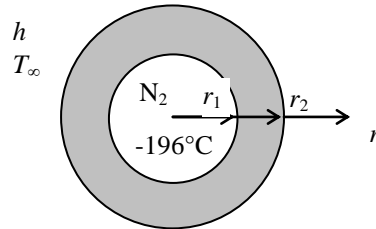
Properties The thermal conductivity of the tank is given to be $k = 18 \text{ W/m}\cdot\text{°C}$. Also, $h_{fg} = 198 \text{ kJ/kg}$ for nitrogen.

Analysis (a) Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = 0$$

and $T(r_1) = T_1 = -196^\circ\text{C}$

$$-k \frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty]$$



(b) Integrating the differential equation once with respect to r gives

$$r^2 \frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r^2} \rightarrow T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad T(r_1) = -\frac{C_1}{r_1} + C_2 = T_1$$

$$r = r_2: \quad -k \frac{C_1}{r_2^2} = h \left(-\frac{C_1}{r_2} + C_2 - T_\infty \right)$$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = \frac{r_2(T_1 - T_\infty)}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \quad \text{and} \quad C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{T_1 - T_\infty}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \frac{r_2}{r_1}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(r) &= -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = C_1 \left(\frac{1}{r_1} - \frac{1}{r} \right) + T_1 = \frac{T_1 - T_\infty}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \left(\frac{r_2}{r_1} - \frac{r_2}{r} \right) + T_1 \\ &= \frac{(-196 - 20)^\circ\text{C}}{1 - \frac{2.1}{2} - \frac{18 \text{ W/m}\cdot\text{°C}}{(25 \text{ W/m}^2\cdot\text{°C})(2.1 \text{ m})}} \left(\frac{2.1}{2} - \frac{2.1}{r} \right) + (-196)^\circ\text{C} = 549.8(1.05 - 2.1/r) - 196 \end{aligned}$$

(c) The rate of heat transfer through the wall and the rate of evaporation of nitrogen are determined from

$$\begin{aligned} \dot{Q} &= -kA \frac{dT}{dx} = -k(4\pi r^2) \frac{C_1}{r^2} = -4\pi k C_1 = -4\pi k \frac{r_2(T_1 - T_\infty)}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \\ &= -4\pi(18 \text{ W/m}\cdot\text{°C}) \frac{(2.1 \text{ m})(-196 - 20)^\circ\text{C}}{1 - \frac{2.1}{2} - \frac{18 \text{ W/m}\cdot\text{°C}}{(25 \text{ W/m}^2\cdot\text{°C})(2.1 \text{ m})}} = -261,200 \text{ W} \quad (\text{to the tank since negative}) \end{aligned}$$

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{261,200 \text{ J/s}}{198,000 \text{ J/kg}} = \mathbf{1.32 \text{ kg/s}}$$

2-127 A spherical liquid oxygen container is subjected to specified temperature on the inner surface and convection on the outer surface. The mathematical formulation, the variation of temperature, and the rate of evaporation of oxygen are to be determined for steady one-dimensional heat transfer.

Assumptions 1 Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the midpoint. **2** Thermal conductivity is constant. **3** There is no heat generation.

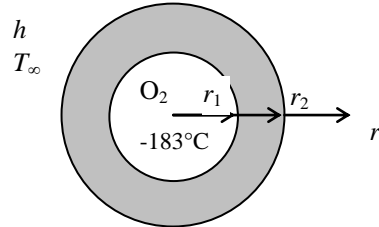
Properties The thermal conductivity of the tank is given to be $k = 18 \text{ W/m}\cdot^\circ\text{C}$. Also, $h_{fg} = 213 \text{ kJ/kg}$ for oxygen.

Analysis (a) Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = 0$$

and $T(r_1) = T_1 = -183^\circ\text{C}$

$$-k \frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty]$$



(b) Integrating the differential equation once with respect to r gives

$$r^2 \frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r^2} \rightarrow T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad T(r_1) = -\frac{C_1}{r_1} + C_2 = T_1$$

$$r = r_2: \quad -k \frac{C_1}{r_2^2} = h \left(-\frac{C_1}{r_2} + C_2 - T_\infty \right)$$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = \frac{r_2(T_1 - T_\infty)}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \quad \text{and} \quad C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{T_1 - T_\infty}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \frac{r_2}{r_1}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(r) &= -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = C_1 \left(\frac{1}{r_1} - \frac{1}{r} \right) + T_1 = \frac{T_1 - T_\infty}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \left(\frac{r_2}{r_1} - \frac{r_2}{r} \right) + T_1 \\ &= \frac{(-183 - 20)^\circ\text{C}}{1 - \frac{2.1}{2} - \frac{18 \text{ W/m}\cdot^\circ\text{C}}{(25 \text{ W/m}^2\cdot^\circ\text{C})(2.1 \text{ m})}} \left(\frac{2.1}{2} - \frac{2.1}{r} \right) + (-183)^\circ\text{C} = 516.7(1.05 - 2.1/r) - 183 \end{aligned}$$

(c) The rate of heat transfer through the wall and the rate of evaporation of nitrogen are determined from

$$\begin{aligned} \dot{Q} &= -kA \frac{dT}{dx} = -k(4\pi r^2) \frac{C_1}{r^2} = -4\pi k C_1 = -4\pi k \frac{r_2(T_1 - T_\infty)}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}} \\ &= -4\pi(18 \text{ W/m}\cdot^\circ\text{C}) \frac{(2.1 \text{ m})(-183 - 20)^\circ\text{C}}{1 - \frac{2.1}{2} - \frac{18 \text{ W/m}\cdot^\circ\text{C}}{(25 \text{ W/m}^2\cdot^\circ\text{C})(2.1 \text{ m})}} = -\mathbf{245,450 \text{ W}} \quad (\text{to the tank since negative}) \end{aligned}$$

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{245,450 \text{ J/s}}{213,000 \text{ J/kg}} = \mathbf{1.15 \text{ kg/s}}$$

2-128 A large plane wall is subjected to convection, radiation, and specified temperature on the right surface and no conditions on the left surface. The mathematical formulation, the variation of temperature in the wall, and the left surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides of the wall are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall.

Properties The thermal conductivity and emissivity are given to be $k = 8.4 \text{ W/m}\cdot\text{°C}$ and $\varepsilon = 0.7$.

Analysis (a) Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, and the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

$$\text{and } -k \frac{dT(L)}{dx} = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{\text{surr}}^4] = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4]$$

$$T(L) = T_2 = 45^\circ\text{C}$$

(b) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$\begin{aligned} \text{Convection at } x = L \quad & -kC_1 = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{surr}}^4] \\ \rightarrow C_1 = & -\{h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 460)^4 - T_{\text{surr}}^4]\} / k \end{aligned}$$

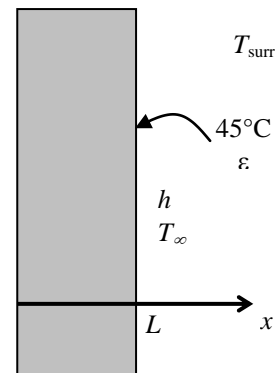
$$\text{Temperature at } x = L: \quad T(L) = C_1 \times L + C_2 = T_2 \rightarrow C_2 = T_2 - C_1L$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= C_1x + (T_2 - C_1L) = T_2 - (L - x)C_1 = T_2 + \frac{h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4]}{k}(L - x) \\ &= 45^\circ\text{C} + \frac{(14 \text{ W/m}^2 \cdot \text{°C})(45 - 25)^\circ\text{C} + 0.7(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(318 \text{ K})^4 - (290 \text{ K})^4]}{8.4 \text{ W/m} \cdot \text{°C}}(0.4 - x) \text{ m} \\ &= 45 + 48.23(0.4 - x) \end{aligned}$$

(c) The temperature at $x = 0$ (the left surface of the wall) is

$$T(0) = 45 + 48.23(0.4 - 0) = \mathbf{64.3^\circ\text{C}}$$



2-129 The base plate of an iron is subjected to specified heat flux on the left surface and convection and radiation on the right surface. The mathematical formulation, and an expression for the outer surface temperature and its value are to be determined for steady one-dimensional heat transfer.

Assumptions 1 Heat conduction is steady and one-dimensional. 2 Thermal conductivity is constant. 3 There is no heat generation. 4 Heat loss through the upper part of the iron is negligible.

Properties The thermal conductivity and emissivity are given to be $k = 2.3 \text{ W/m}\cdot^\circ\text{C}$ and $\varepsilon = 0.7$.

Analysis (a) Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be

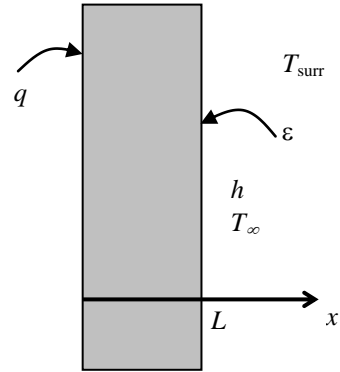
$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{1000 \text{ W}}{150 \times 10^{-4} \text{ m}^2} = 66,667 \text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and $-k \frac{dT(0)}{dx} = \dot{q}_0 = 66,667 \text{ W/m}^2$

$$-k \frac{dT(L)}{dx} = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{\text{surr}}^4] = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4]$$



(b) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \quad -kC_1 = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4]$$

Eliminating the constant C_1 from the two relations above gives the following expression for the outer surface temperature T_2 ,

$$h(T_2 - T_\infty) + \varepsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4] = \dot{q}_0$$

(c) Substituting the known quantities into the implicit relation above gives

$$(30 \text{ W/m}^2 \cdot ^\circ\text{C})(T_2 - 22) + 0.7(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(T_2 + 273)^4 - 290^4] = 66,667 \text{ W/m}^2$$

Using an equation solver (or a trial and error approach), the outer surface temperature is determined from the relation above to be

$$T_2 = 758^\circ\text{C}$$

2-130 The base plate of an iron is subjected to specified heat flux on the left surface and convection and radiation on the right surface. The mathematical formulation, and an expression for the outer surface temperature and its value are to be determined for steady one-dimensional heat transfer.

Assumptions 1 Heat conduction is steady and one-dimensional. 2 Thermal conductivity is constant. 3 There is no heat generation. 4 Heat loss through the upper part of the iron is negligible.

Properties The thermal conductivity and emissivity are given to be $k = 2.3 \text{ W/m}\cdot\text{C}$ and $\epsilon = 0.7$.

Analysis (a) Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be

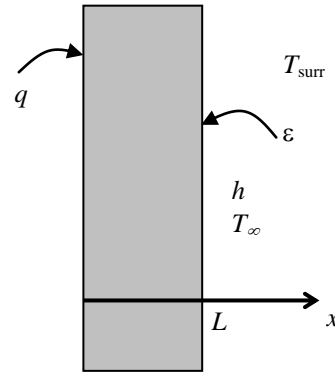
$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{1500 \text{ W}}{150 \times 10^{-4} \text{ m}^2} = 100,000 \text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2 T}{dx^2} = 0$$

and $-k \frac{dT(0)}{dx} = \dot{q}_0 = 80,000 \text{ W/m}^2$

$$-k \frac{dT(L)}{dx} = h[T(L) - T_\infty] + \epsilon\sigma[T(L)^4 - T_{\text{surr}}^4] = h[T_2 - T_\infty] + \epsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4]$$



(b) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \quad -kC_1 = h[T_2 - T_\infty] + \epsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4]$$

Eliminating the constant C_1 from the two relations above gives the following expression for the outer surface temperature T_2 ,

$$h(T_2 - T_\infty) + \epsilon\sigma[(T_2 + 273)^4 - T_{\text{surr}}^4] = \dot{q}_0$$

(c) Substituting the known quantities into the implicit relation above gives

$$(30 \text{ W/m}^2 \cdot \text{C})(T_2 - 22) + 0.7(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(T_2 + 273)^4 - 290^4] = 100,000 \text{ W/m}^2$$

Using an equation solver (or a trial and error approach), the outer surface temperature is determined from the relation above to be

$$T_2 = \mathbf{895.8^\circ\text{C}}$$

2-131E The concrete slab roof of a house is subjected to specified temperature at the bottom surface and convection and radiation at the top surface. The temperature of the top surface of the roof and the rate of heat transfer are to be determined when steady operating conditions are reached.

Assumptions 1 Steady operating conditions are reached. 2 Heat transfer is one-dimensional since the roof area is large relative to its thickness, and the thermal conditions on both sides of the roof are uniform. 3 Thermal properties are constant. 4 There is no heat generation in the wall.

Properties The thermal conductivity and emissivity are given to be $k = 1.1 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$ and $\varepsilon = 0.9$.

Analysis In steady operation, heat conduction through the roof must be equal to net heat transfer from the outer surface. Therefore, taking the outer surface temperature of the roof to be T_2 (in $^\circ\text{F}$),

$$kA \frac{T_1 - T_2}{L} = hA(T_2 - T_\infty) + \varepsilon A \sigma [(T_2 + 460)^4 - T_{\text{sky}}^4]$$

Canceling the area A and substituting the known quantities,

$$(1.1 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}) \frac{(62 - T_2)^\circ\text{F}}{0.8 \text{ ft}} = (3.2 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F})(T_2 - 50)^\circ\text{F} + 0.8(0.1714 \times 10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(T_2 + 460)^4 - 310^4] \text{ R}^4$$

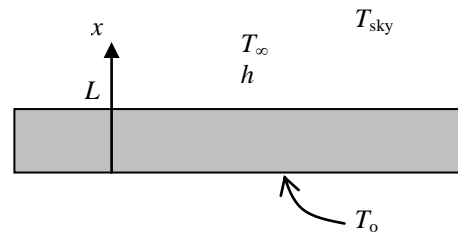
Using an equation solver (or the trial and error method), the outer surface temperature is determined to be

$$T_2 = 38^\circ\text{F}$$

Then the rate of heat transfer through the roof becomes

$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (1.1 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F})(25 \times 35 \text{ ft}^2) \frac{(62 - 38)^\circ\text{F}}{0.8 \text{ ft}} = 28,875 \text{ Btu/h}$$

Discussion The positive sign indicates that the direction of heat transfer is from the inside to the outside. Therefore, the house is losing heat as expected.



2-132 The surface and interface temperatures of a resistance wire covered with a plastic layer are to be determined.

Assumptions 1 Heat transfer is steady since there is no change with time. 2 Heat transfer is one-dimensional since this two-layer heat transfer problem possesses symmetry about the center line and involves no change in the axial direction, and thus $T = T(r)$. 3 Thermal conductivities are constant. 4 Heat generation in the wire is uniform.

Properties It is given that $k_{\text{wire}} = 15 \text{ W/m}\cdot^\circ\text{C}$ and $k_{\text{plastic}} = 1.2 \text{ W/m}\cdot^\circ\text{C}$.

Analysis Letting T_I denote the unknown interface temperature, the mathematical formulation of the heat transfer problem in the wire can be expressed as

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{g}}{k} = 0$$

with $T(r_1) = T_I$ and $\frac{dT(0)}{dr} = 0$

Multiplying both sides of the differential equation by r , rearranging, and integrating give

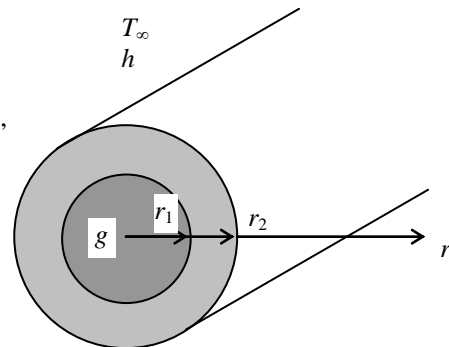
$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = -\frac{\dot{g}}{k} r \rightarrow r \frac{dT}{dr} = -\frac{\dot{g}}{k} \frac{r^2}{2} + C_1 \quad (a)$$

Applying the boundary condition at the center ($r = 0$) gives

B.C. at $r = 0$: $0 \times \frac{dT(0)}{dr} = -\frac{\dot{g}}{2k} \times 0 + C_1 \rightarrow C_1 = 0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr} = -\frac{\dot{g}}{2k} r \rightarrow T(r) = -\frac{\dot{g}}{4k} r^2 + C_2 \quad (b)$$



Applying the other boundary condition at $r = r_1$,

$$\text{B. C. at } r = r_1: \quad T_I = -\frac{\dot{g}}{4k}r_1^2 + C_2 \quad \rightarrow \quad C_2 = T_I + \frac{\dot{g}}{4k}r_1^2$$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T_{\text{wire}}(r) = T_I + \frac{\dot{g}}{4k_{\text{wire}}}(r_1^2 - r^2) \quad (c)$$

Plastic layer The mathematical formulation of heat transfer problem in the plastic can be expressed as

$$\frac{d}{dr}\left(r \frac{dT}{dr}\right) = 0$$

$$\text{with } T(r_1) = T_I \quad \text{and} \quad -k \frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty]$$

The solution of the differential equation is determined by integration to be

$$r \frac{dT}{dr} = C_1 \quad \rightarrow \quad \frac{dT}{dr} = \frac{C_1}{r} \quad \rightarrow \quad T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad C_1 \ln r_1 + C_2 = T_I \quad \rightarrow \quad C_2 = T_I - C_1 \ln r_1$$

$$r = r_2: \quad -k \frac{C_1}{r_2} = h[(C_1 \ln r_2 + C_2) - T_\infty] \quad \rightarrow \quad C_1 = \frac{T_\infty - T_I}{\ln \frac{r_2}{r_1} + \frac{k}{hr_2}}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature in plastic is determined to be

$$T_{\text{plastic}}(r) = C_1 \ln r + T_I - C_1 \ln r_1 = T_I + \frac{T_\infty - T_I}{\ln \frac{r_2}{r_1} + \frac{k_{\text{plastic}}}{hr_2}} \ln \frac{r}{r_1}$$

We have already utilized the first interface condition by setting the wire and ceramic layer temperatures equal to T_I at the interface $r = r_1$. The interface temperature T_I is determined from the second interface condition that the heat flux in the wire and the plastic layer at $r = r_1$ must be the same:

$$-k_{\text{wire}} \frac{dT_{\text{wire}}(r_1)}{dr} = -k_{\text{plastic}} \frac{dT_{\text{plastic}}(r_1)}{dr} \quad \rightarrow \quad \frac{\dot{g}r_1}{2} = -k_{\text{plastic}} \frac{T_\infty - T_I}{\ln \frac{r_2}{r_1} + \frac{k}{hr_2}} \frac{1}{r_1}$$

Solving for T_I and substituting the given values, the interface temperature is determined to be

$$\begin{aligned} T_I &= \frac{\dot{g}r_1^2}{2k_{\text{plastic}}} \left(\ln \frac{r_2}{r_1} + \frac{k_{\text{plastic}}}{hr_2} \right) + T_\infty \\ &= \frac{(1.5 \times 10^6 \text{ W/m}^3)(0.003 \text{ m})^2}{2(1.8 \text{ W/m} \cdot \text{ }^\circ\text{C})} \left(\ln \frac{0.007 \text{ m}}{0.003 \text{ m}} + \frac{1.8 \text{ W/m} \cdot \text{ }^\circ\text{C}}{(14 \text{ W/m}^2 \cdot \text{ }^\circ\text{C})(0.007 \text{ m})} \right) + 25^\circ\text{C} = \mathbf{97.1^\circ\text{C}} \end{aligned}$$

Knowing the interface temperature, the temperature at the center line ($r = 0$) is obtained by substituting the known quantities into Eq. (c),

$$T_{\text{wire}}(0) = T_I + \frac{\dot{g}r_1^2}{4k_{\text{wire}}} = 97.1^\circ\text{C} + \frac{(1.5 \times 10^6 \text{ W/m}^3)(0.003 \text{ m})^2}{4 \times (18 \text{ W/m} \cdot \text{ }^\circ\text{C})} = \mathbf{97.3^\circ\text{C}}$$

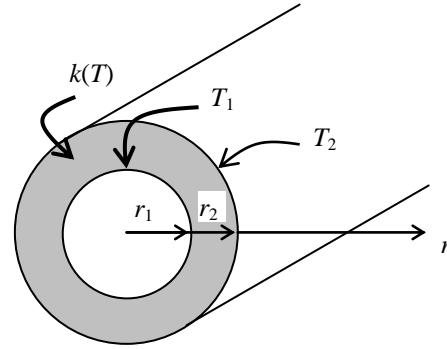
Thus the temperature of the centerline will be slightly above the interface temperature.

2-133 A cylindrical shell with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer through the shell is to be determined.

Assumptions 1 Heat transfer is given to be steady and one-dimensional. 2 Thermal conductivity varies quadratically. 3 There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T^2)$.

Analysis When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 is determined from



$$k_{ave} = \frac{\int_{T_1}^{T_2} k(T) dT}{T_2 - T_1} = \frac{\int_{T_1}^{T_2} k_0(1 + \beta T^2) dT}{T_2 - T_1} = \frac{k_0 \left(T + \frac{\beta}{3} T^3 \right) \Big|_{T_1}^{T_2}}{T_2 - T_1} = \frac{k_0 \left[(T_2 - T_1) + \frac{\beta}{3} (T_2^3 - T_1^3) \right]}{T_2 - T_1}$$

$$= k_0 \left[1 + \frac{\beta}{3} (T_2^2 + T_1 T_2 + T_1^2) \right]$$

This relation is based on the requirement that the rate of heat transfer through a medium with constant average thermal conductivity k_{ave} equals the rate of heat transfer through the same medium with variable conductivity $k(T)$.

Then the rate of heat conduction through the cylindrical shell can be determined from Eq. 2-77 to be

$$\dot{Q}_{cylinder} = 2\pi k_{ave} L \frac{T_1 - T_2}{\ln(r_2 / r_1)} = 2\pi k_0 \left[1 + \frac{\beta}{3} (T_2^2 + T_1 T_2 + T_1^2) \right] L \frac{T_1 - T_2}{\ln(r_2 / r_1)}$$

Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq. 2-77, and performed the indicated integration.

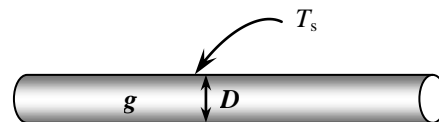
2-134 Heat is generated uniformly in a cylindrical uranium fuel rod. The temperature difference between the center and the surface of the fuel rod is to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. 3 Thermal conductivity is constant. 4 Heat generation is uniform.

Properties The thermal conductivity of uranium at room temperature is $k = 27.6 \text{ W/m}\cdot\text{°C}$ (Table A-3).

Analysis The temperature difference between the center and the surface of the fuel rods is determined from

$$T_o - T_s = \frac{\dot{g} r_o^2}{4k} = \frac{(4 \times 10^7 \text{ W/m}^3)(0.016 \text{ m})^2}{4(27.6 \text{ W/m}\cdot\text{°C})} = \mathbf{92.8\text{°C}}$$



2-135 A large plane wall is subjected to convection on the inner and outer surfaces. The mathematical formulation, the variation of temperature, and the temperatures at the inner and outer surfaces to be determined for steady one-dimensional heat transfer.

Assumptions 1 Heat conduction is steady and one-dimensional. 2 Thermal conductivity is constant. 3 There is no heat generation.

Properties The thermal conductivity is given to be $k = 0.77 \text{ W/m}\cdot\text{°C}$.

Analysis (a) Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the inner surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and

$$h_1[T_{\infty 1} - T(0)] = -k \frac{dT(0)}{dx}$$

$$-k \frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

(b) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad h_1[T_{\infty 1} - (C_1 \times 0 + C_2)] = -kC_1$$

$$x = L: \quad -kC_1 = h_2[(C_1L + C_2) - T_{\infty 2}]$$

Substituting the given values, these equations can be written as

$$5(27 - C_2) = -0.77C_1$$

$$-0.77C_1 = (12)(0.2C_1 + C_2 - 8)$$

Solving these equations simultaneously give

$$C_1 = -45.44 \quad C_2 = 20$$

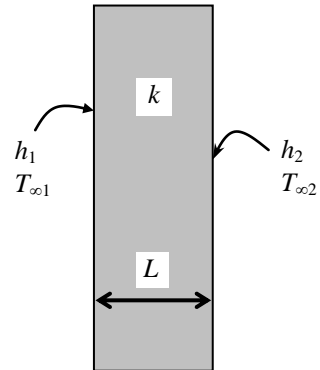
Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = 20 - 45.44x$$

(c) The temperatures at the inner and outer surfaces are

$$T(0) = 20 - 45.44 \times 0 = \mathbf{20^\circ\text{C}}$$

$$T(L) = 20 - 45.44 \times 0.2 = \mathbf{10.9^\circ\text{C}}$$



2-136 A hollow pipe is subjected to specified temperatures at the inner and outer surfaces. There is also heat generation in the pipe. The variation of temperature in the pipe and the center surface temperature of the pipe are to be determined for steady one-dimensional heat transfer.

Assumptions 1 Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the centerline. **2** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 20 \text{ W/m}\cdot^\circ\text{C}$.

Analysis The rate of heat generation is determined from

$$\dot{g} = \frac{\dot{W}}{V} = \frac{\dot{W}}{\pi(D_2^2 - D_1^2)L/4} = \frac{25,000 \text{ W}}{\pi[(0.4 \text{ m})^2 - (0.3 \text{ m})^2](12 \text{ m})/4} = 37,894 \text{ W/m}^3$$

Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{g}}{k} = 0$$

and $T(r_1) = T_1 = 60^\circ\text{C}$

$T(r_2) = T_2 = 80^\circ\text{C}$

Rearranging the differential equation

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = \frac{-\dot{g}r}{k} = 0$$

and then integrating once with respect to r ,

$$r \frac{dT}{dr} = \frac{-\dot{g}r^2}{2k} + C_1$$

Rearranging the differential equation again

$$\frac{dT}{dr} = \frac{-\dot{g}r}{2k} + \frac{C_1}{r}$$

and finally integrating again with respect to r , we obtain

$$T(r) = \frac{-\dot{g}r^2}{4k} + C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad T(r_1) = \frac{-\dot{g}r_1^2}{4k} + C_1 \ln r_1 + C_2$$

$$r = r_2: \quad T(r_2) = \frac{-\dot{g}r_2^2}{4k} + C_1 \ln r_2 + C_2$$

Substituting the given values, these equations can be written as

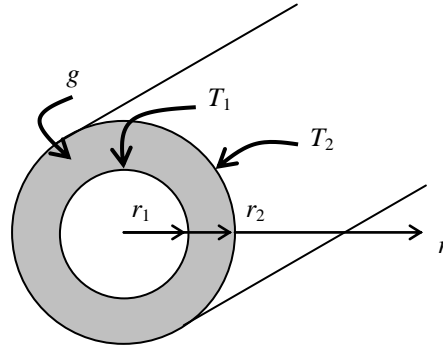
$$60 = \frac{-(37,894)(0.15)^2}{4(20)} + C_1 \ln(0.15) + C_2$$

$$80 = \frac{-(37,894)(0.20)^2}{4(20)} + C_1 \ln(0.20) + C_2$$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = 98.34 \quad C_2 = 257.2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be



$$T(r) = \frac{-37,894r^2}{4(20)} + 98.34 \ln r + 257.2 = 257.2 - 473.68r^2 + 98.34 \ln r$$

The temperature at the center surface of the pipe is determined by setting radius r to be 17.5 cm, which is the average of the inner radius and outer radius.

$$T(r) = 257.2 - 473.68(0.175)^2 + 98.34 \ln(0.175) = 71.2^\circ\text{C}$$

2-137 A spherical ball in which heat is generated uniformly is exposed to iced-water. The temperatures at the center and at the surface of the ball are to be determined.

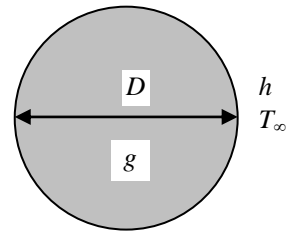
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional, and there is thermal symmetry about the center point. **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be $k = 45 \text{ W/m}\cdot^\circ\text{C}$.

Analysis The temperatures at the center and at the surface of the ball are determined directly from

$$T_s = T_\infty + \frac{\dot{g}r_0}{3h} = 0^\circ\text{C} + \frac{(2.6 \times 10^6 \text{ W/m}^3)(0.15 \text{ m})}{3(1200 \text{ W/m}^2 \cdot ^\circ\text{C})} = 108.3^\circ\text{C}$$

$$T_0 = T_s + \frac{\dot{g}r_0^2}{6k} = 108.3^\circ\text{C} + \frac{(2.6 \times 10^6 \text{ W/m}^3)(0.15 \text{ m})^2}{6(45 \text{ W/m}\cdot^\circ\text{C})} = 325^\circ\text{C}$$



2-138 2-141 Design and Essay Problems
