
Heat Conduction in Cylinders and Spheres

3-64C When the diameter of cylinder is very small compared to its length, it can be treated as an indefinitely long cylinder. Cylindrical rods can also be treated as being infinitely long when dealing with heat transfer at locations far from the top or bottom surfaces. However, it is not proper to use this model when finding temperatures near the bottom and the top of the cylinder.

3-65C Heat transfer in this short cylinder is one-dimensional since there will be no heat transfer in the axial and tangential directions.

3-66C No. In steady-operation the temperature of a solid cylinder or sphere does not change in radial direction (unless there is heat generation).

3-67 A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be k = 15 W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7 \text{ kJ/kg}$. The outer surface of the tank is black and thus its emissivity is ε = 1.

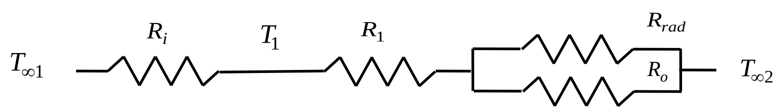
Analysis (*a*) The inner and the outer surface areas of sphere are

$$A_i = \pi D_i^2 = \pi(5\text{ m})^2 = 78.54\text{ m}^2 \qquad A_o = \pi D_o^2 = \pi(5.03\text{ m})^2 = 79.49\text{ m}^2$$

We assume the outer surface temperature T_2 to be 5°C after comparing convection heat transfer coefficients at the inner and the outer surfaces of the tank. With this assumption, the radiation heat transfer coefficient can be determined from

$$h_{rad} = \varepsilon\sigma(T_2^2 + T_{surr}^2)(T_2 + T_{surr})$$
$$= 1(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(273+5\text{ K})^2 + (273+30\text{ K})^2](273+30\text{ K})(273+5\text{ K})] = 5.570\text{ W/m}^2.\text{K}$$

The individual thermal resistances are



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(80\text{ W/m}^2.\text{°C})(78.54\text{ m}^2)} = 0.000159\text{°C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(2.515-2.5)\text{ m}}{4\pi(15\text{ W/m.°C})(2.515\text{ m})(2.5\text{ m})} = 0.000013\text{°C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(10\text{ W/m}^2.\text{°C})(79.49\text{ m}^2)} = 0.00126\text{°C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A} = \frac{1}{(5.57\text{ W/m}^2.\text{°C})(79.54\text{ m}^2)} = 0.00226\text{°C/W}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{conv,o}} + \frac{1}{R_{rad}} = \frac{1}{0.00126} + \frac{1}{0.00226} \longrightarrow R_{eqv} = 0.000809\text{°C/W}$$

$$R_{total} = R_{conv,i} + R_1 + R_{eqv} = 0.000159 + 0.000013 + 0.000809 = 0.000981\text{°C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(30-0)\text{°C}}{0.000981\text{°C/W}} = \mathbf{30{,}581\text{ W}}$$

(*b*) The total amount of heat transfer during a 24-hour period and the amount of ice that will melt during this period are

$$Q = \dot{Q}\Delta t = (30.581\text{ kJ/s})(24\times3600\text{ s}) = 2.642\times10^6\text{ kJ}$$

$$m_{ice} = \frac{Q}{h_{if}} = \frac{2.642\times10^6\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{7918\text{ kg}}$$

Check: The outer surface temperature of the tank is

$$\dot{Q} = h_{conv+rad} A_o (T_{\infty1} - T_s) \to T_s = T_{\infty1} - \frac{\dot{Q}}{h_{conv+rad} A_o} = 30\text{°C} - \frac{30{,}581\text{ W}}{(10+5.57\text{ W/m}^2.\text{°C})(79.54\text{ m}^2)} = 5.3\text{°C}$$

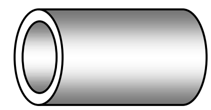
which is very close to the assumed temperature of 5°C for the outer surface temperature used in the evaluation of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations.

3-68 A steam pipe covered with 3-cm thick glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

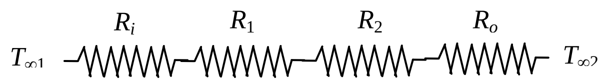
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 15 W/m·°C for steel and k = 0.038 W/m·°C for glass wool insulation

Analysis The inner and the outer surface areas of the insulated pipe per unit length are



$$A_i = \pi D_i L = \pi(0.05\text{ m})(1\text{ m}) = 0.157\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.055 + 0.06\text{ m})(1\text{ m}) = 0.361\text{ m}^2$$



The individual thermal resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(80\text{ W/m}^2.°\text{C})(0.157\text{ m}^2)} = 0.08\,°\text{C/W}$$
$$R_1 = R_{pipe} = \frac{\ln(r_2 / r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(15\text{ W/m.°C})(1\text{ m})} = 0.00101°\text{C/W}$$
$$R_2 = R_{insulation} = \frac{\ln(r_3 / r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.038\text{ W/m.°C})(1\text{ m})} = 3.089\,°\text{C/W}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(15\text{ W/m}^2.°\text{C})(0.361\text{ m}^2)} = 0.1847\,°\text{C/W}$$
$$R_{total} = R_i + R_1 + R_2 + R_o = 0.08 + 0.00101 + 3.089 + 0.1847 = 3.355\,°\text{C/W}$$

Then the steady rate of heat loss from the steam per m. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(320-5)°\text{C}}{3.355\ °\text{C / W}} = \mathbf{93.9\ W}$$

The temperature drops across the pipe and the insulation are

$$\Delta T_{pipe} = \dot{Q} R_{pipe} = (93.9\text{ W})(0.00101\ °\text{C / W}) = \mathbf{0.095°\,C}$$
$$\Delta T_{insulation} = \dot{Q} R_{insulation} = (93.9\text{ W})(3.089\ °\text{C / W}) = \mathbf{290°\,C}$$

3-69

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"GIVEN"
T_infinity_1=320 "[C]"
T_infinity_2=5 "[C]"
k_steel=15 "[W/m-C]"
D_i=0.05 "[m]"
D_o=0.055 "[m]"
r_1=D_i/2
r_2=D_o/2
"t_ins=3 [cm], parameter to be varied"
k_ins=0.038 "[W/m-C]"
h_o=15 "[W/m^2-C]"
h_i=80 "[W/m^2-C]"
L=1 "[m]"

"ANALYSIS"
A_i=pi*D_i*L
A_o=pi*(D_o+2*t_ins*Convert(cm, m))*L
R_conv_i=1/(h_i*A_i)
R_pipe=ln(r_2/r_1)/(2*pi*k_steel*L)
R_ins=ln(r_3/r_2)/(2*pi*k_ins*L)
r_3=r_2+t_ins*Convert(cm, m) "t_ins is in cm"
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_pipe+R_ins+R_conv_o
Q_dot=(T_infinity_1-T_infinity_2)/R_total
DELTAT_pipe=Q_dot*R_pipe
DELTAT_ins=Q_dot*R_ins
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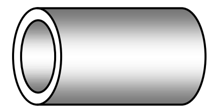
T_{ins} [cm]	Q [W]	ΔT_{ins} [C]
1	189.5	246.1
2	121.5	278.1
3	93.91	290.1
4	78.78	296.3
5	69.13	300
6	62.38	302.4
7	57.37	304.1
8	53.49	305.4
9	50.37	306.4
10	47.81	307.2

3-70 A 50-m long section of a steam pipe passes through an open space at 15°C. The rate of heat loss from the steam pipe, the annual cost of this heat loss, and the thickness of fiberglass insulation needed to save 90 percent of the heat lost are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivity is constant. **4** The thermal contact resistance at the interface is negligible. **5** The pipe temperature remains constant at about 150°C with or without insulation. **6** The combined heat transfer coefficient on the outer surface remains constant even after the pipe is insulated.

Properties The thermal conductivity of fiberglass insulation is given to be k = 0.035 W/m·°C.

Analysis (a) The rate of heat loss from the steam pipe is



$$A_o = \pi DL = \pi(0.1\,\text{m})(50\,\text{m}) = 15.71\,\text{m}^2$$

$$\dot{Q}_{bare} = h_o A(T_s - T_{air}) = (20\,\text{W/m}^2.°\text{C})(15.71\,\text{m}^2)(150-15)°\text{C} = \mathbf{42{,}412\,W}$$

(b) The amount of heat loss per year is

$$Q = \dot{Q}\Delta t = (42.412\,\text{kJ/s})(365\times 24\times 3600\,\text{s/yr}) = 1.337\times 10^9\ \text{kJ/yr}$$

The amount of gas consumption from the natural gas furnace that has an efficiency of 75% is

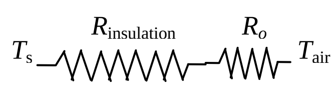
$$Q_{gas} = \frac{1.337\times 10^9\ \text{kJ/yr}}{0.75}\left(\frac{1\,\text{therm}}{105{,}500\,\text{kJ}}\right) = 16{,}903\,\text{therms/yr}$$

The annual cost of this energy lost is

$$\begin{aligned}\text{Energy cost} &= (\text{Energy used})(\text{Unit cost of energy})\\ &= (16{,}903\,\text{therms/yr})(\$0.52/\text{therm}) = \mathbf{\$8790/yr}\end{aligned}$$

(c) In order to save 90% of the heat loss and thus to reduce it to 0.1×42,412 = 4241 W, the thickness of insulation needed is determined from

$$\dot{Q}_{insulated} = \frac{T_s - T_{air}}{R_o + R_{insulation}} = \frac{T_s - T_{air}}{\dfrac{1}{h_o A_o} + \dfrac{\ln(r_2/r_1)}{2\pi kL}}$$



Substituting and solving for r_2, we get

$$4241\,\text{W} = \frac{(150-15)°\text{C}}{\dfrac{1}{(20\,\text{W/m}^2.°\text{C})[(2\pi r_2(50\,\text{m})]} + \dfrac{\ln(r_2/0.05)}{2\pi(0.035\,\text{W/m.°C})(50\,\text{m})}} \longrightarrow r_2 = 0.0692\,\text{m}$$

Then the thickness of insulation becomes

$$t_{insulation} = r_2 - r_1 = 6.92 - 5 = \mathbf{1.92cm}$$

3-71 An electric hot water tank is made of two concentric cylindrical metal sheets with foam insulation in between. The fraction of the hot water cost that is due to the heat loss from the tank and the payback period of the do-it-yourself insulation kit are to be determined.

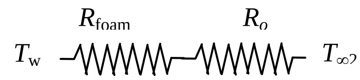
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal resistances of the water tank and the outer thin sheet metal shell are negligible. **5** Heat loss from the top and bottom surfaces is negligible.

Properties The thermal conductivities are given to be k = 0.03 W/m·°C for foam insulation and k = 0.035 W/m·°C for fiber glass insulation

Analysis We consider only the side surfaces of the water heater for simplicity, and disregard the top and bottom surfaces (it will make difference of about 10 percent). The individual thermal resistances are

$$A_o = \pi D_o L = \pi(0.46\text{ m})(2\text{ m}) = 2.89\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.°\text{C})(2.89\text{ m}^2)} = 0.029\,°\text{C/W}$$



$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2.°\text{C})(2\text{ m})} = 0.37\,°\text{C/W}$$

$$R_{total} = R_o + R_{foam} = 0.029 + 0.37 = 0.40\,°\text{C/W}$$

The rate of heat loss from the hot water tank is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)°\text{C}}{0.40\text{ °C / W}} = 70\text{ W}$$

The amount and cost of heat loss per year are

$$Q = \dot{Q}\Delta t = (0.07\text{ kW})(365 \times 24\text{ h / yr}) = 613.2\text{ kWh / yr}$$

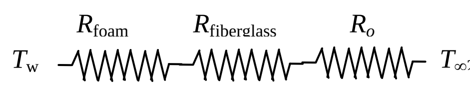
$$\text{Cost of Energy} = (\text{Amount of energy})(\text{Unit cost}) = (613.2\text{ kWh})(\$0.08/\text{kWh}) = \$49.056$$

$$f = \frac{\$49.056}{\$280} = 0.1752 = \mathbf{17.5\%}$$

If 3 cm thick fiber glass insulation is used to wrap the entire tank, the individual resistances becomes

$$A_o = \pi D_o L = \pi(0.52\text{ m})(2\text{ m}) = 3.267\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.^\text{o}\text{C})(3.267\text{ m}^2)} = 0.026\ ^\text{o}\text{C/W}$$



$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2.°\text{C})(2\text{ m})} = 0.371°\text{C/W}$$

$$R_{fiberglass} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(26/23)}{2\pi(0.035\text{ W/m}^2.°\text{C})(2\text{ m})} = 0.279°\text{C/W}$$

$$R_{total} = R_o + R_{foam} + R_{fiberglass} = 0.026 + 0.371 + 0.279 = 0.676°\text{C/W}$$

The rate of heat loss from the hot water heater in this case is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)°\text{C}}{0.676°\text{C/W}} = 41.42\text{ W}$$

The energy saving is

saving = 70 - 41.42 = 28.58 W

The time necessary for this additional insulation to pay for its cost of $30 is then determined to be

$$\text{Cost} = (0.02858\text{ kW})(\text{Time period})(\$0.08/\text{kWh}) = \$30$$

Then, $\text{Time period} = 13{,}121\text{ hours} = \mathbf{547\ days} \approx \mathbf{1.5\ years}$

3-72

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"GIVEN"
L=2 "[m]"
D_i=0.40 "[m]"
D_o=0.46 "[m]"
r_1=D_i/2
r_2=D_o/2
"T_w=55 [C], parameter to be varied"
T_infinity_2=27 "[C]"
h_i=50 "[W/m^2-C]"
h_o=12 "[W/m^2-C]"
k_ins=0.03 "[W/m-C]"
Price_electric=0.08 "[$/kWh]"
Cost_heating=280 "[$/year]"

"ANALYSIS"
A_i=pi*D_i*L
A_o=pi*D_o*L
R_conv_i=1/(h_i*A_i)
R_ins=ln(r_2/r_1)/(2*pi*k_ins*L)
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_ins+R_conv_o
Q_dot=(T_w-T_infinity_2)/R_total
Q=(Q_dot*Convert(W, kW))*time
time=365*24 "[h/year]"
Cost_HeatLoss=Q*Price_electric
f_HeatLoss=Cost_HeatLoss/Cost_heating*Convert(, %)
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T_w [C]	$f_{HeatLoss}$ [%]
40	7.984
45	11.06
50	14.13
55	17.2
60	20.27
65	23.34
70	26.41
75	29.48
80	32.55
85	35.62
90	38.69

3-73 A cold aluminum canned drink that is initially at a uniform temperature of 3°C is brought into a room air at 25°C. The time it will take for the average temperature of the drink to rise to 10°C with and without rubber insulation is to be determined.

Assumptions **1** The drink is at a uniform temperature at all times. **2** The thermal resistance of the can and the internal convection resistance are negligible so that the can is at the same temperature as the drink inside. **3** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **4** Thermal properties are constant. **5** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of rubber insulation is given to be k = 0.13 W/m·°C. For the drink, we use the properties of water at room temperature, ρ = 1000 kg/m^3 and C_p = 4180 J/kg.°C.

Analysis This is a transient heat conduction, and the rate of heat transfer will decrease as the drink warms up and the temperature difference between the drink and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (3+10)/2 = 6.5°C during the process. Then the average rate of heat transfer into the drink is

$$A_o = \pi D_o L + 2\frac{\pi D^2}{4} = \pi(0.06\text{ m})(0.125\text{ m}) + 2\frac{\pi(0.06\text{ m})^2}{4} = 0.0292\text{ m}^2$$

$$\dot{Q}_{bare,ave} = h_o A(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.°\text{C})(0.0292\text{ m}^2)(25-6.5)°\text{C} = 5.40\text{ W}$$

The amount of heat that must be supplied to the drink to raise its temperature to 10 °C is

$$m = \rho V = \rho\pi r^2 L = (1000\text{ kg/m}^3)\pi(0.03\text{ m})^2(0.125\text{ m}) = 0.353\text{ kg}$$

$$Q = mC_p \Delta T = (0.353\text{ kg})(4180\text{ J/kg})(10-3)°\text{C} = 10{,}329\text{ J}$$

Then the time required for this much heat transfer to take place is

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{5.4\text{ J/s}} = 1912\text{ s} = \mathbf{31.9\text{ min}}$$

We now repeat calculations after wrapping the can with 1-cm thick rubber insulation, except the top surface. The rate of heat transfer from the top surface is

$$\dot{Q}_{top,ave} = h_o A_{top}(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.°\text{C})[\pi(0.03\text{ m})^2](25-6.5)°\text{C} = 0.52\text{ W}$$

Heat transfer through the insulated side surface is

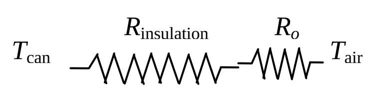
$$A_o = \pi D_o L = \pi(0.08\text{ m})(0.125\text{ m}) = 0.03142\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(10\text{ W/m}^2.°\text{C})(0.03142\text{ m}^2)} = 3.183°\text{C/W}$$

$$R_{insulation,side} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(4/3)}{2\pi(0.13\text{ W/m}^2.°\text{C})(0.125\text{ m})} = 2.818°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 3.183 + 2.818 = 6.001°\text{C/W}$$

$$\dot{Q}_{side} = \frac{T_{air} - T_{can,ave}}{R_{conv,o}} = \frac{(25-6.5)°\text{C}}{6.001°\text{C/W}} = 3.08\text{ W}$$



The ratio of bottom to the side surface areas is $(\pi r^2)/(2\pi rL) = r/(2L) = 3/(2\times 12.5) = 0.12$. Therefore, the effect of heat transfer through the bottom surface can be accounted for approximately by increasing the heat transfer from the side surface by 12%. Then,

$$\dot{Q}_{insulated} = \dot{Q}_{side+bottom} + \dot{Q}_{top} = 1.12\times 3.08 + 0.52 = 3.97\text{ W}$$

Then the time of heating becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{3.97\text{ J/s}} = 2602\text{ s} = \mathbf{43.4\text{ min}}$$

3-74 A cold aluminum canned drink that is initially at a uniform temperature of 3°C is brought into a room air at 25°C. The time it will take for the average temperature of the drink to rise to 10°C with and without rubber insulation is to be determined.

Assumptions **1** The drink is at a uniform temperature at all times. **2** The thermal resistance of the can and the internal convection resistance are negligible so that the can is at the same temperature as the drink inside. **3** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **4** Thermal properties are constant. **5** The thermal contact resistance at the interface is to be considered.

Properties The thermal conductivity of rubber insulation is given to be k = 0.13 W/m·°C. For the drink, we use the properties of water at room temperature, ρ = 1000 kg/m^3 and C_p = 4180 J/kg.°C.

Analysis This is a transient heat conduction, and the rate of heat transfer will decrease as the drink warms up and the temperature difference between the drink and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (3+10)/2 = 6.5°C during the process. Then the average rate of heat transfer into the drink is

$$A_o = \pi D_o L + 2\frac{\pi D^2}{4} = \pi(0.06\text{ m})(0.125\text{ m}) + 2\frac{\pi(0.06\text{ m})^2}{4} = 0.0292\text{ m}^2$$

$$\dot{Q}_{bare,ave} = h_o A(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.^\circ\text{C})(0.0292\text{ m}^2)(25-6.5)^\circ\text{C} = 5.40\text{ W}$$

The amount of heat that must be supplied to the drink to raise its temperature to 10 °C is

$$m = \rho V = \rho\pi r^2 L = (1000\text{ kg/m}^3)\pi(0.03\text{ m})^2(0.125\text{ m}) = 0.353\text{ kg}$$

$$Q = mC_p\Delta T = (0.353\text{ kg})(4180\text{ J/kg})(10\text{-}3)^\circ\text{C} = 10{,}329\text{ J}$$

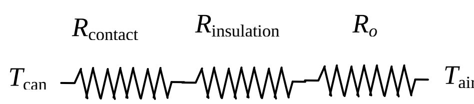
Then the time required for this much heat transfer to take place is

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{5.4\text{ J/s}} = 1912\text{ s} = \mathbf{31.9\text{ min}}$$

We now repeat calculations after wrapping the can with 1-cm thick rubber insulation, except the top surface. The rate of heat transfer from the top surface is

$$\dot{Q}_{top,ave} = h_o A_{top}(T_{air} - T_{can,ave}) = (10\text{ W/m}^2.^\circ\text{C})[\pi(0.03\text{ m})^2](25-6.5)^\circ\text{C} = 0.52\text{ W}$$

Heat transfer through the insulated side surface is



$$A_o = \pi D_o L = \pi(0.08\text{ m})(0.125\text{ m}) = 0.03142\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(10\text{ W/m}^2.^\circ\text{C})(0.03142\text{m}^2)} = 3.183^\circ\text{C/W}$$

$$R_{insulation,side} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(4/3)}{2\pi(0.13\text{ W/m}^2.^\circ\text{C})(0.125\text{m})} = 2.818^\circ\text{C/W}$$

$$R_{contact} = \frac{0.00008\text{m}^2.^\circ\text{C/W}}{\pi(0.06\text{m})(0.125\text{m})} = 0.0034^\circ\text{C/W}$$

$$R_{total} = R_o + R_{insulation} + R_{contact} = 3.183 + 2.818 + 0.0034 = 6.004^\circ\text{C/W}$$

$$\dot{Q}_{\text{side}} = \frac{T_{\text{air}} - T_{\text{can,ave}}}{R_{\text{conv,o}}} = \frac{(25-6.5)^\circ\text{C}}{6.004^\circ\text{C/W}} = 3.08\text{ W}$$

The ratio of bottom to the side surface areas is $(\pi r^2)/(2\pi rL) = r/(2L) = 3/(2\times 12.5) = 0.12$. Therefore, the effect of heat transfer through the bottom surface can be accounted for approximately by increasing the heat transfer from the side surface by 12%. Then,

$$\dot{Q}_{insulated} = \dot{Q}_{side+bottom} + \dot{Q}_{top} = 1.12\times 3.08 + 0.52 = 3.97\text{ W}$$

Then the time of heating becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{10{,}329\text{ J}}{3.97\text{ J/s}} = 2602\text{ s} = \mathbf{43.4\text{ min}}$$

Discussion The thermal contact resistance did not have any effect on heat transfer.

3-75E A steam pipe covered with 2-in thick fiberglass insulation is subjected to convection on its surfaces. The rate of heat loss from the steam per unit length and the error involved in neglecting the thermal resistance of the steel pipe in calculations are to be determined.

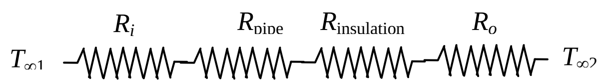
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 8.7 Btu/h·ft·°F for steel and k = 0.020 Btu/h·ft·°F for fiberglass insulation.

Analysis The inner and outer surface areas of the insulated pipe are

$$A_i = \pi D_i L = \pi(3.5/12\text{ ft})(1\text{ ft}) = 0.916\text{ ft}^2$$

$$A_o = \pi D_o L = \pi(8/12\text{ ft})(1\text{ ft}) = 2.094\text{ ft}^2$$



The individual resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(30\text{ Btu/h.ft}^2.°\text{F})(0.916\text{ ft}^2)} = 0.036\text{ h}\cdot°\text{F/Btu}$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2/1.75)}{2\pi(8.7\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.002\text{ h}\cdot°\text{F/Btu}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(4/2)}{2\pi(0.020\text{ Btu/h.ft.°F})(1\text{ ft})} = 5.516\text{ h}\cdot°\text{F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(5\text{ Btu/h.ft}^2.°\text{F})(2.094\text{ ft}^2)} = 0.096\text{ h}\cdot°\text{F/Btu}$$

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.036 + 0.002 + 5.516 + 0.096 = 5.65\text{ h}\cdot°\text{F/Btu}$$

Then the steady rate of heat loss from the steam per ft. pipe length becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(450-55)°\text{F}}{5.65\text{ h°F/Btu}} = \mathbf{69.91\ Btu/h}$$

If the thermal resistance of the steel pipe is neglected, the new value of total thermal resistance will be

$$R_{total} = R_i + R_2 + R_o = 0.036 + 5.516 + 0.096 = 5.648\text{ h°F/Btu}$$

Then the percentage error involved in calculations becomes

$$error\% = \frac{(5.65 - 5.648)\text{h° F / Btu}}{5.65\text{ h° F / Btu}} \times 100 = \mathbf{0.035\%}$$

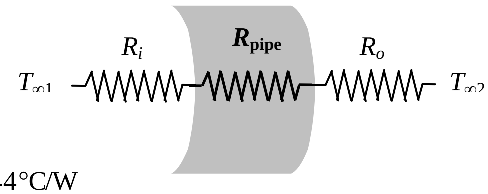
which is insignificant.

3-76 Hot water is flowing through a 3-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement. The rate of heat loss from the hot water and the average velocity of the water in the pipe as it passes through the basement are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity and emissivity of cast iron are given to be k = 52 W/m·°C and ε = 0.7.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.04 \text{ m})(15 \text{ m}) = 1.885 \text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.046 \text{ m})(15 \text{ m}) = 2.168 \text{ m}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(120 \text{ W/m}^2.°\text{C})(1.885 \text{ m}^2)} = 0.0044°\text{C/W}$$

$$R_{pipe} = \frac{\ln(r_2 / r_1)}{2\pi k_1 L} = \frac{\ln(2.3/2)}{2\pi(52 \text{ W/m.°C})(15 \text{ m})} = 0.00003°\text{C/W}$$

The outer surface temperature of the pipe will be somewhat below the water temperature. Assuming the outer surface temperature of the pipe to be 80°C (we will check this assumption later), the radiation heat transfer coefficient is determined to be

$$h_{rad} = \varepsilon\sigma(T_2^{\ 2} + T_{surr}^{\ 2})(T_2 + T_{surr})$$
$$= (0.7)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(353\text{K})^2 + (283\text{K})^2](353+283) = 5.167 \text{ W/m}^2.\text{K}$$

Since the surrounding medium and surfaces are at the same temperature, the radiation and convection heat transfer coefficients can be added and the result can be taken as the combined heat transfer coefficient. Then,

$$h_{combined} = h_{rad} + h_{conv,2} = 5.167 + 15 = 20.167 \text{ W/m}^2.°\text{C}$$

$$R_o = \frac{1}{h_{combined} A_o} = \frac{1}{(20.167 \text{ W/m}^2.°\text{C})(2.168 \text{ m}^2)} = 0.0229°\text{C/W}$$

$$R_{total} = R_i + R_{pipe} + R_o = 0.0044 + 0.00003 + 0.0229 = 0.0273°\text{C/W}$$

The rate of heat loss from the hot water pipe then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(90-10)°\text{C}}{0.0273°\text{C/W}} = \mathbf{2927\ W}$$

For a temperature drop of 3°C, the mass flow rate of water and the average velocity of water must be

$$\dot{Q} = \dot{m} C_p \Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p \Delta T} = \frac{2927 \text{ J/s}}{(4180 \text{ J/kg.°C})(3\text{ °C})} = 0.233 \text{ kg/s}$$

$$\dot{m} = \rho V A_c \longrightarrow V = \frac{\dot{m}}{\rho A_c} = \frac{0.233 \text{ kg/s}}{(1000 \text{ kg/m}^3)\dfrac{\pi(0.04 \text{ m})^2}{4}} = \mathbf{0.186\ m/s}$$

Discussion The outer surface temperature of the pipe is

$$\dot{Q} = \frac{T_{\infty 1} - T_s}{R_i + R_{pipe}} \rightarrow 2927 \text{ W} = \frac{(90 - T_s)°\text{C}}{(0.0044 + 0.00003)°\text{C/W}} \rightarrow T_s = 77°\text{C}$$

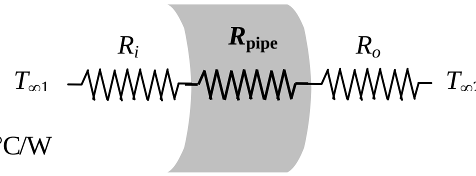
which is very close to the value assumed for the surface temperature in the evaluation of the radiation resistance. Therefore, there is no need to repeat the calculations.

3-77 Hot water is flowing through a 15 m section of a copper pipe. The pipe is exposed to cold air and surfaces in the basement. The rate of heat loss from the hot water and the average velocity of the water in the pipe as it passes through the basement are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity and emissivity of copper are given to be k = 386 W/m·°C and ε = 0.7.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.04 \text{ m})(15 \text{ m}) = 1.885 \text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.046 \text{ m})(15 \text{ m}) = 2.168 \text{ m}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(120 \text{ W/m}^2.°\text{C})(1.885 \text{ m}^2)} = 0.0044°\text{C/W}$$

$$R_{pipe} = \frac{\ln(r_2 / r_1)}{2\pi k L} = \frac{\ln(2.3/2)}{2\pi(386 \text{ W/m.°C})(15 \text{ m})} = 0.0000038°\text{C/W}$$

The outer surface temperature of the pipe will be somewhat below the water temperature. Assuming the outer surface temperature of the pipe to be 80°C (we will check this assumption later), the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2{}^2 + T_{surr}{}^2)(T_2 + T_{surr}) \\ &= (0.7)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(353\text{K})^2 + (283\text{K})^2](353+283) = 5.167 \text{ W/m}^2.\text{K} \end{aligned}$$

Since the surrounding medium and surfaces are at the same temperature, the radiation and convection heat transfer coefficients can be added and the result can be taken as the combined heat transfer coefficient. Then,

$$h_{combined} = h_{rad} + h_{conv,2} = 5.167 + 15 = 20.167 \text{ W/m}^2.°\text{C}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(20.167 \text{ W/m}^2.°\text{C})(2.168 \text{ m}^2)} = 0.0229°\text{C/W}$$

$$R_{total} = R_i + R_{pipe} + R_o = 0.004 + 0.0000038 + 0.0229 = 0.0273°\text{C/W}$$

The rate of heat loss from the hot tank water then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(90-10)°\text{C}}{0.0273°\text{C/W}} = \mathbf{2930 \text{ W}}$$

For a temperature drop of 3°C, the mass flow rate of water and the average velocity of water must be

$$\dot{Q} = \dot{m} C_p \Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{C_p \Delta T} = \frac{2930 \text{ J/s}}{(4180 \text{ J/kg.}^\circ\text{C})(3\ ^\circ\text{C})} = 0.234 \text{ kg/s}$$

$$\dot{m} = \rho V A_c \longrightarrow V = \frac{\dot{m}}{\rho A_c} = \frac{0.234 \text{ kg/s}}{(1000 \text{ kg/m}^3)\left[\frac{\pi(0.04 \text{ m})^2}{4}\right]} = \mathbf{0.186 \text{ m/s}}$$

Discussion The outer surface temperature of the pipe is

$$\dot{Q} = \frac{T_{\infty 1} - T_s}{R_i + R_{pipe}} \rightarrow 2930 \text{ W} = \frac{(90 - T_s)°\text{C}}{(0.0044 + 0.0000)°\text{C/W}} \rightarrow T_s = 77°\text{C}$$

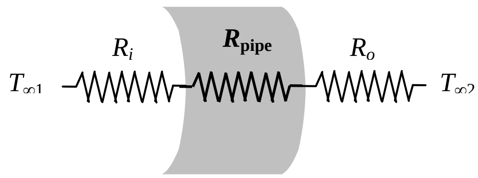
which is very close to the value assumed for the surface temperature in the evaluation of the radiation resistance. Therefore, there is no need to repeat the calculations.

3-78E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients, the length of the tube required to condense steam at a rate of 400 lbm/h is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivity of copper tube is given to be k = 223 Btu/h·ft·°F. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.4/12\text{ ft})(1\text{ ft}) = 0.105\text{ ft}^2$$

$$A_o = \pi D_o L = \pi(0.6/12\text{ ft})(1\text{ ft}) = 0.157\text{ ft}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35\text{ Btu/h.ft}^2.°\text{F})(0.105\text{ ft}^2)} = 0.27211\text{h°F/Btu}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3/2)}{2\pi(223\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.00029\text{h°F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(1500\text{ Btu/h.ft}^2.°\text{F})(0.157\text{ ft}^2)} = 0.00425\text{h°F/Btu}$$

$$R_{total} = R_i + R_{pipe} + R_o = 0.27211 + 0.00029 + 0.00425 = 0.27665\text{h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(100-70)°\text{F}}{0.27665°\text{F/Btu}} = 108.44\text{Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 400 lbm/h and the length of the tube required is determined to be

$$\dot{Q}_{total} = \dot{m}h_{fg} = (120\text{ lbm/h})(1037\text{ Btu/lbm}) = 124{,}440\text{ Btu/h}$$

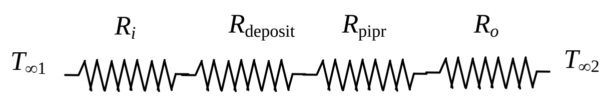
$$\text{Tube length} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{124{,}440}{108.44} = \mathbf{1148\ ft}$$

3-79E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients and 0.01-in thick scale build up on the inner surface, the length of the tube required to condense steam at a rate of 400 lbm/h is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tube and be k = 0.5 Btu/h·ft·°F for the mineral deposit. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis When a 0.01-in thick layer of deposit forms on the inner surface of the pipe, the inner diameter of the pipe will reduce from 0.4 in to 0.38 in. The individual thermal resistances are



$$A_i = \pi D_i L = \pi(0.4/12 \text{ ft})(1 \text{ ft}) = 0.105 \text{ ft}^2$$

$$A_o = \pi D_o L = \pi(0.6/12 \text{ ft})(1 \text{ ft}) = 0.157 \text{ ft}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35 \text{ Btu/h.ft}^2.°\text{F})(0.105 \text{ ft}^2)} = 0.2711 \text{h°F/Btu}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3/2)}{2\pi(223 \text{ Btu/h.ft.°F})(1 \text{ ft})} = 0.00029 \text{h°F/Btu}$$

$$R_{deposit} = \frac{\ln(r_1/r_{dep})}{2\pi k_2 L} = \frac{\ln(0.2/0.19)}{2\pi(0.5 \text{ Btu/h.ft.°F})(1 \text{ ft})} = 0.01633 \text{h.°F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(1500 \text{ Btu/h.ft}^2.°\text{F})(0.157 \text{ ft}^2)} = 0.00425 \text{h°F/Btu}$$

$$R_{total} = R_i + R_{pipe} + R_{deposit} + R_o = 0.27211 + 0.00029 + 0.01633 + 0.00425 = 0.29298 \text{h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(100-70)°\text{F}}{0.29298 °\text{F/Btu}} = 102.40 \text{ Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 400 lbm/h and the length of the tube required can be determined to be

$$\dot{Q}_{total} = \dot{m} h_{fg} = (120 \text{ lbm/h})(1037 \text{ Btu/lbm}) = 124{,}440 \text{ Btu/h}$$

$$\text{Tube length} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{124{,}440}{102.40} = \mathbf{1215 \text{ ft}}$$

3-80E

```
"GIVEN"
T_infinity_1=100 "[F]"
T_infinity_2=70 "[F]"
k_pipe=223 "[Btu/h-ft-F], parameter to be varied"
D_i=0.4 "[in]"
"D_o=0.6 [in], parameter to be varied"
r_1=D_i/2
r_2=D_o/2
h_fg=1037 "[Btu/lbm]"
h_o=1500 "[Btu/h-ft^2-F]"
h_i=35 "[Btu/h-ft^2-F]"
m_dot=120 "[lbm/h]"
"ANALYSIS"
L=1 "[ft], for 1 ft length of the tube"
A_i=pi*(D_i*Convert(in, ft))*L
A_o=pi*(D_o*Convert(in, ft))*L
R_conv_i=1/(h_i*A_i)
R_pipe=ln(r_2/r_1)/(2*pi*k_pipe*L)
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_pipe+R_conv_o
Q_dot=(T_infinity_1-T_infinity_2)/R_total
Q_dot_total=m_dot*h_fg
L_tube=Q_dot_total/Q_dot
```

k_{pipe} [Btu/h.ft.F]	L_{tube} [ft]
10	1176
30.53	1158
51.05	1155
71.58	1153
92.11	1152
112.6	1152
133.2	1151
153.7	1151
174.2	1151
194.7	1151
215.3	1151
235.8	1150
256.3	1150
276.8	1150
297.4	1150
317.9	1150
338.4	1150
358.9	1150
379.5	1150
400	1150

D_o[in]	L_{tube} [ft]
0.5	1154
0.525	1153
0.55	1152
0.575	1151
0.6	1151
0.625	1150
0.65	1149
0.675	1149
0.7	1148
0.725	1148
0.75	1148
0.775	1147
0.8	1147
0.825	1147
0.85	1146
0.875	1146
0.9	1146
0.925	1146
0.95	1145
0.975	1145
1	1145

3-81 A 3-m diameter spherical tank filled with liquid nitrogen at 1 atm and -196°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid nitrogen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

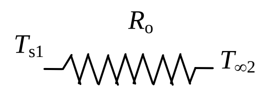
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the nitrogen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid nitrogen at 1 atm are given to be 198 kJ/kg and 810 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (*a*) The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\text{ m})^2 = 28.27\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(28.27\text{ m}^2)} = 0.00101°\text{C/W}$$



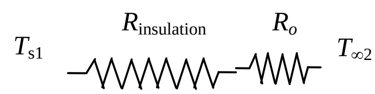
$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-196)]°\text{C}}{0.00101°\text{C/W}} = 208{,}910\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{208.910\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{1.055\ kg/s}$$

(*b*) The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\text{ m})^2 = 30.19\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(30.19\text{ m}^2)} = 0.000946°\text{C/W}$$



$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\text{ m}}{4\pi(0.035\text{ W/m.°C})(1.55\text{ m})(1.5\text{ m})} = 0.0489°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498°\text{C/W}$$

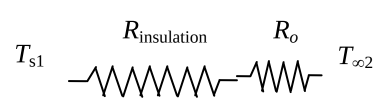
$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{0.0498°\text{C/W}} = 4233\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{4.233\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{0.0214\ kg/s}$$

(*c*) The heat transfer rate and the rate of evaporation of the liquid with 2-cm thick layer of superinsulation is

$$A = \pi D^2 = \pi(3.04\text{ m})^2 = 29.03\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(29.03\text{ m}^2)} = 0.000984°\text{C/W}$$



$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\text{ m}}{4\pi(0.00005\text{ W/m.°C})(1.52\text{ m})(1.5\text{ m})} = 13.96°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{13.96\text{ °C / W}} = 15.11\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01511\text{ kJ / s}}{198\text{ kJ / kg}} = \mathbf{0.000076\ kg/s}$$

3-82 A 3-m diameter spherical tank filled with liquid oxygen at 1 atm and -183°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid oxygen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

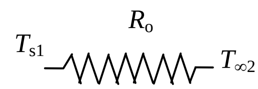
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the oxygen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid oxygen at 1 atm are given to be 213 kJ/kg and 1140 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (*a*) The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\text{ m})^2 = 28.27\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(28.27\text{ m}^2)} = 0.00101°\text{C/W}$$



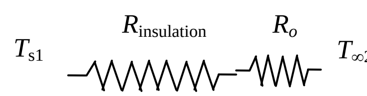
$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15 - (-183)]°\text{C}}{0.00101°\text{C/W}} = 196{,}040\text{ W}$$

$$\dot{Q} = \dot{m} h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{196.040\text{ kJ/s}}{213\text{ kJ/kg}} = \mathbf{0.920\ kg/s}$$

(*b*) The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\text{ m})^2 = 30.19\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(30.19\text{ m}^2)} = 0.000946°\text{C/W}$$



$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55 - 1.5)\text{ m}}{4\pi(0.035\text{ W/m.°C})(1.55\text{ m})(1.5\text{ m})} = 0.0489°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498°\text{C/W}$$

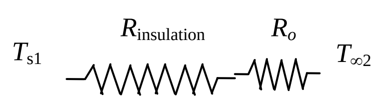
$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15 - (-183)]°\text{C}}{0.0498\ °\text{C / W}} = 3976\text{ W}$$

$$\dot{Q} = \dot{m} h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{3.976\text{ kJ / s}}{213\text{ kJ / kg}} = \mathbf{0.0187\ kg / s}$$

(*c*) The heat transfer rate and the rate of evaporation of the liquid with a 2-cm superinsulation is

$$A = \pi D^2 = \pi(3.04\text{ m})^2 = 29.03\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(29.03\text{ m}^2)} = 0.000984°\text{C/W}$$



$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.5)\text{ m}}{4\pi(0.00005\text{ W/m.°C})(1.52\text{ m})(1.5\text{ m})} = 13.96\ °\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96\ °\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15 - (-183)]°\text{C}}{13.96\ °\text{C / W}} = 14.18\text{ W}$$

$$\dot{Q} = \dot{m} h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01418\text{ kJ / s}}{213\text{ kJ / kg}} = \mathbf{0.000067\ kg / s}$$