Thermal Contact Resistance

3-39C The resistance that an interface offers to heat transfer per unit interface area is called thermal contact resistance**,** R_c. The inverse of thermal contact resistance is called the thermal contact conductance.

3-40C The thermal contact resistance will be greater for rough surfaces because an interface with rough surfaces will contain more air gaps whose thermal conductivity is low.

3-41C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance has significance only for highly conducting materials like metals. Therefore, the thermal contact resistance can be ignored for two layers of insulation pressed against each other.

3-42C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance is significant for highly conducting materials like metals. Therefore, the thermal contact resistance must be considered for two layers of metals pressed against each other.

3-43C Heat transfer through the voids at an interface is by conduction and radiation. Evacuating the interface eliminates heat transfer by conduction, and thus increases the thermal contact resistance.

3-44C Thermal contact resistance can be minimized by (1) applying a thermally conducting liquid on the surfaces before they are pressed against each other, (2) by replacing the air at the interface by a better conducting gas such as helium or hydrogen, (3) by increasing the interface pressure, and (4) by inserting a soft metallic foil such as tin, silver, copper, nickel, or aluminum between the two surfaces.

3-45 The thickness of copper plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties The thermal conductivity of copper is given to be k = 386 W/m·°C (Table A-2).

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is determined to be

$$R_c = \frac{1}{h_c} = \frac{1}{18{,}000\ \text{W/m}^2.^\circ\text{C}} = 5.556\times 10^{-5}\ \text{m}^2.^\circ\text{C/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as $R = \frac{L}{k}$ where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR = kR_c = (386\ \text{W/m.}^\circ\text{C})(5.556\times 10^{-5}\ \text{m}^2.^\circ\text{C/W}) = 0.0214\ \text{m} = \mathbf{2.14\ cm}$$

Therefore, the interface between the two plates offers as much resistance to heat transfer as a 2.14 cm thick copper. Note that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

3-46 Six identical power transistors are attached on a copper plate. For a maximum case temperature of 85°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. **3** All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick plexiglas layer. **4** Thermal conductivities are constant.

Properties The thermal conductivity of copper is given to be k = 386 W/m·°C. The contact conductance at the interface of copper-aluminum plates for the case of 1.3-1.4 μm roughness and 10 MPa pressure is h_c = 49,000 W/m^2·°C (Table 3-2).

Analysis The contact area between the case and the plate is given to be 9 cm^2, and the plate area for each transistor is 100 cm^2. The thermal resistance network of this problem consists of three resistances in series (contact, plate, and convection) which are determined to be

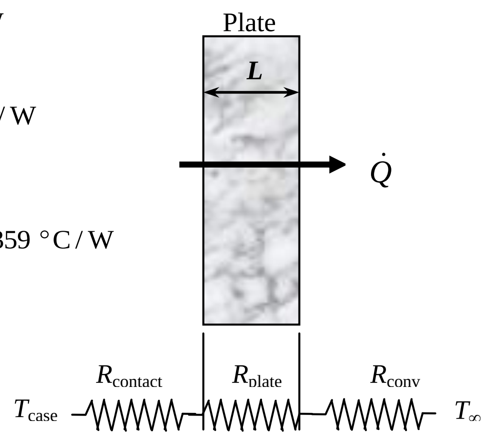
$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(49{,}000\ \text{W/m}^2.^\circ\text{C})(9\times 10^{-4}\ \text{m}^2)} = 0.0227\ ^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.012\ \text{m}}{(386\ \text{W/m}.^\circ\text{C})(0.01\ \text{m}^2)} = 0.0031\ ^\circ\text{C/W}$$

$$R_{\text{convection}} = \frac{1}{h_o A} = \frac{1}{(30\ \text{W/m}^2.^\circ\text{C})(0.01\ \text{m}^2)} = 3.333\ ^\circ\text{C/W}$$

The total thermal resistance is then

$$R_{\text{total}} = R_{\text{contact}} + R_{\text{plate}} + R_{\text{convection}} = 0.0227 + 0.0031 + 3.333 = 3.359\ ^\circ\text{C/W}$$



Note that the thermal resistance of copper plate is very small and can be ignored all together. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(85-15)^\circ\text{C}}{3.359\ ^\circ\text{C/W}} = \mathbf{20.8\ W}$$

Therefore, the power transistor should not be operated at power levels greater than 20.8 W if the case temperature is not to exceed 85°C.

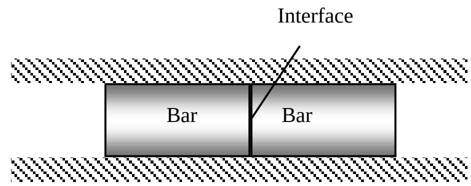
The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q}R_{\text{contact}} = (20.8\ \text{W})(0.0227\ ^\circ\text{C/W}) = \mathbf{0.47^\circ C}$$

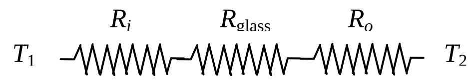
which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we will lower the operating temperature of the transistor in this case by less than 1°C.

3-47 Two cylindrical aluminum bars with ground surfaces are pressed against each other in an insulation sleeve. For specified top and bottom surface temperatures, the rate of heat transfer along the cylinders and the temperature drop at the interface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional in the axial direction since the lateral surfaces of both cylinders are well-insulated. **3** Thermal conductivities are constant.



Properties The thermal conductivity of aluminum bars is given to be k = 176 W/m·°C. The contact conductance at the interface of aluminum-aluminum plates for the case of ground surfaces and of 20 atm ≈ 2 MPa pressure is h_c = 11,400 W/m^2·°C (Table 3-2).



Analysis (*a*) The thermal resistance network in this case consists of two conduction resistance and the contact resistance, and are determined to be

$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(11{,}400\ \text{W/m}^2.°\text{C})[\pi(0.05\text{m})^2/4]} = 0.0447°\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.15\ \text{m}}{(176\ \text{W/m.°C})[\pi(0.05\ \text{m})^2/4]} = 0.4341\ °\text{C/W}$$

Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{\text{contact}} + 2R_{\text{bar}}} = \frac{(150-20)°\text{C}}{(0.0447 + 2\times 0.4341)\ °\text{C/W}} = \mathbf{142.4\ W}$$

Therefore, the rate of heat transfer through the bars is 142.4 W.

(*b*) The temperature drop at the interface is determined to be

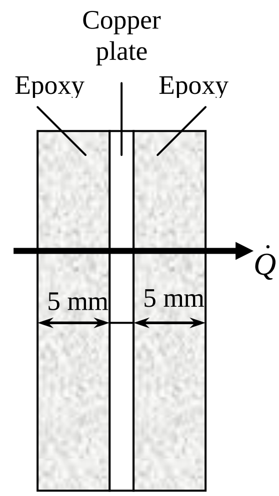
$$\Delta T_{\text{interface}} = \dot{Q}R_{\text{contact}} = (142.4\ \text{W})(0.0447°\text{C/W}) = \mathbf{6.4°C}$$

3-48 A thin copper plate is sandwiched between two epoxy boards. The error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the plate is large. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m·°C for copper plates and k = 0.26 W/m·°C for epoxy boards. The contact conductance at the interface of copper-epoxy layers is given to be h_c = 6000 W/m^2·°C.

Analysis The thermal resistances of different layers for unit surface area of 1 m^2 are



$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(6000\,\text{W/m}^2.°\text{C})(1\,\text{m}^2)} = 0.00017°\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.001\text{ m}}{(386\text{ W/m.°C})(1\text{ m}^2)} = 2.6\times10^{-6}\ °\text{C/W}$$

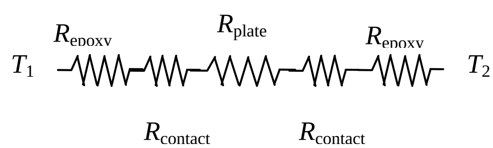
$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.005\text{ m}}{(0.26\text{ W/m.°C})(1\text{ m}^2)} = 0.01923\ °\text{C/W}$$

The total thermal resistance is

$$\begin{aligned} R_{\text{total}} &= 2R_{\text{contact}} + R_{\text{plate}} + 2R_{\text{epoxy}} \\ &= 2\times0.00017 + 2.6\times10^{-6} + 2\times0.01923 = 0.03914°\text{C/W} \end{aligned}$$

Then the percent error involved in the total thermal resistance of the plate if the thermal contact resistances are ignored is determined to be

$$\%\text{Error} = \frac{2R_{\text{contact}}}{R_{\text{total}}}\times100 = \frac{2\times0.00017}{0.03914}\times100 = \mathbf{0.87\%}$$



which is negligible.

Generalized Thermal Resistance Networks

3-49C Parallel resistances indicate simultaneous heat transfer (such as convection and radiation on a surface). Series resistances indicate sequential heat transfer (such as two homogeneous layers of a wall).

3-50C The thermal resistance network approach will give adequate results for multi-dimensional heat transfer problems if heat transfer occurs predominantly in one direction.

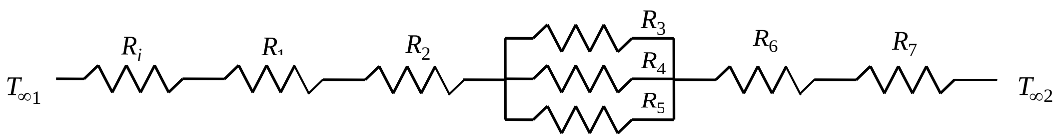
3-51C Two approaches used in development of the thermal resistance network in the x-direction for multi-dimensional problems are (1) to assume any plane wall normal to the x-axis to be isothermal and (2) to assume any plane parallel to the x-axis to be adiabatic.

3-52 A wall consists of horizontal bricks separated by plaster layers. There are also plaster layers on each side of the wall, and a rigid foam on the inner side of the wall. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.72 W/m·°C for bricks, k = 0.22 W/m·°C for plaster layers, and k = 0.026 W/m·°C for the rigid foam.

Analysis We consider 1 m deep and 0.33 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are



$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2.^\circ\text{C})(0.33\times 1 \text{ m}^2)} = 0.303\ ^\circ\text{C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.026 \text{ W/m.}^\circ\text{C})(0.33\times 1 \text{ m}^2)} = 2.33\ ^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\substack{plaster\\side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m.}^\circ\text{C})(0.30\times 1 \text{ m}^2)} = 0.303\ ^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\substack{plaster\\center}} = \frac{L}{h_o A} = \frac{0.18 \text{ m}}{(0.22 \text{ W/m.}^\circ\text{C})(0.015\times 1 \text{ m}^2)} = 54.55\ ^\circ\text{C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.18 \text{ m}}{(0.72 \text{ W/m.}^\circ\text{C})(0.30\times 1 \text{ m}^2)} = 0.833\ ^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20 \text{ W/m.}^\circ\text{C})(0.33\times 1 \text{ m}^2)} = 0.152\ ^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{54.55} + \frac{1}{0.833} + \frac{1}{54.55} \longrightarrow R_{mid} = 0.81\ ^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.303 + 2.33 + 2(0.303) + 0.81 + 0.152$$
$$= 4.201\ ^\circ\text{C/W}$$

The steady rate of heat transfer through the wall per 0.33 m^2 is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[(22-(-4)]^\circ\text{C}}{4.201^\circ\text{C/W}} = 6.19 \text{ W}$$

Then steady rate of heat transfer through the entire wall becomes

$$\dot{Q}_{total} = (6.19 \text{ W})\frac{(4\times 6)\text{m}^2}{0.33 \text{ m}^2} = \mathbf{450\ W}$$

3-53

```
"GIVEN"
A=4*6 "[m^2]"
L_brick=0.18 "[m]"
L_plaster_center=0.18 "[m]"
L_plaster_side=0.02 "[m]"
"L_foam=2 [cm], parameter to be varied"
k_brick=0.72 "[W/m-C]"
k_plaster=0.22 "[W/m-C]"
k_foam=0.026 "[W/m-C]"
T_infinity_1=22 "[C]"
T_infinity_2=-4 "[C]"
h_1=10 "[W/m^2-C]"
h_2=20 "[W/m^2-C]"

"ANALYSIS"
R_conv_1=1/(h_1*A_1)
A_1=0.33*1 "[m^2]"
R_foam=(L_foam*Convert(cm, m))/(k_foam*A_1) "L_foam is in cm"
R_plaster_side=L_plaster_side/(k_plaster*A_2)
A_2=0.30*1 "[m^2]"
R_plaster_center=L_plaster_center/(k_plaster*A_3)
A_3=0.015*1 "[m^2]"
R_brick=L_brick/(k_brick*A_2)
R_conv_2=1/(h_2*A_1)

1/R_mid=2*1/R_plaster_center+1/R_brick
R_total=R_conv_1+R_foam+2*R_plaster_side+R_mid+R_conv_2
Q_dot=(T_infinity_1-T_infinity_2)/R_total
Q_dot_total=Q_dot*A/A_1
```

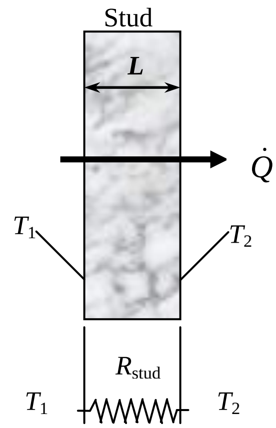
L_{foam} [cm]	Q_{total} [W]
1	623.1
2	450.2
3	352.4
4	289.5
5	245.7
6	213.4
7	188.6
8	168.9
9	153
10	139.8

3-54 A wall is to be constructed of 10-cm thick wood studs or with pairs of 5-cm thick wood studs nailed to each other. The rate of heat transfer through the solid stud and through a stud pair nailed to each other, as well as the effective conductivity of the nailed stud pair are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x* direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance between the two layers is negligible. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.11 W/m·°C for wood studs and k = 50 W/m·°C for manganese steel nails.

Analysis (*a*) The heat transfer area of the stud is A = (0.1 m)(2.5 m) = 0.25 m^2. The thermal resistance and heat transfer rate through the solid stud are



$$R_{stud} = \frac{L}{kA} = \frac{0.1\,\text{m}}{(0.11\,\text{W/m.°C})(0.25\,\text{m}^2)} = 3.636\,\text{°C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{stud}} = \frac{8\text{°C}}{3.636\,\text{°C/W}} = \mathbf{2.2\,W}$$

(*b*) The thermal resistances of stud pair and nails are in parallel

$$A_{nails} = 50\frac{\pi D^2}{4} = 50\left[\frac{\pi(0.004\,\text{m})^2}{4}\right] = 0.000628\text{m}^2$$

$$R_{nails} = \frac{L}{kA} = \frac{0.1\,\text{m}}{(50\,\text{W/m.°C})(0.000628\text{m}^2)} = 3.18\,\text{°C/W}$$

$$R_{stud} = \frac{L}{kA} = \frac{0.1\,\text{m}}{(0.11\,\text{W/m.°C})(0.25-0.000628\text{m}^2)} = 3.65\,\text{°C/W}$$

$$\frac{1}{R_{total}} = \frac{1}{R_{stud}} + \frac{1}{R_{nails}} = \frac{1}{3.65} + \frac{1}{3.18} \longrightarrow R_{total} = 1.70\,\text{°C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{stud}} = \frac{8\text{°C}}{1.70\,\text{°C/W}} = \mathbf{4.7\,W}$$

(*c*) The effective conductivity of the nailed stud pair can be determined from

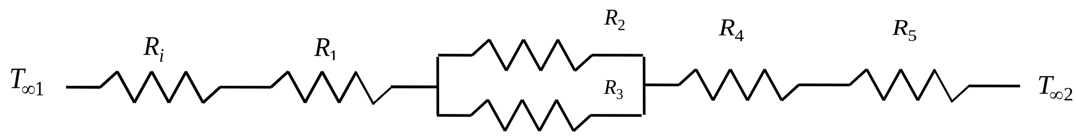
$$\dot{Q} = k_{eff} A\frac{\Delta T}{L} \longrightarrow k_{eff} = \frac{\dot{Q}L}{\Delta TA} = \frac{(4.7\,\text{W})(0.1\,\text{m})}{(8\text{°C})(0.25\text{m}^2)} = \mathbf{0.235\ W/m.°C}$$

3-55 A wall is constructed of two layers of sheetrock spaced by 5 cm × 12 cm wood studs. The space between the studs is filled with fiberglass insulation. The thermal resistance of the wall and the rate of heat transfer through the wall are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.17 W/m·°C for sheetrock, k = 0.11 W/m·°C for wood studs, and k = 0.034 W/m·°C for fiberglass insulation.

Analysis (*a*) The representative surface area is $A = 1 \times 0.65 = 0.65\ \text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_i = \frac{1}{h_i A} = \frac{1}{(8.3\ \text{W/m}^2.°\text{C})(0.65\ \text{m}^2)} = 0.185\ °\text{C/W}$$

$$R_1 = R_4 = R_{sheetrock} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(0.17\ \text{W/m.°C})(0.65\ \text{m}^2)} = 0.090\ °\text{C/W}$$

$$R_2 = R_{stud} = \frac{L}{kA} = \frac{0.12\ \text{m}}{(0.11\ \text{W/m.°C})(0.05\ \text{m}^2)} = 21.818\ °\text{C/W}$$

$$R_3 = R_{fiberglass} = \frac{L}{kA} = \frac{0.12\ \text{m}}{(0.034\ \text{W/m.°C})(0.60\ \text{m}^2)} = 5.882\ °\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(34\ \text{W/m}^2.^\circ\text{C})(0.65\ \text{m}^2)} = 0.045\ °\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{21.818} + \frac{1}{5.882} \longrightarrow R_{mid} = 4.633°\text{C/W}$$

$$R_{total} = R_i + R_1 + R_{mid} + R_4 + R_o = 0.185 + 0.090 + 4.633 + 0.090 + 0.045 = \mathbf{4.858°C/W}\ (\text{for a } 1\ \text{m} \times 0.65\ \text{m section})$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-5)]°\text{C}}{4.858\ °\text{C/W}} = 5.15\ \text{W}$$

(*b*) Then steady rate of heat transfer through entire wall becomes

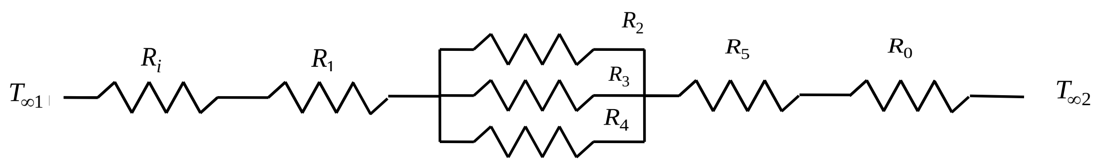
$$\dot{Q}_{total} = (5.15\ \text{W})\frac{(12\ \text{m})(5\ \text{m})}{0.65\ \text{m}^2} = \mathbf{475\ W}$$

3-56E A wall is to be constructed using solid bricks or identical size bricks with 9 square air holes. There is a 0.5 in thick sheetrock layer between two adjacent bricks on all four sides, and on both sides of the wall. The rates of heat transfer through the wall constructed of solid bricks and of bricks with air holes are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.40 Btu/h·ft·°F for bricks, k = 0.015 Btu/h·ft·°F for air, and k = 0.10 Btu/h·ft·°F for sheetrock.

Analysis (*a*) The representative surface area is $A = (7.5/12)(7.5/12) = 0.3906 \text{ ft}^2$. The thermal resistance network and the individual thermal resistances if the wall is constructed of solid bricks are



$$R_i = \frac{1}{h_i A} = \frac{1}{(1.5 \text{ Btu/h.ft}^2.°\text{F})(0.3906 \text{ ft}^2)} = 1.7068 \text{ h°F/Btu}$$

$$R_1 = R_5 = R_{plaster} = \frac{L}{kA} = \frac{0.5/12 \text{ ft}}{(0.10 \text{ Btu/h.ft.°F})(0.3906 \text{ ft}^2)} = 1.0667 \text{ h°F/Btu}$$

$$R_2 = R_{plaster} = \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.10 \text{ Btu/h.ft.°F})[(7.5/12)\times(0.5/12)]\text{ft}^2} = 288 \text{ h°F/Btu}$$

$$R_3 = R_{plaster} = \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.10 \text{ Btu/h.ft.}^\text{o}\text{ F})[(7/12)\times(0.5/12)]\text{ft}^2} = 308.57 \text{ h°F/Btu}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.40 \text{ Btu/h.ft.°F})[(7/12)\times(7/12)]\text{ft}^2} = 5.51 \text{ h°F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4 \text{ Btu/h.ft}^2.°\text{F})(0.3906 \text{ ft}^2)} = 0.64 \text{ h°F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{5.51} \longrightarrow R_{mid} = 5.3135 \text{ h°F/Btu}$$

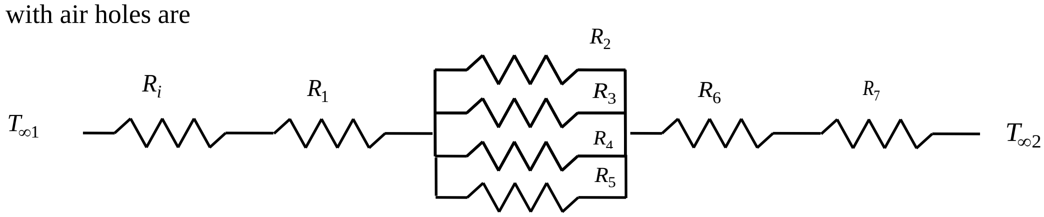
$$R_{total} = R_i + R_1 + R_{mid} + R_5 + R_o = 1.7068 + 1.0667 + 5.3135 + 1.0667 + 0.64 = 9.7937 \text{ h°F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(80-30)°\text{F}}{9.7937 \text{ h°F/Btu}} = 5.1053 \text{Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (5.1053 \text{Btu/h}) \frac{(30 \text{ ft})(10 \text{ ft})}{0.3906 \text{ m}^2} = \mathbf{3921 \text{ Btu/h}}$$

(*b*) The thermal resistance network and the individual thermal resistances if the wall is constructed of bricks with air holes are



$$A_{airholes} = 9(1.25/12)\times(1.25/12) = 0.0977\,\text{ft}^2$$

$$A_{bricks} = (7/12\,\text{ft})^2 - 0.0977 = 0.2426\,\text{ft}^2$$

$$R_4 = R_{airholes} = \frac{L}{kA} = \frac{9/12\,\text{ft}}{(0.015\,\text{Btu/h.ft.°F})(0.0977\text{ft}^2)} = 511.77\,\text{h°F/Btu}$$

$$R_5 = R_{brick} = \frac{L}{kA} = \frac{9/12\,\text{ft}}{(0.40\,\text{Btu/h.ft.°F})(0.2426\text{ft}^2)} = 7.729\,\text{h°F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{511.77} + \frac{1}{7.729} \longrightarrow R_{mid} = 7.244\,\text{h°F/Btu}$$

$$R_{total} = R_i + R_1 + R_{mid} + R_6 + R_o = 1.7068 + 1.0667 + 7.244 + 1.0677 + 0.64 = 11.7252\,\text{h°F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(80-30)\text{°F}}{11.7252\,\text{h°F/Btu}} = 4.2643\,\text{Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

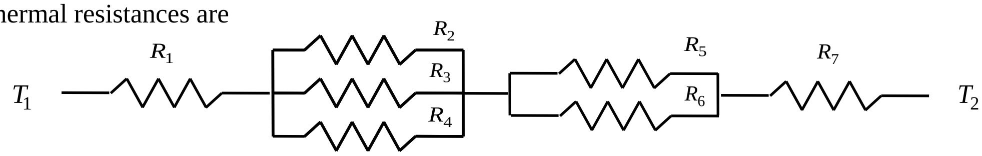
$$\dot{Q}_{total} = (4.2643\,\text{Btu/h})\frac{(30\,\text{ft})(10\,\text{ft})}{0.3906\,\text{ft}^2} = \mathbf{3275\ Btu/h}$$

3-57 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistances at the interfaces are disregarded.

Properties The thermal conductivities are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, $k_E = 35$ W/m·°C.

Analysis (*a*) The representative surface area is $A = 0.12\times 1 = 0.12\ \text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_1 = R_A = \left(\frac{L}{kA}\right)_A = \frac{0.01\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.04\ \text{°C/W}$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA}\right)_C = \frac{0.05\ \text{m}}{(20\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.06\ \text{°C/W}$$

$$R_3 = R_B = \left(\frac{L}{kA}\right)_B = \frac{0.05\ \text{m}}{(8\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.16\ \text{°C/W}$$

$$R_5 = R_D = \left(\frac{L}{kA}\right)_D = \frac{0.1\ \text{m}}{(15\ \text{W/m.}^\circ\text{C})(0.06\ \text{m}^2)} = 0.11\ \text{°C/W}$$

$$R_6 = R_E = \left(\frac{L}{kA}\right)_E = \frac{0.1\ \text{m}}{(35\ \text{W/m.°C})(0.06\ \text{m}^2)} = 0.05\ ^\circ\text{C/W}$$

$$R_7 = R_F = \left(\frac{L}{kA}\right)_F = \frac{0.06\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.25\ \text{°C/W}$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.06} \longrightarrow R_{mid,1} = 0.025\ \text{°C/W}$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{mid,2} = 0.034\ \text{°C/W}$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 = 0.04 + 0.025 + 0.034 + 0.25 = 0.349\ \text{°C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300-100)\text{°C}}{0.349\ \text{°C/W}} = 572\ \text{W}\ \ \text{(for a 0.12 m}\times\text{1 m section)}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (572\ \text{W})\frac{(5\ \text{m})(8\ \text{m})}{0.12\ \text{m}^2} = \mathbf{1.91\times 10^5\ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.025 = 0.065\ \text{°C/W}$$

Then the temperature at the point where the sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300\text{°C} - (572\ \text{W})(0.065\text{°C/W}) = \mathbf{263°C}$$

(*c*) The temperature drop across the section F can be determined from

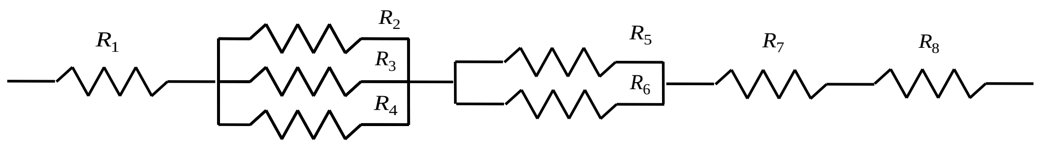
$$\dot{Q} = \frac{\Delta T}{R_F} \rightarrow \Delta T = \dot{Q}R_F = (572\ \text{W})(0.25\text{°C/W}) = \mathbf{143°C}$$

3-58 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistances at the interfaces are to be considered.

Properties The thermal conductivities of various materials used are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$ W/m·°C.

Analysis The representative surface area is $A = 0.12 \times 1 = 0.12\ \text{m}^2$



(*a*) The thermal resistance network and the individual thermal resistances are

$$R_1 = R_A = \left(\frac{L}{kA}\right)_A = \frac{0.01\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.04\ \text{°C/W}$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA}\right)_C = \frac{0.05\ \text{m}}{(20\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.06\ \text{°C/W}$$

$$R_3 = R_B = \left(\frac{L}{kA}\right)_B = \frac{0.05\ \text{m}}{(8\ \text{W/m.°C})(0.04\ \text{m}^2)} = 0.16\ \text{°C/W}$$

$$R_5 = R_D = \left(\frac{L}{kA}\right)_D = \frac{0.1\ \text{m}}{(15\ \text{W/m.}^\circ\text{C})(0.06\ \text{m}^2)} = 0.11\ \text{°C/W}$$

$$R_6 = R_E = \left(\frac{L}{kA}\right)_E = \frac{0.1\ \text{m}}{(35\ \text{W/m.°C})(0.06\ \text{m}^2)} = 0.05\ ^\circ\text{C/W}$$

$$R_7 = R_F = \left(\frac{L}{kA}\right)_F = \frac{0.06\ \text{m}}{(2\ \text{W/m.°C})(0.12\ \text{m}^2)} = 0.25\ \text{°C/W}$$

$$R_8 = \frac{0.00012\ \text{m}^2\ \text{°C/W}}{0.12\ \text{m}^2} = 0.001\ \text{°C/W}$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.06} \longrightarrow R_{mid,1} = 0.025\ \text{°C/W}$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{mid,2} = 0.034\ \text{°C/W}$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 + R_8 = 0.04 + 0.025 + 0.034 + 0.25 + 0.001$$
$$= 0.350\ \text{°C/W}$$

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(300-100)\text{°C}}{0.350\ \text{°C/W}} = 571\ \text{W}\ \ (\text{for a } 0.12\ \text{m} \times 1\ \text{m section})$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (571\ \text{W})\frac{(5\ \text{m})(8\ \text{m})}{0.12\ \text{m}^2} = \mathbf{1.90 \times 10^5\ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.025 = 0.065\ °\text{C/W}$$

Then the temperature at the point where The sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300°\text{C} - (571\,\text{W})(0.065°\text{C/W}) = \mathbf{263°C}$$

(*c*) The temperature drop across the section F can be determined from

$$\dot{Q} = \frac{\Delta T}{R_F} \longrightarrow \Delta T = \dot{Q}R_F = (571\,\text{W})(0.25°\text{C/W}) = \mathbf{143°C}$$

3-59 A coat is made of 5 layers of 0.1 mm thick synthetic fabric separated by 1.5 mm thick air space. The rate of heat loss through the jacket is to be determined, and the result is to be compared to the heat loss through a jackets without the air space. Also, the equivalent thickness of a wool coat is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the jacket is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.13 W/m·°C for synthetic fabric, k = 0.026 W/m·°C for air, and k = 0.035 W/m·°C for wool fabric.

Analysis The thermal resistance network and the individual thermal resistances are

$$R_{fabric} = R_1 = R_3 = R_5 = R_7 = R_9 = \frac{L}{kA} = \frac{0.0001\ \text{m}}{(0.13\ \text{W/m.°C})(1.1\ \text{m}^2)} = 0.0007\ °\text{C/W}$$

$$R_{air} = R_2 = R_4 = R_6 = R_8 = \frac{L}{kA} = \frac{0.0015\ \text{m}}{(0.026\ \text{W/m.°C})(1.1\ \text{m}^2)} = 0.0524\ °\text{C/W}$$

$$R_o = \frac{1}{hA} = \frac{1}{(25\ \text{W/m}^2.°\text{C})(1.1\ \text{m}^2)} = 0.0364\ °\text{C/W}$$

$$R_{total} = 5R_{fabric} + 4R_{air} + R_o = 5\times 0.0007 + 4\times 0.0524 + 0.0364 = 0.2495\ °\text{C/W}$$

and

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[(28-(-5)]°\text{C}}{0.2495\ °\text{C/W}} = \mathbf{132.3\ W}$$

If the jacket is made of a single layer of 0.5 mm thick synthetic fabric, the rate of heat transfer would be

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{T_{s1} - T_{\infty 2}}{5\times R_{fabric} + R_o} = \frac{[(28-(-5)]°\text{C}}{(5\times 0.0007 + 0.0364)\ °\text{C/W}} = 827\ \text{W}$$

The thickness of a wool fabric that has the same thermal resistance is determined from

$$R_{total} = R_{\substack{wool\\fabric}} + R_o = \frac{L}{kA} + \frac{1}{hA}$$

$$0.2495\ °\text{C/W} = \frac{L}{(0.035\ \text{W/m.°C})(1.1\ \text{m}^2)} + 0.0364 \longrightarrow L = 0.00820\ \text{m} = \mathbf{8.2\ mm}$$

3-60 A coat is made of 5 layers of 0.1 mm thick cotton fabric separated by 1.5 mm thick air space. The rate of heat loss through the jacket is to be determined, and the result is to be compared to the heat loss through a jackets without the air space. Also, the equivalent thickness of a wool coat is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the jacket is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.06 W/m·°C for cotton fabric, k = 0.026 W/m·°C for air, and k = 0.035 W/m·°C for wool fabric.

Analysis The thermal resistance network and the individual thermal resistances are

$$R_{\text{cotton}} = R_1 = R_3 = R_5 = R_7 = R_9 = \frac{L}{kA} = \frac{0.0001 \text{ m}}{(0.06 \text{ W/m.°C})(1.1 \text{ m}^2)} = 0.00152 \text{ °C/W}$$

$$R_{air} = R_2 = R_4 = R_6 = R_8 = \frac{L}{kA} = \frac{0.0015 \text{ m}}{(0.026 \text{ W/m.}^{\text{o}}\text{C})(1.1 \text{ m}^2)} = 0.0524 \text{ °C/W}$$

$$R_o = \frac{1}{hA} = \frac{1}{(25 \text{ W/m}^2\text{.°C})(1.1 \text{ m}^2)} = 0.0364 \text{ °C/W}$$

$$R_{total} = 5R_{fabric} + 4R_{air} + R_o = 5\times 0.00152 + 4\times 0.0524 + 0.0364 = 0.2536 \text{ °C/W}$$

and

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[(28-(-5)]\text{°C}}{0.2536\text{°C/W}} = \mathbf{130\ W}$$

If the jacket is made of a single layer of 0.5 mm thick cotton fabric, the rate of heat transfer will be

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{T_{s1} - T_{\infty 2}}{5\times R_{fabric} + R_o} = \frac{[(28-(-5)]\text{° C}}{(5\times 0.00152 + 0.0364) \text{ °C/W}} = \mathbf{750\ W}$$

The thickness of a wool fabric for that case can be determined from

$$R_{\text{total}} = R_{\substack{\text{wool}\\ \text{fabric}}} + R_o = \frac{L}{kA} + \frac{1}{hA}$$

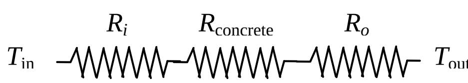
$$0.2536 \text{ °C/W} = \frac{L}{(0.035 \text{ W/m.°C})(1.1 \text{ m}^2)} + 0.0364 \longrightarrow L = 0.0084 \text{ m} = \mathbf{8.4\ mm}$$

3-61 A kiln is made of 20 cm thick concrete walls and ceiling. The two ends of the kiln are made of thin sheet metal covered with 2-cm thick styrofoam. For specified indoor and outdoor temperatures, the rate of heat transfer from the kiln is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the walls and ceiling is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer. **5** Heat loss through the floor is negligible. **6** Thermal resistance of sheet metal is negligible.

Properties The thermal conductivities are given to be k = 0.9 W/m·°C for concrete and k = 0.033 W/m·°C for styrofoam insulation.

Analysis In this problem there is a question of which surface area to use. We will use the outer surface area for outer convection resistance, the inner surface area for inner convection resistance, and the average area for the conduction resistance. Or we could use the inner or the outer surface areas in the calculation of all thermal resistances with little loss in accuracy. For top and the two side surfaces:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2.°\text{C})[(40\ \text{m})(13-0.6)\ \text{m}]} = 0.0067\times10^{-4}\ °\text{C/W}$$

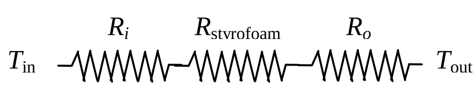
$$R_{concrete} = \frac{L}{kA_{ave}} = \frac{0.2\ \text{m}}{(0.9\ \text{W/m}.°\text{C})[(40\ \text{m})(13-0.3)\ \text{m}]} = 4.37\times10^{-4}\ °\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2.°\text{C})[(40\ \text{m})(13\text{m})]} = 0.769\times10^{-4}\ °\text{C/W}$$

$$R_{total} = R_i + R_{concrete} + R_o = (0.0067+4.37+0.769)\times10^{-4} = 5.146\times10^{-4}\ °\text{C/W}$$

and $$\dot{Q}_{top+sides} = \frac{T_{in}-T_{out}}{R_{total}} = \frac{[40-(-4)]°\text{C}}{5.146\times10^{-4}\ °\text{C/W}} = 85{,}500\ \text{W}$$

Heat loss through the end surface of the kiln with styrofoam:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2.°\text{C})[(4-0.4)(5-0.4)\ \text{m}^2]} = 0.201\times10^{-4}\ °\text{C/W}$$

$$R_{styrofoam} = \frac{L}{kA_{ave}} = \frac{0.02\ \text{m}}{(0.033\ \text{W/m}.°\text{C})[(4-0.2)(5-0.2)\ \text{m}^2]} = 0.0332°\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2.°\text{C})[4\times5\ \text{m}^2]} = 0.0020°\text{C/W}$$

$$R_{total} = R_i + R_{styrpfoam} + R_o = 0.201\times10^{-4} + 0.0332 + 0.0020 = 0.0352°\text{C/W}$$

and $$\dot{Q}_{end\ surface} = \frac{T_{in}-T_{out}}{R_{total}} = \frac{[40-(-4)]°\text{C}}{0.0352\ °\text{C/W}} = 1250\ \text{W}$$

Then the total rate of heat transfer from the kiln becomes

$$\dot{Q}_{total} = \dot{Q}_{top+sides} + 2\dot{Q}_{side} = 85{,}500 + 2\times1250 = \mathbf{88{,}000\ W}$$

3-62

```
"GIVEN"
width=5 "[m]"
height=4 "[m]"
length=40 "[m]"
```

```
L_wall=0.2 "[m], parameter to be varied"
k_concrete=0.9 "[W/m-C]"
T_in=40 "[C]"
T_out=-4 "[C]"
L_sheet=0.003 "[m]"
L_styrofoam=0.02 "[m]"
k_styrofoam=0.033 "[W/m-C]"
h_i=3000 "[W/m^2-C]"
"h_o=25 [W/m^2-C], parameter to be varied"

"ANALYSIS"
R_conv_i=1/(h_i*A_1)
A_1=(2*height+width-3*L_wall)*length
R_concrete=L_wall/(k_concrete*A_2)
A_2=(2*height+width-1/2*3*L_wall)*length
R_conv_o=1/(h_o*A_3)
A_3=(2*height+width)*length
R_total_top_sides=R_conv_i+R_concrete+R_conv_o
Q_dot_top_sides=(T_in-T_out)/R_total_top_sides "Heat loss from top and the two side
surfaces"

R_conv_i_end=1/(h_i*A_4)
A_4=(height-2*L_wall)*(width-2*L_wall)
R_styrofoam=L_styrofoam/(k_styrofoam*A_5)
A_5=(height-L_wall)*(width-L_wall)
R_conv_o_end=1/(h_o*A_6)
A_6=height*width
R_total_end=R_conv_i_end+R_styrofoam+R_conv_o_end
Q_dot_end=(T_in-T_out)/R_total_end "Heat loss from one end surface"

Q_dot_total=Q_dot_top_sides+2*Q_dot_end
```

L_{wall} [m]	Q_{total} [W]
0.1	152397
0.12	132921
0.14	117855
0.16	105852
0.18	96063
0.2	87927
0.22	81056
0.24	75176
0.26	70087
0.28	65638
0.3	61716

h_o [W/m^2.C]	**Q_{total} [W]**
5	55515
10	72095
15	80100
20	84817
25	87927
30	90132
35	91776
40	93050
45	94065
50	94894

3-63E The thermal resistance of an epoxy glass laminate across its thickness is to be reduced by planting cylindrical copper fillings throughout. The thermal resistance of the epoxy board for heat conduction across its thickness as a result of this modification is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the plate is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 0.10 Btu/h·ft·°F for epoxy glass laminate and k = 223 Btu/h·ft·°F for copper fillings.

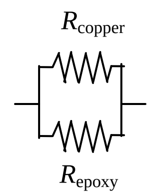
Analysis The thermal resistances of copper fillings and the epoxy board are in parallel. The number of copper fillings in the board and the area they comprise are

$$A_{total} = (6/12\text{ ft})(8/12\text{ ft}) = 0.333\text{ m}^2$$

$$n_{copper} = \frac{0.33\text{ ft}^2}{(0.06/12\text{ ft})(0.06/12\text{ ft})} = 13{,}333 \text{ (number of copper fillings)}$$

$$A_{copper} = n\frac{\pi D^2}{4} = 13{,}333\frac{\pi(0.02/12\text{ ft})^2}{4} = 0.0291\text{ ft}^2$$

$$A_{epoxy} = A_{total} - A_{copper} = 0.3333 - 0.0291 = 0.3042\text{ ft}^2$$



The thermal resistances are evaluated to be

$$R_{copper} = \frac{L}{kA} = \frac{0.05/12\text{ ft}}{(223\text{ Btu/h.ft.°F})(0.0291\text{ ft}^2)} = 0.00064\text{ h.°F/Btu}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.05/12\text{ ft}}{(0.10\text{ Btu/h.ft.°F})(0.3042\text{ ft}^2)} = 0.137\text{ h.°F/Btu}$$

Then the thermal resistance of the entire epoxy board becomes

$$\frac{1}{R_{board}} = \frac{1}{R_{copper}} + \frac{1}{R_{epoxy}} = \frac{1}{0.00064} + \frac{1}{0.137} \longrightarrow R_{board} = \mathbf{0.00064\ h.°F/Btu}$$