Chapter 3
STEADY HEAT CONDUCTION

Steady Heat Conduction In Plane Walls

3-1C (a) If the lateral surfaces of the rod are insulated, the heat transfer surface area of the cylindrical rod is the bottom or the top surface area of the rod, $A_s = \pi D^2 / 4$. (b) If the top and the bottom surfaces of the rod are insulated, the heat transfer area of the rod is the lateral surface area of the rod, $A = \pi DL$.

3-2C In steady heat conduction, the rate of heat transfer into the wall is equal to the rate of heat transfer out of it. Also, the temperature at any point in the wall remains constant. Therefore, the energy content of the wall does not change during steady heat conduction. However, the temperature along the wall and thus the energy content of the wall will change during transient conduction.

3-3C The temperature distribution in a plane wall will be a straight line during steady and one dimensional heat transfer with constant wall thermal conductivity.

3-4C The thermal resistance of a medium represents the resistance of that medium against heat transfer.

3-5C The combined heat transfer coefficient represents the combined effects of radiation and convection heat transfers on a surface, and is defined as $h_{\text{combined}} = h_{\text{convection}} + h_{\text{radiation}}$. It offers the convenience of incorporating the effects of radiation in the convection heat transfer coefficient, and to ignore radiation in heat transfer calculations.

3-6C Yes. The convection resistance can be defined as the inverse of the convection heat transfer coefficient per unit surface area since it is defined as $R_{conv} = 1/(hA)$.

3-7C The convection and the radiation resistances at a surface are parallel since both the convection and radiation heat transfers occur simultaneously.

3-8C For a surface of A at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad}, the single equivalent heat transfer coefficient is $h_{eqv} = h_{conv} + h_{rad}$ when the medium and the surrounding surfaces are at the same temperature. Then the equivalent thermal resistance will be $R_{eqv} = 1/(h_{eqv} A)$.

3-9C The thermal resistance network associated with a five-layer composite wall involves five single-layer resistances connected in series.

3-10C Once the rate of heat transfer $\dot{Q}$ is known, the temperature drop across any layer can be determined by multiplying heat transfer rate by the thermal resistance across that layer, $\Delta T_{layer} = \dot{Q} R_{layer}$

3-11C The temperature of each surface in this case can be determined from

$$\dot{Q} = (T_{\infty 1} - T_{s1}) / R_{\infty 1 - s1} \longrightarrow T_{s1} = T_{\infty 1} - (\dot{Q} R_{\infty 1 - s1})$$

$$\dot{Q} = (T_{s2} - T_{\infty 2}) / R_{s2 - \infty 2} \longrightarrow T_{s2} = T_{\infty 2} + (\dot{Q} R_{s2 - \infty 2})$$

where $R_{\infty - i}$ is the thermal resistance between the environment ∞ and surface i.

3-12C Yes, it is.

3-13C The window glass which consists of two 4 mm thick glass sheets pressed tightly against each other will probably have thermal contact resistance which serves as an additional thermal resistance to heat transfer through window, and thus the heat transfer rate will be smaller relative to the one which consists of a single 8 mm thick glass sheet.

3-14C Convection heat transfer through the wall is expressed as $\dot{Q} = hA_s(T_s - T_\infty)$. In steady heat transfer, heat transfer rate to the wall and from the wall are equal. Therefore at the outer surface which has convection heat transfer coefficient three times that of the inner surface will experience three times smaller temperature drop compared to the inner surface. Therefore, at the outer surface, the temperature will be closer to the surrounding air temperature.

3-15C The new design introduces the thermal resistance of the copper layer in addition to the thermal resistance of the aluminum which has the same value for both designs. Therefore, the new design will be a poorer conductor of heat.

3-16C The blanket will introduce additional resistance to heat transfer and slow down the heat gain of the drink wrapped in a blanket. Therefore, the drink left on a table will warm up faster.

3-17 The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

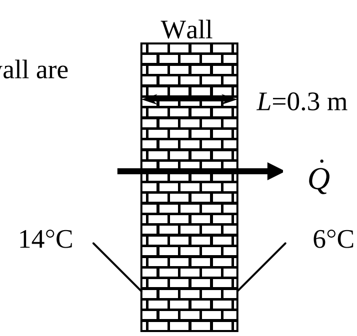
Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be k = 0.8 W/m·°C.

Analysis The surface area of the wall and the rate of heat loss through the wall are

$$A = (4 \text{ m}) \times (6 \text{ m}) = 24 \text{ m}^2$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8 \text{ W/m.°C})(24 \text{ m}^2)\frac{(14-6)\text{°C}}{0.3 \text{ m}} = \mathbf{512\ W}$$

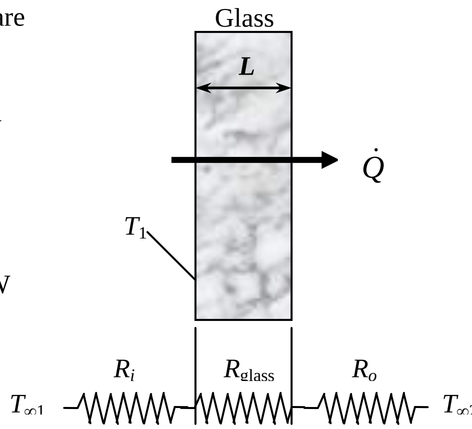


3-18 The two surfaces of a window are maintained at specified temperatures. The rate of heat loss through the window and the inner surface temperature are to be determined.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be k = 0.78 W/m·°C.

Analysis The area of the window and the individual resistances are



$$A = (1.2\text{ m}) \times (2\text{ m}) = 2.4\text{ m}^2$$

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.\text{°C})(2.4\text{ m}^2)} = 0.04167\text{°C/W}$$

$$R_{glass} = \frac{L}{k_1 A} = \frac{0.006\text{ m}}{(0.78\text{ W/m.°C})(2.4\text{ m}^2)} = 0.00321\text{°C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.\text{°C})(2.4\text{ m}^2)} = 0.01667\text{°C/W}$$

$$R_{total} = R_{conv,1} + R_{glass} + R_{conv,2} = 0.04167 + 0.00321 + 0.01667 = 0.06155\text{°C/W}$$

The steady rate of heat transfer through window glass is then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24 - (-5)]\text{°C}}{0.06155\text{ °C/W}} = \mathbf{471\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24\text{°C} - (471\text{ W})(0.04167\text{ °C/W}) = \mathbf{4.4°C}$$

3-19 A double-pane window consists of two 3-mm thick layers of glass separated by a 12-mm wide stagnant air space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass and air are given to be k_{glass} = 0.78 W/m·°C and k_{air} = 0.026 W/m·°C.

Analysis The area of the window and the individual resistances are

$$A = (1.2\text{ m}) \times (2\text{ m}) = 2.4\text{ m}^2$$

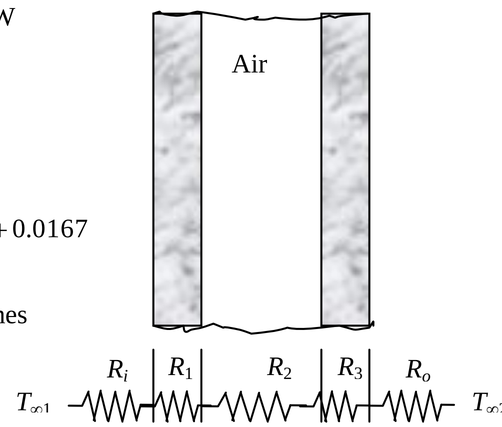
$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.°\text{C})(2.4\text{ m}^2)} = 0.0417°\text{C/W}$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.003\text{ m}}{(0.78\text{ W/m}.°\text{C})(2.4\text{ m}^2)} = 0.0016°\text{C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.012\text{ m}}{(0.026\text{ W/m}.°\text{C})(2.4\text{ m}^2)} = 0.1923°\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^\circ\text{C})(2.4\text{ m}^2)} = 0.0167\ ^\circ\text{C/W}$$

$$R_{total} = R_{conv,1} + 2R_1 + R_2 + R_{conv,2} = 0.0417 + 2(0.0016) + 0.1923 + 0.0167$$
$$= 0.2539°\text{C/W}$$



The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[24 - (-5)]°\text{C}}{0.2539°\text{C/W}} = \mathbf{114\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty1} - \dot{Q}R_{conv,1} = 24^\circ\text{C} - (114\text{ W})(0.0417°\text{C/W}) = \mathbf{19.2°C}$$

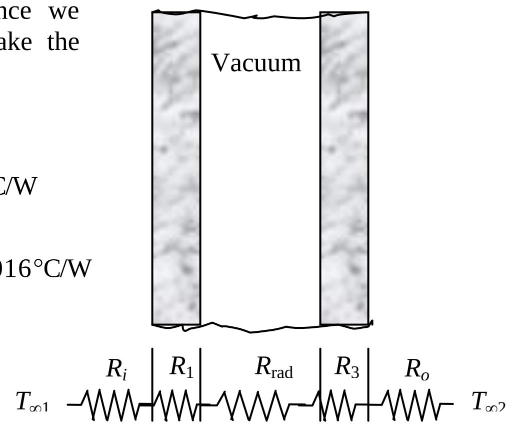
3-20 A double-pane window consists of two 3-mm thick layers of glass separated by an evacuated space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be k_{glass} = 0.78 W/m·°C.

Analysis Heat cannot be conducted through an evacuated space since the thermal conductivity of vacuum is zero (no medium to conduct heat) and thus its thermal resistance is zero. Therefore, if radiation is disregarded, the heat transfer through the window will be zero. Then the answer of this problem is **zero** since the problem states to disregard radiation.

Discussion In reality, heat will be transferred between the glasses by radiation. We do not know the inner surface temperatures of windows. In order to determine radiation heat resistance we assume them to be 5°C and 15°C, respectively, and take the emissivity to be 1. Then individual resistances are



$$A = (1.2 \text{ m}) \times (2 \text{ m}) = 2.4 \text{ m}^2$$

$$R_{\text{i}} = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2.\text{°C})(2.4 \text{ m}^2)} = 0.0417\text{°C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.003 \text{m}}{(0.78 \text{ W/m.°C})(2.4 \text{ m}^2)} = 0.0016\text{°C/W}$$

$$\begin{aligned} R_{rad} &= \frac{1}{\varepsilon\sigma A(T_s{}^2 + T_{surr}{}^2)(T_s + T_{surr})} \\ &= \frac{1}{1(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)(2.4 \text{ m}^2)[288^2 + 278^2][288+278]K^3} \\ &= 0.0810\text{°C/W} \end{aligned}$$

$$R_{\text{o}} = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2.\text{°C})(2.4 \text{ m}^2)} = 0.0167\text{°C/W}$$

$$\begin{aligned} R_{total} &= R_{conv,1} + 2R_1 + R_{rad} + R_{conv,2} = 0.0417 + 2(0.0016) + 0.0810 + 0.0167 \\ &= 0.1426\text{°C/W} \end{aligned}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[24 - (-5)]\text{°C}}{0.1426\text{°C / W}} = \mathbf{203\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty1} - \dot{Q}R_{conv,1} = 24\text{° C} - (203 \text{ W})(0.0417\text{° C / W}) = \mathbf{15.5°\ C}$$

Similarly, the inner surface temperatures of the glasses are calculated to be 15.2 and -1.2°C (we had assumed them to be 15 and 5°C when determining the radiation resistance). We can improve the result obtained by reevaluating the radiation resistance and repeating the calculations.

3-21

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"GIVEN"
A=1.2*2 "[m^2]"
L_glass=3 "[mm]"
k_glass=0.78 "[W/m-C]"
"L_air=12 [mm], parameter to be varied"
T_infinity_1=24 "[C]"
T_infinity_2=-5 "[C]"
h_1=10 "[W/m^2-C]"
h_2=25 "[W/m^2-C]"

"PROPERTIES"
k_air=conductivity(Air,T=25)

"ANALYSIS"
R_conv_1=1/(h_1*A)
R_glass=(L_glass*Convert(mm, m))/(k_glass*A)
R_air=(L_air*Convert(mm, m))/(k_air*A)
R_conv_2=1/(h_2*A)
R_total=R_conv_1+2*R_glass+R_air+R_conv_2
Q_dot=(T_infinity_1-T_infinity_2)/R_total
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L_{air} [mm]	Q [W]
2	307.8
4	228.6
6	181.8
8	150.9
10	129
12	112.6
14	99.93
16	89.82
18	81.57
20	74.7

3-22E The inner and outer surfaces of the walls of an electrically heated house remain at specified temperatures during a winter day. The amount of heat lost from the house that day and its its cost are to be determined.

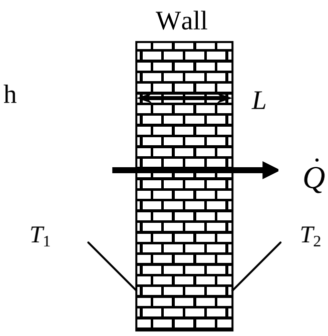
Assumptions **1** Heat transfer through the walls is steady since the surface temperatures of the walls remain constant at the specified values during the time period considered. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity of the walls is constant.

Properties The thermal conductivity of the brick wall is given to be k = 0.40 Btu/h·ft·°F.

Analysis We consider heat loss through the walls only. The total heat transfer area is

$$A = 2(40\times 9 + 30\times 9) = 1260 \text{ ft}^2$$

The rate of heat loss during the daytime is



$$\dot{Q}_{\text{day}} = kA\frac{T_1 - T_2}{L} = (0.40 \text{ Btu/h.ft.°F})(1260 \text{ ft}^2)\frac{(55-45)\text{°F}}{1 \text{ ft}} = 5040 \text{ Btu/h}$$

The rate of heat loss during nighttime is

$$\dot{Q}_{\text{night}} = kA\frac{T_1 - T_2}{L}$$

$$= (0.40 \text{ Btu/h.ft.°F})(1260 \text{ft}^2)\frac{(55-35)\text{°C}}{1 \text{ft}} = 10{,}080 \text{ Btu/h}$$

The amount of heat loss from the house that night will be

$$\dot{Q} = \frac{Q}{\Delta t} \longrightarrow Q = \dot{Q}\Delta t = 10\dot{Q}_{day} + 14\dot{Q}_{night} = (10 \text{ h})(5040 \text{ Btu/h}) + (14 \text{ h})(10{,}080 \text{ Btu/h})$$

$$= \mathbf{191{,}520 \text{ Btu}}$$

Then the cost of this heat loss for that day becomes

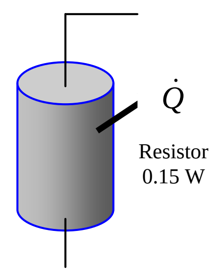
$$Cost = (191{,}520/3412 \text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$5.05}$$

3-23 A cylindrical resistor on a circuit board dissipates 0.15 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the resistor.

Analysis (*a*) The amount of heat this resistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.15 \text{ W})(24 \text{ h}) = \mathbf{3.6 \text{ Wh}}$$



(*b*) The heat flux on the surface of the resistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.003 \text{m})^2}{4} + \pi(0.003 \text{m})(0.012 \text{m}) = 0.000127 \text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.15 \text{ W}}{0.000127 \text{m}^2} = \mathbf{1179 \text{ W/m}^2}$$

(*c*) The surface temperature of the resistor can be determined from

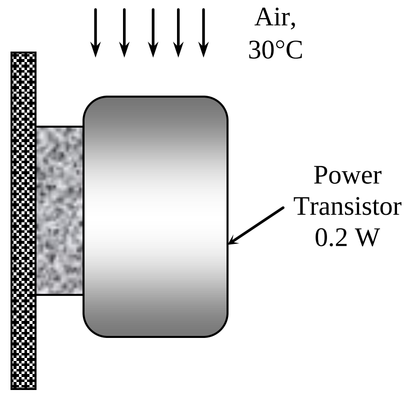
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = \frac{0.15 \text{ W}}{(1179 \text{W/m}^2.\text{°C})(0.000127 \text{ m}^2)} = \mathbf{171°C}$$

3-24 A power transistor dissipates 0.2 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the transistor.

Analysis (*a*) The amount of heat this transistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.2\text{ W})(24\text{ h}) = 4.8\text{ Wh} = \mathbf{0.0048\ kWh}$$



(*b*) The heat flux on the surface of the transistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL$$

$$= 2\frac{\pi(0.005\text{ m})^2}{4} + \pi(0.005\text{ m})(0.004\text{ m}) = 0.0001021\text{ m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.2\text{ W}}{0.0001021\text{ m}^2} = \mathbf{1959\ W/m^2}$$

(*c*) The surface temperature of the transistor can be determined from

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = \frac{0.2\text{ W}}{(18\text{ W/m}^2.°\text{C})(0.0001021\text{ m}^2)} = \mathbf{193°C}$$

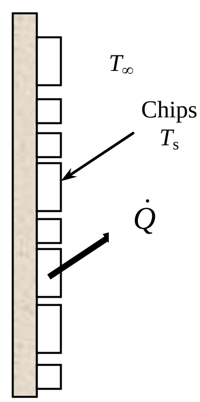
3-25 A circuit board houses 100 chips, each dissipating 0.07 W. The surface heat flux, the surface temperature of the chips, and the thermal resistance between the surface of the board and the cooling medium are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer from the back surface of the board is negligible. **2** Heat is transferred uniformly from the entire front surface.

Analysis (*a*) The heat flux on the surface of the circuit board is

$$A_s = (0.12\text{ m})(0.18\text{ m}) = 0.0216\text{ m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{(100\times 0.07)\text{ W}}{0.0216\text{ m}^2} = \mathbf{324\ W/m^2}$$



(*b*) The surface temperature of the chips is

$$\dot{Q} = hA_s(T_s - T_\infty)$$

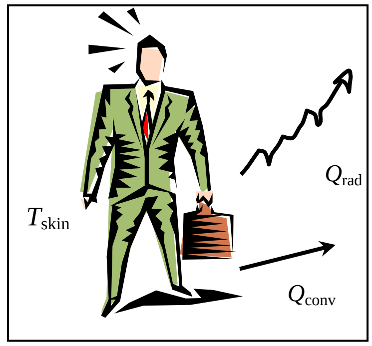
$$\longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 40°\text{C} + \frac{(100\times 0.07)\text{ W}}{(10\text{ W/m}^2.°\text{C})(0.0216\text{ m}^2)} = \mathbf{72.4°C}$$

(*c*) The thermal resistance is

$$R_{conv} = \frac{1}{hA_s} = \frac{1}{(10\text{ W/m}^2.°\text{C})(0.0216\text{ m}^2)} = \mathbf{4.63°C/W}$$

3-26 A person is dissipating heat at a rate of 150 W by natural convection and radiation to the surrounding air and surfaces. For a given deep body temperature, the outer skin temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire exposed surface of the person. **3** The surrounding surfaces are at the same temperature as the indoor air temperature. **4** Heat generation within the 0.5-cm thick outer layer of the tissue is negligible.



Properties The thermal conductivity of the tissue near the skin is given to be k = 0.3 W/m·°C.

Analysis The skin temperature can be determined directly from

$$\dot{Q} = kA\frac{T_1 - T_{skin}}{L}$$

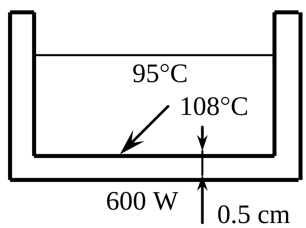
$$T_{skin} = T_1 - \frac{\dot{Q}L}{kA} = 37°\text{C} - \frac{(150\text{ W})(0.005\text{ m})}{(0.3\text{ W/m.°C})(1.7\text{ m}^2)} = \mathbf{35.5°C}$$

3-27 Heat is transferred steadily to the boiling water in an aluminum pan. The inner surface temperature of the bottom of the pan is given. The boiling heat transfer coefficient and the outer surface temperature of the bottom of the pan are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the thickness of the bottom of the pan is small relative to its diameter. **3** The thermal conductivity of the pan is constant.

Properties The thermal conductivity of the aluminum pan is given to be k = 237 W/m·°C.

Analysis (a) The boiling heat transfer coefficient is



$$A_s = \frac{\pi D^2}{4} = \frac{\pi(0.25\text{ m})^2}{4} = 0.0491\text{m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{800\text{ W}}{(0.0491\text{m}^2)(108-95)°\text{C}} = \mathbf{1254\ W/m^2.°C}$$

(b) The outer surface temperature of the bottom of the pan is

$$\dot{Q} = kA\frac{T_{s,outer} - T_{s,inner}}{L}$$

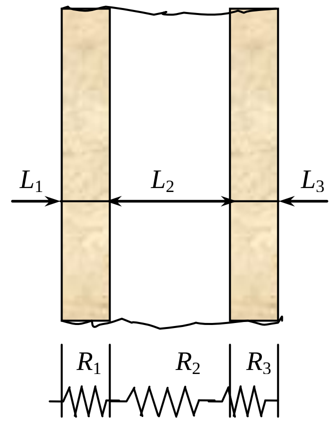
$$T_{s,outer} = T_{s,inner1} + \frac{\dot{Q}L}{kA} = 108°\text{C} + \frac{(800\text{ W})(0.005\text{ m})}{(237\text{ W/m.°C})(0.0491\text{m}^2)} = \mathbf{108.3°C}$$

3-28E A wall is constructed of two layers of sheetrock with fiberglass insulation in between. The thermal resistance of the wall and its R-value of insulation are to be determined.

Assumptions **1** Heat transfer through the wall is one-dimensional. **2** Thermal conductivities are constant.

Properties The thermal conductivities are given to be $k_{\text{sheetrock}}$ = 0.10 Btu/h·ft·°F and $k_{\text{insulation}}$ = 0.020 Btu/h·ft·°F.

Analysis (*a*) The surface area of the wall is not given and thus we consider a unit surface area (A = 1 ft^2). Then the R-value of insulation of the wall becomes equivalent to its thermal resistance, which is determined from.



$$R_{sheetrock} = R_1 = R_3 = \frac{L_1}{k_1} = \frac{0.5/12\text{ ft}}{(0.10\text{ Btu/h.ft.°F})} = 0.417\text{ ft}^2\text{.°F.h/Btu}$$

$$R_{fiberglass} = R_2 = \frac{L_2}{k_2} = \frac{5/12\text{ ft}}{(0.020\text{ Btu/h.ft.°F})} = 20.83\text{ ft}^2\text{.°F.h/Btu}$$

$$R_{total} = 2R_1 + R_2 = 2\times 0.417 + 20.83 = \mathbf{21.66\ \ ft^2.°F.h/Btu}$$

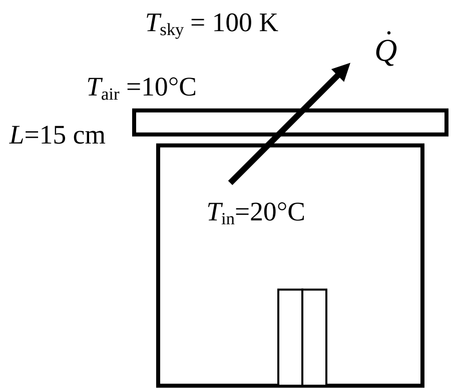
(*b*) Therefore, this is approximately a ***R*-22** wall in English units.

3-29 The roof of a house with a gas furnace consists of 3-cm thick concrete that is losing heat to the outdoors by radiation and convection. The rate of heat transfer through the roof and the money lost through the roof that night during a 14 hour period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The emissivity and thermal conductivity of the roof are constant.

Properties The thermal conductivity of the concrete is given to be k = 2 W/m·°C. The emissivity of both surfaces of the roof is given to be 0.9.

Analysis When the surrounding surface temperature is different than the ambient temperature, the thermal resistances network approach becomes cumbersome in problems that involve radiation. Therefore, we will use a different but intuitive approach.



In steady operation, heat transfer from the room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), that must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{room to roof,conv+rad}} = \dot{Q}_{\text{roof,cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

Taking the inner and outer surface temperatures of the roof to be $T_{s,in}$ and $T_{s,out}$, respectively, the quantities above can be expressed as

$$\dot{Q}_{\text{room to roof,conv+rad}} = h_i A(T_{room} - T_{s,in}) + \varepsilon A\sigma(T_{room}{}^4 - T_{s,in}{}^4) = (5\text{ W/m}^2.°\text{C})(300\text{ m}^2)(20 - T_{s,in})°\text{C}$$
$$+ (0.9)(300\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(20+273\text{K})^4 - (T_{s,in}+273\text{K})^4\right]$$

$$\dot{Q}_{\text{roof,cond}} = kA\frac{T_{s,in} - T_{s,out}}{L} = (2\text{ W/m.°C})(300\text{ m}^2)\frac{T_{s,in} - T_{s,out}}{0.15\text{ m}}$$

$$\dot{Q}_{\text{roof to surr, conv+rad}} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}{}^4 - T_{surr}{}^4) = (12\text{ W/m}^2.°\text{C})(300\text{ m}^2)(T_{s,out} - 10)°\text{C}$$
$$+ (0.9)(300\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\ (T_{s,out} + 273\text{ K})^4 - (100\text{ K})^4$$

Solving the equations above simultaneously gives

$$\dot{Q} = \mathbf{37{,}440\,W},\ T_{s,in} = \mathbf{7.3°C},\ \text{and}\ T_{s,\text{out}} = -2.1°\text{C}$$

The total amount of natural gas consumption during a 14-hour period is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{\dot{Q}\Delta t}{0.80} = \frac{(37.440\text{ kJ/s})(14\times3600\text{s})}{0.80}\left(\frac{1\text{ therm}}{105{,}500\text{kJ}}\right) = 22.36\text{ therms}$$

Finally, the money lost through the roof during that period is

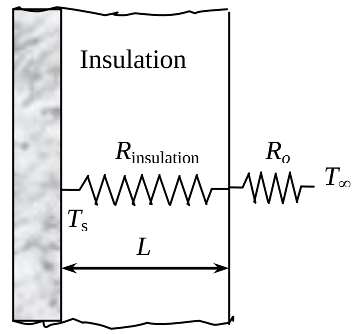
$$\text{Money lost} = (22.36\text{ therms})(\$0.60/\text{therm}) = \mathbf{\$13.4}$$

3-30 An exposed hot surface of an industrial natural gas furnace is to be insulated to reduce the heat loss through that section of the wall by 90 percent. The thickness of the insulation that needs to be used is to be determined. Also, the length of time it will take for the insulation to pay for itself from the energy it saves will be determined.

Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficient accounts for the radiation effects.

Properties The thermal conductivity of the glass wool insulation is given to be k = 0.038 W/m·°C.

Analysis The rate of heat transfer without insulation is



$$A = (2\text{ m})(1.5\text{ m}) = 3\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (10\text{ W/m}^2.^\circ\text{C})(3\text{ m}^2)(80-30)^\circ\text{C} = 1500\text{ W}$$

In order to reduce heat loss by 90%, the new heat transfer rate and thermal resistance must be

$$\dot{Q} = 0.10 \times 1500\text{ W} = 150\text{ W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} \longrightarrow R_{total} = \frac{\Delta T}{\dot{Q}} = \frac{(80-30)^\circ\text{C}}{150\text{ W}} = 0.333\ ^\circ\text{C/W}$$

and in order to have this thermal resistance, the thickness of insulation must be

$$R_{total} = R_{\text{conv}} + R_{insulation} = \frac{1}{hA} + \frac{L}{kA}$$

$$= \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(3\text{ m}^2)} + \frac{L}{(0.038\text{ W/m}.^\circ\text{C})(3\text{ m}^2)} = 0.333^\circ\text{C/W}$$

$$L = 0.034\text{ m} = \mathbf{3.4\,cm}$$

Noting that heat is saved at a rate of 0.9×1500 = 1350 W and the furnace operates continuously and thus 365×24 = 8760 h per year, and that the furnace efficiency is 78%, the amount of natural gas saved per year is

$$\text{Energy Saved} = \frac{\dot{Q}_{saved}\Delta t}{Efficiency} = \frac{(1.350\text{ kJ/s})(8760\text{ h})}{0.78}\left(\frac{3600\text{s}}{1\text{ h}}\right)\left(\frac{1\text{ therm}}{105{,}500\text{kJ}}\right) = 517.4\text{ therms}$$

The money saved is

$$\text{Money saved} = (\text{Energy Saved})(\text{Cost of energy}) = (517.4\text{ therms})(\$0.55/\text{ therm}) = \$284.5\text{ (per year)}$$

The insulation will pay for its cost of $250 in

$$\text{Payback period} = \frac{\text{Money spent}}{\text{Money saved}} = \frac{\$250}{\$284.5/\text{ yr}} = \mathbf{0.88\ yr}$$

which is less than one year.

3-31 An exposed hot surface of an industrial natural gas furnace is to be insulated to reduce the heat loss through that section of the wall by 90 percent. The thickness of the insulation that needs to be used is to be determined. Also, the length of time it will take for the insulation to pay for itself from the energy it saves will be determined.

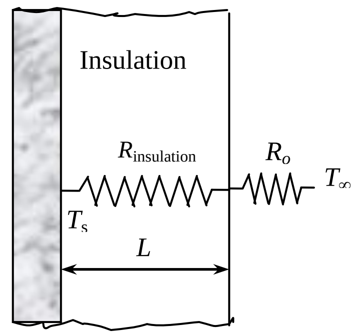
Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficients accounts for the radiation effects.

Properties The thermal conductivity of the expanded perlite insulation is given to be k = 0.052 W/m·°C.

Analysis The rate of heat transfer without insulation is

$$A = (2\text{ m})(1.5\text{ m}) = 3\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (10\text{ W/m}^2.°\text{C})(3\text{ m}^2)(80-30)°\text{C} = 1500\text{ W}$$



In order to reduce heat loss by 90%, the new heat transfer rate and thermal resistance must be

$$\dot{Q} = 0.10 \times 1500\text{ W} = 150\text{ W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} \longrightarrow R_{total} = \frac{\Delta T}{\dot{Q}} = \frac{(80-30)°\text{C}}{150\text{ W}} = 0.333\text{ °C/W}$$

and in order to have this thermal resistance, the thickness of insulation must be

$$R_{total} = R_{\text{conv}} + R_{insulation} = \frac{1}{hA} + \frac{L}{kA}$$

$$= \frac{1}{(10\text{ W/m}^2.°\text{C})(3\text{ m}^2)} + \frac{L}{(0.052\text{ W/m.°C})(3\text{ m}^2)} = 0.333°\text{C/W}$$

$$L = 0.047\text{ m} = \mathbf{4.7\ cm}$$

Noting that heat is saved at a rate of 0.9×1500 = 1350 W and the furnace operates continuously and thus 365×24 = 8760 h per year, and that the furnace efficiency is 78%, the amount of natural gas saved per year is

$$\text{Energy Saved} = \frac{\dot{Q}_{saved}\Delta t}{Efficiency} = \frac{(1.350\text{ kJ/s})(8760\text{ h})}{0.78}\left(\frac{3600\text{s}}{1\text{ h}}\right)\left(\frac{1\text{ therm}}{105{,}500\text{kJ}}\right) = 517.4\text{ therms}$$

The money saved is

$$\text{Money saved} = (\text{Energy Saved})(\text{Cost of energy}) = (517.4\text{ therms})(\$0.55/\text{therm}) = \$284.5\text{ (per year)}$$

The insulation will pay for its cost of $250 in

$$\text{Payback period} = \frac{\text{Money spent}}{\text{Money saved}} = \frac{\$250}{\$284.5/\text{yr}} = \mathbf{0.88\ yr}$$

which is less than one year.

3-32

```
"GIVEN"
A=2*1.5 "[m^2]"
T_s=80 "[C]"
T_infinity=30 "[C]"
h=10 "[W/m^2-C]"
"k_ins=0.038 [W/m-C], parameter to be varied"
f_reduce=0.90

"ANALYSIS"
Q_dot_old=h*A*(T_s-T_infinity)
Q_dot_new=(1-f_reduce)*Q_dot_old
Q_dot_new=(T_s-T_infinity)/R_total
R_total=R_conv+R_ins
R_conv=1/(h*A)
R_ins=(L_ins*Convert(cm, m))/(k_ins*A) "L_ins is in cm"
```

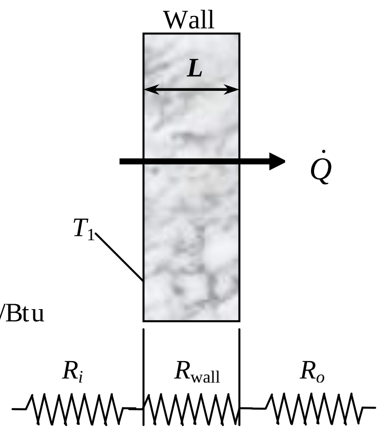
k_{ins} [W/m.C]	L_{ins} [cm]
0.02	1.8
0.025	2.25
0.03	2.7
0.035	3.15
0.04	3.6
0.045	4.05
0.05	4.5
0.055	4.95
0.06	5.4
0.065	5.85
0.07	6.3
0.075	6.75
0.08	7.2

3-33E Two of the walls of a house have no windows while the other two walls have 4 windows each. The ratio of heat transfer through the walls with and without windows is to be determined.

Assumptions **1** Heat transfer through the walls and the windows is steady and one-dimensional. **2** Thermal conductivities are constant. **3** Any direct radiation gain or loss through the windows is negligible. **4** Heat transfer coefficients are constant and uniform over the entire surface.

Properties The thermal conductivity of the glass is given to be k_{glass} = 0.45 Btu/h.ft·°F. The R-value of the wall is given to be 19 h.ft²·°F/Btu.

Analysis The thermal resistances through the wall without windows are



$$A = (12\,\text{ft})(40\text{ft}) = 480\,\text{m}^2$$

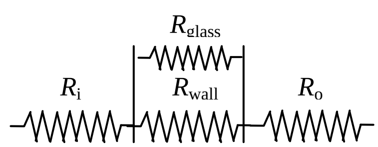
$$R_i = \frac{1}{h_i A} = \frac{1}{(2\,\text{Btu/h.ft}^2.°\text{F})(480\,\text{ft}^2)} = 0.0010417\,\text{h}\cdot°\text{F/Btu}$$

$$R_{\text{wall}} = \frac{L}{kA} = \frac{19\,\text{hft}^2\,°\text{F/Btu}}{(480\,\text{m}^2)} = 0.03958\,\text{h}\cdot°\text{F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4\,\text{Btu/h.ft}^2.°\text{F})(480\,\text{ft}^2)} = 0.00052\,\text{h}\cdot°\text{F/Btu}$$

$$R_{total,1} = R_i + R_{wall} + R_o = 0.0010417 + 0.03958 + 0.00052 = 0.0411417\,\text{h}\cdot°\text{F/Btu}$$

The thermal resistances through the wall with windows are



$$A_{windows} = 4(3\times5) = 60\,\text{ft}^2$$

$$A_{wall} = A_{total} - A_{windows} = 480 - 60 = 420\,\text{ft}^2$$

$$R_2 = R_{glass} = \frac{L}{kA} = \frac{0.25/12\,\text{ft}}{(0.45\,\text{Btu/h.ft.°F})(60\,\text{ft}^2)} = 0.0007716\,\text{h}\cdot°\text{F/Btu}$$

$$R_4 = R_{wall} = \frac{L}{kA} = \frac{19\,\text{h.ft}^2.°\text{F/Btu}}{(420\,\text{ft}^2)} = 0.04524\,\text{h}\cdot°\text{F/Btu}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{glass}} + \frac{1}{R_{wall}} = \frac{1}{0.0007716} + \frac{1}{0.04524} \longrightarrow R_{eqv} = 0.00076\cdot\text{h°F/Btu}$$

$$R_{total,2} = R_i + R_{eqv} + R_o = 0.001047 + 0.00076 + 0.00052 = 0.002327\,\text{h}\cdot°\text{F/Btu}$$

Then the ratio of the heat transfer through the walls with and without windows becomes

$$\frac{\dot{Q}_{total,2}}{\dot{Q}_{total,1}} = \frac{\Delta T / R_{total,2}}{\Delta T / R_{total,1}} = \frac{R_{total,1}}{R_{total,2}} = \frac{0.0411417}{0.002327} = \mathbf{17.7}$$

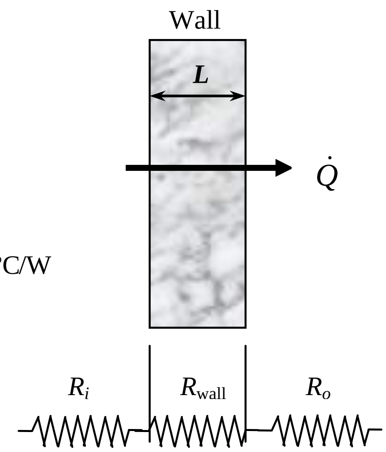
3-34 Two of the walls of a house have no windows while the other two walls have single- or double-pane windows. The average rate of heat transfer through each wall, and the amount of money this household will save per heating season by converting the single pane windows to double pane windows are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.026 W/m·°C for air, and 0.78 W/m·°C for glass.

Analysis The rate of heat transfer through each wall can be determined by applying thermal resistance network. The convection resistances at the inner and outer surfaces are common in all cases.

Walls without windows :



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2.°\text{C})(10\times 4\ \text{m}^2)} = 0.003571°\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\,°\text{C/W}}{(10\times 4\ \text{m}^2)} = 0.05775°\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(15\ \text{W/m}^2.°\text{C})(10\times 4\ \text{m}^2)} = 0.001667°\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{wall}} + R_o = 0.003571 + 0.05775 + 0.001667 = 0.062988°\text{C/W}$$

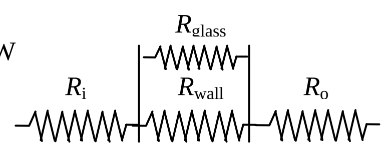
Then $$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(22-8)°\text{C}}{0.062988°\text{C/W}} = \mathbf{222.3\ W}$$

Wall with single pane windows:

$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2.°\text{C})(20\times 4\ \text{m}^2)} = 0.001786°\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\,°\text{C/W}}{(20\times 4) - 5(1.2\times 1.8)\ \text{m}^2} = 0.033382°\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\,\text{m}}{(0.78\ \text{W/m}^2.^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968°\text{C/W}$$



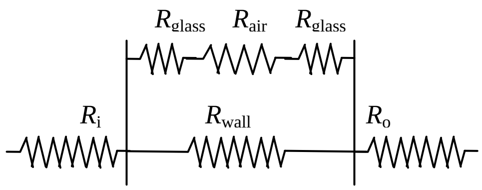
$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{glass}}} = \frac{1}{0.033382} + 5\frac{1}{0.002968} \rightarrow R_{eqv} = 0.00058\ ^\text{o}\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(15\ \text{W/m}^2.°\text{C})(20\times 4\ \text{m}^2)} = 0.000833°\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.000583 + 0.000833 = 0.003202°\text{C/W}$$

Then $$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(22-8)°\text{C}}{0.003202°\text{C/W}} = \mathbf{4372\ W}$$

4th wall with double pane windows:



$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\,°\text{C/W}}{(20\times 4)-5(1.2\times 1.8)\text{m}^2} = 0.033382°\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\,\text{m}}{(0.78\,\text{W/m}^2.°\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968°\text{C/W}$$

$$R_{\text{air}} = \frac{L_{\text{air}}}{kA} = \frac{0.015\,\text{m}}{(0.026\,\text{W/m}^2.^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.267094°\text{C/W}$$

$$R_{\text{window}} = 2R_{\text{glass}} + R_{\text{air}} = 2\times 0.002968 + 0.267094 = 0.27303°\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{window}}} = \frac{1}{0.033382} + 5\frac{1}{0.27303} \longrightarrow R_{\text{eqv}} = 0.020717°\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.020717 + 0.000833 = 0.023336°\text{C/W}$$

Then $\dot{Q} = \dfrac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \dfrac{(22-8)°\text{C}}{0.023336°\text{C/W}} = \mathbf{600\,W}$

The rate of heat transfer which will be saved if the single pane windows are converted to double pane windows is

$$\dot{Q}_{\text{save}} = \dot{Q}_{\substack{\text{single}\\\text{pane}}} - \dot{Q}_{\substack{\text{double}\\\text{pane}}} = 4372 - 600 = 3772\,\text{W}$$

The amount of energy and money saved during a 7-month long heating season by switching from single pane to double pane windows become

$$Q_{save} = \dot{Q}_{save}\Delta t = (3.772\,\text{kW})(7\times 30\times 24\,\text{h}) = 19{,}011\text{kWh}$$

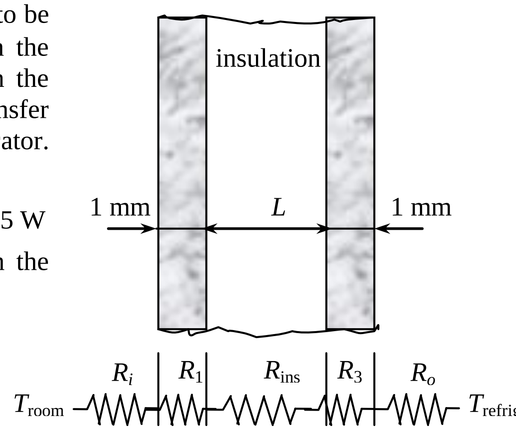
Money savings = (Energy saved)(Unit cost of energy) = (19,011 kWh)($0.08/kWh) = **$1521**

3-35 The wall of a refrigerator is constructed of fiberglass insulation sandwiched between two layers of sheet metal. The minimum thickness of insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces is to be determined.

Assumptions **1** Heat transfer through the refrigerator walls is steady since the temperatures of the food compartment and the kitchen air remain constant at the specified values. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation effects.

Properties The thermal conductivities are given to be k = 15.1 W/m·°C for sheet metal and 0.035 W/m·°C for fiberglass insulation.

Analysis The minimum thickness of insulation can be determined by assuming the outer surface temperature of the refrigerator to be 10°C. In steady operation, the rate of heat transfer through the refrigerator wall is constant, and thus heat transfer between the room and the refrigerated space is equal to the heat transfer between the room and the outer surface of the refrigerator. Considering a unit surface area,



$$\dot{Q} = h_o A(T_{room} - T_{s,out}) = (9\ \text{W/m}^2.°\text{C})(1\ \text{m}^2)(25-20)°\text{C} = 45\ \text{W}$$

Using the thermal resistance network, heat transfer between the room and the refrigerated space can be expressed as

$$\dot{Q} = \frac{T_{room} - T_{refrig}}{R_{total}}$$

$$\dot{Q}/A = \frac{T_{room} - T_{refrig}}{\dfrac{1}{h_o} + 2\left(\dfrac{L}{k}\right)_{metal} + \left(\dfrac{L}{k}\right)_{insulation} + \dfrac{1}{h_i}}$$

Substituting,

$$45\ \text{W/m}^2 = \frac{(25-3)°\text{C}}{\dfrac{1}{9\ \text{W/m}^2.°\text{C}} + \dfrac{2\times 0.001\ \text{m}}{15.1\ \text{W/m}^2.°\text{C}} + \dfrac{L}{0.035\ \text{W/m}^2.°\text{C}} + \dfrac{1}{4\ \text{W/m}^2.°\text{C}}}$$

Solv ing for L, the minimum thickness of insulation is determined to be

L = 0.0045 m = **0.45 cm**

3-36

```
"GIVEN"
k_ins=0.035 "[W/m-C], parameter to be varied"
L_metal=0.001 "[m]"
k_metal=15.1 "[W/m-C], parameter to be varied"
T_refrig=3 "[C]"
T_kitchen=25 "[C]"
h_i=4 "[W/m^2-C]"
h_o=9 "[W/m^2-C]"
T_s_out=20 "[C]"

"ANALYSIS"
A=1 "[m^2], a unit surface area is considered"
Q_dot=h_o*A*(T_kitchen-T_s_out)
Q_dot=(T_kitchen-T_refrig)/R_total
R_total=R_conv_i+2*R_metal+R_ins+R_conv_o
R_conv_i=1/(h_i*A)
R_metal=L_metal/(k_metal*A)
R_ins=(L_ins*Convert(cm, m))/(k_ins*A) "L_ins is in cm"
R_conv_o=1/(h_o*A)
```

k_{ins} [W/m.C]	L_{ins} [cm]
0.02	0.2553
0.025	0.3191
0.03	0.3829
0.035	0.4468
0.04	0.5106
0.045	0.5744
0.05	0.6382
0.055	0.702
0.06	0.7659
0.065	0.8297
0.07	0.8935
0.075	0.9573
0.08	1.021

k_{metal} [W/m.C]	L_{ins} [cm]
10	0.4465
30.53	0.447
51.05	0.4471
71.58	0.4471
92.11	0.4471
112.6	0.4472
133.2	0.4472
153.7	0.4472
174.2	0.4472
194.7	0.4472
215.3	0.4472
235.8	0.4472
256.3	0.4472
276.8	0.4472
297.4	0.4472
317.9	0.4472
338.4	0.4472
358.9	0.4472
379.5	0.4472

400	0.4472

3-37 Heat is to be conducted along a circuit board with a copper layer on one side. The percentages of heat conduction along the copper and epoxy layers as well as the effective thermal conductivity of the board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces is disregarded **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m·°C for copper and 0.26 W/m·°C for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer board can be expressed as

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}} = \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right]w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" board of thickness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}(t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff}(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fractions of heat conducted along the copper and epoxy layers as well as the effective thermal conductivity of the board are determined to be

$$(kt)_{copper} = (386\text{ W/m.°C})(0.0001\text{ m}) = 0.0386\text{ W/°C}$$

$$(kt)_{epoxy} = (0.26\text{ W/m.°C})(0.0012\text{ m}) = 0.000312\text{ W/°C}$$

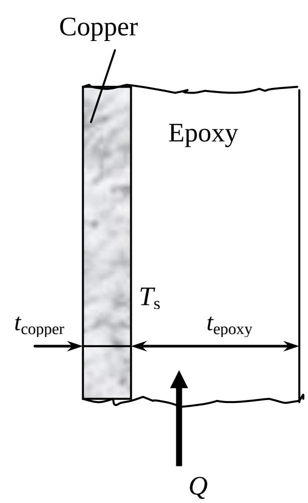
$$(kt)_{total} = (kt)_{copper} + (kt)_{epoxy} = 0.0386 + 0.000312 = 0.038912\text{ W/°C}$$

$$f_{epoxy} = \frac{(kt)_{epoxy}}{(kt)_{total}} = \frac{0.000312}{0.038912} = 0.008 = \mathbf{0.8\%}$$

$$f_{copper} = \frac{(kt)_{copper}}{(kt)_{total}} = \frac{0.0386}{0.038912} = 0.992 = \mathbf{99.2\%}$$

and

$$k_{eff} = \frac{(386 \times 0.0001 + 0.26 \times 0.0012)\text{ W/°C}}{(0.0001 + 0.0012)\text{ m}} = \mathbf{29.9\text{ W/m.°C}}$$



3-38E A thin copper plate is sandwiched between two layers of epoxy boards. The effective thermal conductivity of the board along its 9 in long side and the fraction of the heat conducted through copper along that side are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces are disregarded **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper and 0.15 Btu/h·ft·°F for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer plate can be expressed as (we treat the two layers of epoxy as a single layer that is twice as thick)

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}} = \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right]w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" plate of thick ness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}(t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

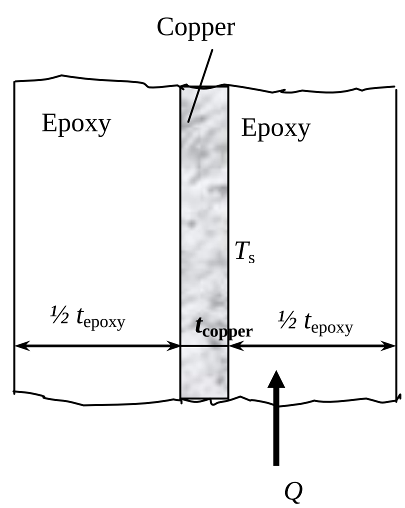
$$k_{eff}(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fraction of heat conducted along the copper layer and the effective thermal conductivity of the plate are determined to be

$$(kt)_{\text{copper}} = (223 \text{ Btu/h.ft.°F})(0.03/12 \text{ ft}) = 0.5575 \text{ Btu/h.°F}$$

$$(kt)_{\text{epoxy}} = 2(0.15 \text{ Btu/h.ft.°F})(0.1/12 \text{ ft}) = 0.0025 \text{ Btu/h.°F}$$

$$(kt)_{\text{total}} = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} = (0.5575 + 0.0025) = 0.56 \text{ Btu/h.°F}$$



and

$$k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}} = \frac{0.56 \text{ Btu/h.°F}}{[(0.03/12) + 2(0.1/12)] \text{ ft}} = \mathbf{29.2 \text{ Btu/h.ft}^2\text{.°F}}$$

$$f_{\text{copper}} = \frac{(kt)_{\text{copper}}}{(kt)_{\text{total}}} = \frac{0.5575}{0.56} = 0.996 = \mathbf{99.6\%}$$