Critical Radius Of Insulation

3-83C In a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of insulation, but decreases the convection resistance of the surface because of the increase in the outer surface area. Due to these opposite effects, a critical radius of insulation is defined as the outer radius that provides maximum rate of heat transfer. For a cylindrical layer, it is defined as $r_{cr} = k/h$ where k is the thermal conductivity of insulation and *h* is the external convection heat transfer coefficient.

3-84C It will decrease.

3-85C Yes, the measurements can be right. If the radius of insulation is less than critical radius of insulation of the pipe, the rate of heat loss will increase.

3-86C No.

3-87C For a cylindrical pipe, the critical radius of insulation is defined as $r_{cr} = k/h$. On windy days, the external convection heat transfer coefficient is greater compared to calm days. Therefore critical radius of insulation will be greater on calm days.

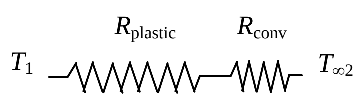
3-88 An electric wire is tightly wrapped with a 1-mm thick plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient accounts for the radiation effects, if any.

Properties The thermal conductivity of plastic cover is given to be k = 0.15 W/m·°C.

Analysis In steady operation, the rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$



The total thermal resistance is

$$R_{conv} = \frac{1}{h_o A_o} = \frac{1}{(24\text{ W/m}^2.\text{°C})[\pi(0.004\text{m})(10\text{m})]} = 0.3316\text{°C/W}$$

$$R_{plastic} = \frac{\ln(r_2/r_1}{2\pi kL} = \frac{\ln(2/1)}{2\pi(0.15\text{ W/m.°C})(10\text{ m})} = 0.0735\text{°C/W}$$

$$R_{total} = R_{conv} + R_{plastic} = 0.3316 + 0.0735 = 0.4051\text{°C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 30\text{°C} + (80\text{ W})(0.4051\text{°C/W}) = \mathbf{62.4°C}$$

The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m.°C}}{24\text{ W/m}^2.\text{°C}} = 0.00625\text{m} = 6.25\text{ mm}$$

Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature.

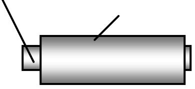
3-89E An electrical wire is covered with 0.02-in thick plastic insulation. It is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be k = 0.075 Btu/h·ft·°F.

Analysis The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.075\,\text{Btu/h.ft.°F}}{2.5\,\text{Btu/h.ft}^2.\text{°F}} = 0.03\,\text{ft} = 0.36\,\text{in} > r_2\,(= 0.0615\text{in})$$



Since the outer radius of the wire with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

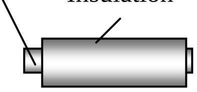
3-90E An electrical wire is covered with 0.02-in thick plastic insulation. By considering the effect of thermal contact resistance, it is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant

Properties The thermal conductivity of plastic cover is given to be k = 0.075 Btu/h·ft·°F.

Analysis Without insulation, the total thermal resistance is (per ft length of the wire)

$$R_{\text{tot}} = R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(2.5\,\text{Btu/h.ft}^2.\text{°F})[\pi(0.083/12\text{ft})(1\text{ft})]} = 18.4\,\text{h.°F/Btu}$$



With insulation, the total thermal resistance is

$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(2.5\,\text{Btu/h.ft}^2.\text{°F})[\pi(0.123/12\text{ft})(1\text{ft})]} = 12.42\,\text{h.°F/Btu}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(0.123/0.083)}{2\pi(0.075\,\text{Btu/h.ft.°F})(1\,\text{ft})} = 0.835\,\text{h.°F/Btu}$$

$$R_{\text{interface}} = \frac{h_c}{A_c} = \frac{0.001\,\text{h.ft}^2.\text{°F/Btu}}{[\pi(0.083/12\text{ft})(1\text{ft})]} = 0.046\,\text{h.°F/Btu}$$

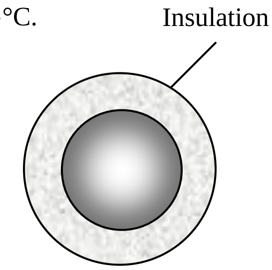
$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} + R_{\text{interface}} = 12.42 + 0.835 + 0.046 = 13.30\,\text{h.°F/Btu}$$

Since the total thermal resistance decreases after insulation, plastic insulation **will increase** heat transfer from the wire. The thermal contact resistance appears to have negligible effect in this case.

3-91 A spherical ball is covered with 1-mm thick plastic insulation. It is to be determined if the plastic insulation on the ball will increase or decrease heat transfer from it.

Assumptions **1** Heat transfer from the ball is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be k = 0.13 W/m·°C.



Analysis The critical radius of plastic insulation for the spherical ball is

$$r_{cr} = \frac{2k}{h} = \frac{2(0.13 \text{ W/m.°C})}{20 \text{ W/m}^2\text{.°C}} = 0.013 \text{ m} = 13 \text{ mm} > r_2 (= 7 \text{ mm})$$

Since the outer temperature of the ball with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

3-92

```
"GIVEN"
D_1=0.005 "[m]"
"t_ins=1 [mm], parameter to be varied"
k_ins=0.13 "[W/m-C]"
T_ball=50 "[C]"
T_infinity=15 "[C]"
h_o=20 "[W/m^2-C]"

"ANALYSIS"
D_2=D_1+2*t_ins*Convert(mm, m)
A_o=pi*D_2^2
R_conv_o=1/(h_o*A_o)
R_ins=(r_2-r_1)/(4*pi*r_1*r_2*k_ins)
r_1=D_1/2
r_2=D_2/2
R_total=R_conv_o+R_ins
Q_dot=(T_ball-T_infinity)/R_total
```

t_{ins} [mm]	Q [W]
0.5	0.07248
1.526	0.1035
2.553	0.1252
3.579	0.139
4.605	0.1474
5.632	0.1523
6.658	0.1552
7.684	0.1569
8.711	0.1577
9.737	0.1581
10.76	0.1581
11.79	0.158
12.82	0.1578
13.84	0.1574
14.87	0.1571
15.89	0.1567
16.92	0.1563
17.95	0.1559
18.97	0.1556
20	0.1552

Heat Transfer From Finned Surfaces

3-93C Increasing the rate of heat transfer from a surface by increasing the heat transfer surface area.

3-94C The fin efficiency is defined as the ratio of actual heat transfer rate from the fin to the ideal heat transfer rate from the fin if the entire fin were at base temperature, and its value is between 0 and 1. Fin effectiveness is defined as the ratio of heat transfer rate from a finned surface to the heat transfer rate from the same surface if there were no fins, and its value is expected to be greater than 1.

3-95C Heat transfer rate will decrease since a fin effectiveness smaller than 1 indicates that the fin acts as insulation.

3-96C Fins enhance heat transfer from a surface by increasing heat transfer surface area for convection heat transfer. However, adding too many fins on a surface can suffocate the fluid and retard convection, and thus it may cause the overall heat transfer coefficient and heat transfer to decrease.

3-97C Effectiveness of a single fin is the ratio of the heat transfer rate from the entire exposed surface of the fin to the heat transfer rate from the fin base area. The overall effectiveness of a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins.

3-98C Fins should be attached on the air side since the convection heat transfer coefficient is lower on the air side than it is on the water side.

3-99C Fins should be attached to the outside since the heat transfer coefficient inside the tube will be higher due to forced convection. Fins should be added to both sides of the tubes when the convection coefficients at the inner and outer surfaces are comparable in magnitude.

3-100C Welding or tight fitting introduces thermal contact resistance at the interface, and thus retards heat transfer. Therefore, the fins formed by casting or extrusion will provide greater enhancement in heat transfer.

3-101C If the fin is too long, the temperature of the fin tip will approach the surrounding temperature and we can neglect heat transfer from the fin tip. Also, if the surface area of the fin tip is very small compared to the total surface area of the fin, heat transfer from the tip can again be neglected.

3-102C Increasing the length of a fin decreases its efficiency but increases its effectiveness.

3-103C Increasing the diameter of a fin will increase its efficiency but decrease its effectiveness.

3-104C The thicker fin will have higher efficiency; the thinner one will have higher effectiveness.

3-105C The fin with the lower heat transfer coefficient will have the higher efficiency and the higher effectiveness.

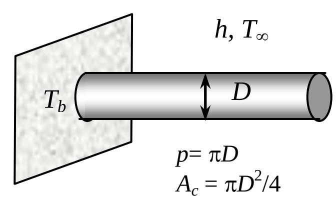
3-106 A relation is to be obtained for the fin efficiency for a fin of constant cross-sectional area A_s, perimeter p, length L, and thermal conductivity k exposed to convection to a medium at T_∞ with a heat transfer coefficient h. The relation is to be simplified for circular fin of diameter D and for a rectangular fin of thickness t.

Assumptions **1** The fins are sufficiently long so that the temperature of the fin at the tip is nearly T_∞. **2** Heat transfer from the fin tips is negligible.

Analysis Taking the temperature of the fin at the base to be T_b and using the heat transfer relation for a long fin, fin efficiency for long fins can be expressed as

$$\eta_{\text{fin}} = \frac{\text{Actual heat transfer rate from the fin}}{\substack{\text{Ideal heat transfer rate from the fin} \\ \text{if the entire fin were at base temperature}}}$$

$$= \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{\sqrt{hpkA_c}}{hpL} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}}$$



This relation can be simplified for a circular fin of diameter D and rectangular fin of thickness t and width w to be

$$\eta_{\text{fin,circular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(\pi D^2/4)}{(\pi D)h}} = \frac{1}{2L}\sqrt{\frac{kD}{h}}$$

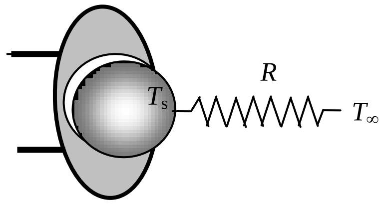
$$\eta_{\text{fin,rectangular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(wt)}{2(w+t)h}} \cong \frac{1}{L}\sqrt{\frac{k(wt)}{2wh}} = \frac{1}{L}\sqrt{\frac{kt}{2h}}$$

3-107 The maximum power rating of a transistor whose case temperature is not to exceed 80°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 80°C.

Properties The case-to-ambient thermal resistance is given to be 20°C/W.

Analysis The maximum power at which this transistor can be operated safely is

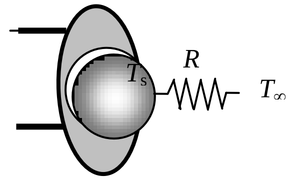


$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} = \frac{T_{\text{case}} - T_\infty}{R_{\text{case-ambient}}} = \frac{(80-40)\ °\text{C}}{25\ °\text{C/W}} = \mathbf{1.6\ W}$$

3-108 A commercially available heat sink is to be selected to keep the case temperature of a transistor below 90°C in an environment at 20°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be



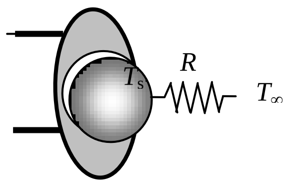
$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} \longrightarrow R_{\text{case-ambient}} = \frac{T_{\text{transistor}} - T_\infty}{\dot{Q}} = \frac{(90-20)°\text{C}}{40\ \text{W}} = \mathbf{1.75\ °C/W}$$

The thermal resistance of the heat sink must be below $1.75\ ^\circ\text{C/W}$. Table 3-4 reveals that HS6071 in vertical position, HS5030 and HS6115 in both horizontal and vertical position can be selected.

3-109 A commercially available heat sink is to be selected to keep the case temperature of a transistor below 80°C in an environment at 35°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 80°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be



$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} \longrightarrow R_{\text{case-ambient}} = \frac{T_{\text{transistor}} - T_\infty}{\dot{Q}} = \frac{(80-35)°\text{C}}{30\ \text{W}} = \mathbf{1.5\ °C/W}$$

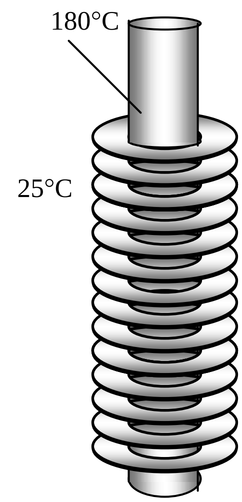
The thermal resistance of the heat sink must be below $1.5\ ^\circ\text{C/W}$. Table 3-4 reveals that HS5030 in both horizontal and vertical positions, HS6071 in vertical position, and HS6115 in both horizontal and vertical positions can be selected.

3-110 Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be k = 186 W/m·°C.

Analysis In case of no fins, heat transfer from the tube per meter of its length is



$$A_{\text{no fin}} = \pi D_1 L = \pi(0.05\text{ m})(1\text{ m}) = 0.1571\text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (40\text{ W/m}^2.°\text{C})(0.1571\text{ m}^2)(180-25)°\text{C} = 974\text{ W}$$

The efficiency of these circular fins is, from the efficiency curve,

$$\left.\begin{aligned} &L = (D_2 - D_1)/2 = (0.06-0.05)/2 = 0.005\text{m} \\ &\frac{r_2 + (t/2)}{r_1} = \frac{0.03+(0.001/2)}{0.025} = 1.22 \\ &\left(L+\frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.005+\frac{0.001}{2}\right)\sqrt{\frac{40\text{ W/m}^2\,{}^\circ\text{C}}{(186\text{ W/m}^\circ\text{C})(0.001\text{m})}} = 0.08 \end{aligned}\right\}\eta_{\text{fin}} = 0.97$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi(r_2^{\,2} - r_1^{\,2}) + 2\pi r_2 t = 2\pi(0.03^2 - 0.025^2) + 2\pi(0.03)(0.001) = 0.001916\text{ m}^2$$

$$\begin{aligned} \dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= 0.97(40\text{ W/m}^2.°\text{C})(0.001916\text{ m}^2)(180-25)°\text{C} \\ &= 11.53\text{ W} \end{aligned}$$

Heat transfer from a single unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 s = \pi(0.05\text{ m})(0.003\text{ m}) = 0.0004712\text{ m}^2$$

$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (40\text{ W/m}^2.°\text{C})(0.0004712\text{ m}^2)(180-25)°\text{C} = 2.92\text{ W}$$

There are 250 fins and thus 250 interfin spacings per meter length of the tube. The total heat transfer from the finned tube is then determined from

$$\dot{Q}_{\text{total,fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 250(11.53+2.92) = 3613\text{ W}$$

Therefore the increase in heat transfer from the tube per meter of its length as a result of the addition of the fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total,fin}} - \dot{Q}_{\text{no fin}} = 3613 - 974 = \mathbf{2639\text{ W}}$$

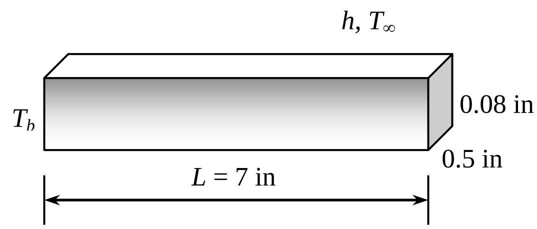
3-111E The handle of a stainless steel spoon partially immersed in boiling water extends 7 in. in the air from the free surface of the water. The temperature difference across the exposed surface of the spoon handle is to be determined.

Assumptions **1** The temperature of the submerged portion of the spoon is equal to the water temperature. **2** The temperature in the spoon varies in the axial direction only (along the spoon), $T(x)$. **3** The heat transfer from the tip of the spoon is negligible. **4** The heat transfer coefficient is constant and uniform over the entire spoon surface. **5** The thermal properties of the spoon are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the spoon..

Properties The thermal conductivity of the spoon is given to be k = 8.7 Btu/h·ft·°F.

Analysis Noting that the cross-sectional area of the spoon is constant and measuring x from the free surface of water, the variation of temperature along the spoon can be expressed as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh a(L-x)}{\cosh aL}$$



where

$$p = 2(0.5/12\ \text{ft} + 0.08/12\ \text{ft}) = 0.0967\ \text{ft}$$

$$A_c = (0.5/12\ \text{ft})(0.08/12\ \text{ft}) = 0.000278\ \text{ft}^2$$

$$a=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{(3\ \text{Btu/h.ft}^2.°\text{F})(0.0967\ \text{ft})}{(8.7\ \text{Btu/h.ft.°F})(0.000278\ \text{ft}^2)}}=10.95\ \text{ft}^{-1}$$

Noting that $x = L$ = 7/12=0.583 ft at the tip and substituting, the tip temperature of the spoon is determined to be

$$T(L)=T_\infty+(T_b-T_\infty)\frac{\cosh a(L-L)}{\cosh aL}$$

$$=75°\text{F}+(200-75)\frac{\cosh 0}{\cosh(10.95\times 0.583)}=75°\text{F}+(200-75)\frac{1}{296}=75.4°\text{F}$$

Therefore, the temperature difference across the exposed section of the spoon handle is

$$\Delta T=T_b-T_{\text{tip}}=(200-75.4)°\text{F}=\mathbf{124.6°F}$$

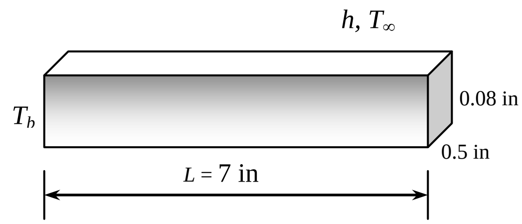
3-112E The handle of a silver spoon partially immersed in boiling water extends 7 in. in the air from the free surface of the water. The temperature difference across the exposed surface of the spoon handle is to be determined.

Assumptions **1** The temperature of the submerged portion of the spoon is equal to the water temperature. **2** The temperature in the spoon varies in the axial direction only (along the spoon), $T(x)$. **3** The heat transfer from the tip of the spoon is negligible. **4** The heat transfer coefficient is constant and uniform over the entire spoon surface. **5** The thermal properties of the spoon are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the spoon..

Properties The thermal conductivity of the spoon is given to be k = 247 Btu/h·ft·°F.

Analysis Noting that the cross-sectional area of the spoon is constant and measuring x from the free surface of water, the variation of temperature along the spoon can be expressed as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh a(L-x)}{\cosh aL}$$



where

$$p=2(0.5/12\text{ ft}+0.08/12\text{ ft})=0.0967\text{ ft}$$

$$A_c=(0.5/12\text{ ft})(0.08/12\text{ ft})=0.000278\text{ ft}^2$$

$$a=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{(3\text{ Btu/h.ft}^2.°\text{F})(0.0967\text{ ft})}{(247\text{ Btu/h.ft.°F})(0.000278\text{ ft}^2)}}=2.055\text{ ft}^{-1}$$

Noting that $x = L$ = 0.7/12=0.583 ft at the tip and substituting, the tip temperature of the spoon is determined to be

$$T(L)=T_\infty+(T_b-T_\infty)\frac{\cosh a(L-L)}{\cosh aL}$$

$$=75°\text{F}+(200-75)\frac{\cosh 0}{\cosh(2.055\times 0.583)}=75°\text{F}+(200-75)\frac{1}{1.81}=\mathbf{144.1°F}$$

Therefore, the temperature difference across the exposed section of the spoon handle is

$$\Delta T=T_b-T_{\text{tip}}=(200-144.1)°\text{C}=\mathbf{55.9°F}$$

3-113

```
"GIVEN"
k_spoon=8.7 "[Btu/h-ft-F], parameter to be varied"
T_w=200 "[F]"
T_infinity=75 "[F]"
A_c=0.08/12*0.5/12 "[ft^2]"
"L=7 [in], parameter to be varied"
h=3 "[Btu/h-ft^2-F]"

"ANALYSIS"
p=2*(0.08/12+0.5/12)
a=sqrt((h*p)/(k_spoon*A_c))
(T_tip-T_infinity)/(T_w-T_infinity)=cosh(a*(L-x)*Convert(in, ft))/cosh(a*L*Convert(in, ft))
x=L "for tip temperature"
DELTAT=T_w-T_tip
```

k_{spoon} [Btu/h.ft.F]	ΔT [F]
5	124.9
16.58	122.6
28.16	117.8
39.74	112.5
51.32	107.1
62.89	102
74.47	97.21
86.05	92.78
97.63	88.69
109.2	84.91
120.8	81.42
132.4	78.19
143.9	75.19
155.5	72.41
167.1	69.82
178.7	67.4
190.3	65.14
201.8	63.02
213.4	61.04
225	59.17

k_{spoon} [Btu/h.ft.F]	ΔT [F]
5	122.4
5.5	123.4
6	124
6.5	124.3
7	124.6
7.5	124.7
8	124.8
8.5	124.9
9	124.9
9.5	125
10	125
10.5	125
11	125
11.5	125
12	125

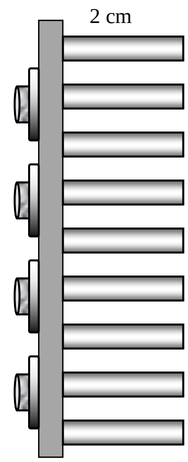
3-114 A circuit board houses 80 logic chips on one side, dissipating 0.04 W each through the back side of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 864 aluminum pin fins on the back surface.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the back side of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

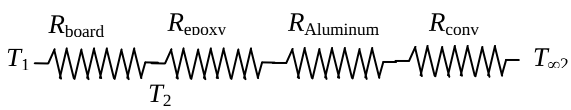
Properties The thermal conductivities are given to be k = 20 W/m·°C for the circuit board, k = 237 W/m·°C for the aluminum plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (*a*) The total rate of heat transfer dissipated by the chips is

$$\dot{Q} = 80 \times (0.04\ \text{W}) = 3.2\ \text{W}$$



The individual resistances are



$$A = (0.12\ \text{m})(0.18\ \text{m}) = 0.0216\ \text{m}^2$$

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.003\ \text{m}}{(20\ \text{W/m.°C})(0.0216\ \text{m}^2)} = 0.00694\ \text{°C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(50\ \text{W/m}^2.\text{°C})(0.0216\ \text{m}^2)} = 0.9259\ \text{°C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.00694 + 0.9259 = 0.93284\ \text{°C/W}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40\text{°C} + (3.2\ \text{W})(0.93284\ \text{°C/W}) = \mathbf{43.0°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 43.0\text{°C} - (3.2\ \text{W})(0.00694\ \text{°C/W}) = 40.5 - 0.02 \cong \mathbf{43.0°C}$$

Therefore, the board is nearly isothermal.

(*b*) Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(50\ \text{W/m}^2.\text{°C})}{(237\ \text{W/m.°C})(0.0025\ \text{m})}} = 18.37\ \text{m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(18.37\ \text{m}^{-1} \times 0.02\ \text{m})}{18.37\ \text{m}^{-1} \times 0.02\ \text{m}} = 0.957$$

The fins can be assumed to be at base temperature provided that the fin area is modified by multiplying it by 0.957. Then the various thermal resistances are

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0002\ \text{m}}{(1.8\ \text{W/m.°C})(0.0216\ \text{m}^2)} = 0.0051\ \text{°C/W}$$

$$R_{\text{Al}} = \frac{L}{kA} = \frac{0.002\ \text{m}}{(237\ \text{W/m.°C})(0.0216\ \text{m}^2)} = 0.00039\ \text{°C/W}$$

$$A_{\text{finned}} = \eta_{\text{fin}} n\pi DL = 0.957 \times 864\pi(0.0025\text{ m})(0.02\text{ m}) = 0.130\text{ m}^2$$

$$A_{\text{unfinned}} = 0.0216 - 864\frac{\pi D^2}{4} = 0.0216 - 864 \times \frac{\pi(0.0025)^2}{4} = 0.0174\text{ m}^2$$

$$A_{\text{total,with fins}} = A_{\text{finned}} + A_{\text{unfinned}} = 0.130 + 0.017 = 0.147\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_{\text{total,with fins}}} = \frac{1}{(50\text{ W/m}^2.°\text{C})(0.147\text{ m}^2)} = 0.1361\ °\text{C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{epoxy}} + R_{\text{aluminum}} + R_{\text{conv}}$$
$$= 0.00694 + 0.0051 + 0.00039 + 0.1361 = 0.1484\ °\text{C/W}$$

Then the temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40°\text{C} + (3.2\text{ W})(0.1484\ °\text{C/W}) = \mathbf{40.5°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 40.5°\text{C} - (3.2\text{ W})(0.00694\ °\text{C/W}) = 40.5 - 0.02 \cong \mathbf{40.5°C}$$

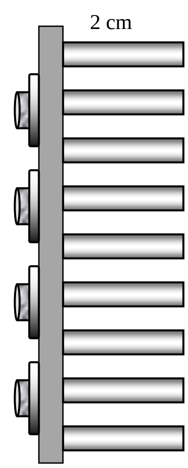
3-115 A circuit board houses 80 logic chips on one side, dissipating 0.04 W each through the back side of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 864 copper pin fins on the back surface.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the back side of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

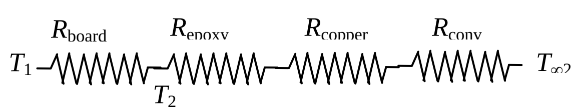
Properties The thermal conductivities are given to be k = 20 W/m·°C for the circuit board, k = 386 W/m·°C for the copper plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (*a*) The total rate of heat transfer dissipated by the chips is

$$\dot{Q} = 80 \times (0.04\text{ W}) = 3.2\text{ W}$$



The individual resistances are



$$A = (0.12\text{ m})(0.18\text{ m}) = 0.0216\text{ m}^2$$

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.003\text{ m}}{(20\text{ W/m.°C})(0.0216\text{ m}^2)} = 0.00694\ °\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(50\text{ W/m}^2.°\text{C})(0.0216\text{ m}^2)} = 0.9259\ °\text{C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.00694 + 0.9259 = 0.93284\ °\text{C/W}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40°\text{C} + (3.2\ \text{W})(0.93284\ °\text{C/W}) = \mathbf{43.0°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 43.0°\text{C} - (3.2\ \text{W})(0.00694\ °\text{C/W}) = 40.5 - 0.02 \cong \mathbf{43.0°C}$$

Therefore, the board is nearly isothermal.

(*b*) Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(50\ \text{W/m}^2.°\text{C})}{(386\ \text{W/m}.°\text{C})(0.0025\ \text{m})}} = 14.40\ \text{m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(14.40\ \text{m}^{-1} \times 0.02\ \text{m})}{14.40\ \text{m}^{-1} \times 0.02\ \text{m}} = 0.973$$

The fins can be assumed to be at base temperature provided that the fin area is modified by multiplying it by 0.973. Then the various thermal resistances are

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0002\ \text{m}}{(1.8\ \text{W/m}.°\text{C})(0.0216\ \text{m}^2)} = 0.0051\ °\text{C/W}$$

$$R_{\text{copper}} = \frac{L}{kA} = \frac{0.002\ \text{m}}{(386\ \text{W/m}.°\text{C})(0.0216\ \text{m}^2)} = 0.00024\ °\text{C/W}$$

$$A_{\text{finned}} = \eta_{\text{fin}} n\pi DL = 0.973 \times 864\pi(0.0025\ \text{m})(0.02\ \text{m}) = 0.132\ \text{m}^2$$

$$A_{\text{unfinned}} = 0.0216 - 864\frac{\pi D^2}{4} = 0.0216 - 864 \times \frac{\pi(0.0025)^2}{4} = 0.0174\ \text{m}^2$$

$$A_{\text{total,with fins}} = A_{\text{finned}} + A_{\text{unfinned}} = 0.132 + 0.017 = 0.149\ \text{m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_{\text{total,with fins}}} = \frac{1}{(50\ \text{W/m}^2.°\text{C})(0.149\ \text{m}^2)} = 0.1342\ °\text{C/W}$$

$$\begin{aligned} R_{\text{total}} &= R_{\text{board}} + R_{\text{epoxy}} + R_{\text{copper}} + R_{\text{conv}} \\ &= 0.00694 + 0.0051 + 0.00024 + 0.1342 = 0.1465\ °\text{C/W} \end{aligned}$$

Then the temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40°\text{C} + (3.2\ \text{W})(0.1465\ °\text{C/W}) = \mathbf{40.5°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 40.5°\text{C} - (3.2\ \text{W})(0.00694\ °\text{C/W}) = 40.5 - 0.02 \cong \mathbf{40.5°C}$$

3-116 A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

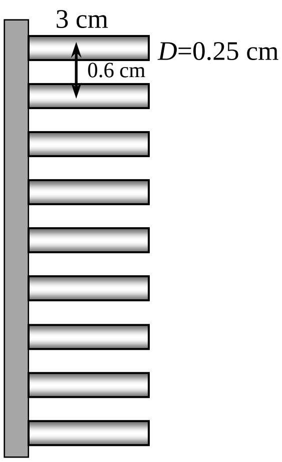
Properties The thermal conductivity of the aluminum plate and fins is given to be k = 237 W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(35\text{ W/m}^2.^\circ\text{C})}{(237\text{ W/m.}^\circ\text{C})(0.0025\text{ m})}} = 15.37\text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(15.37\text{ m}^{-1}\times 0.03\text{ m})}{15.37\text{ m}^{-1}\times 0.03\text{ m}} = 0.935$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are



$$n = \frac{1\text{ m}^2}{(0.006\text{ m})(0.006\text{ m})} = 27,777$$

$$A_{\text{fin}} = 27777\left[\pi DL + \frac{\pi D^2}{4}\right] = 27777\left[\pi(0.0025)(0.03) + \frac{\pi(0.0025)^2}{4}\right] = 6.68\text{ m}^2$$

$$A_{\text{unfinned}} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left[\frac{\pi(0.0025)^2}{4}\right] = 0.86\text{ m}^2$$

$$\begin{aligned}\dot{Q}_{\text{finned}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}}hA_{\text{fin}}(T_b - T_\infty)\\ &= 0.935(35\text{ W/m}^2.^\circ\text{C})(6.68\text{ m}^2)(100-30)^\circ\text{C}\\ &= 15,300\text{ W}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\text{unfinned}} &= hA_{\text{unfinned}}(T_b - T_\infty) = (35\text{ W/m}^2.^\circ\text{C})(0.86\text{ m}^2)(100-30)^\circ\text{C}\\ &= 2107\text{ W}\end{aligned}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 15,300 + 2107 = 1.74\times 10^4\text{ W} = \mathbf{17.4\,kW}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (1\text{ m})(1\text{ m}) = 1\text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (35\text{ W/m}^2.^\circ\text{C})(1\text{ m}^2)(100-30)^\circ\text{C} = 2450\text{ W}$$

Then the fin effectiveness becomes

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{17,400}{2450} = \mathbf{7.10}$$

3-117 A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

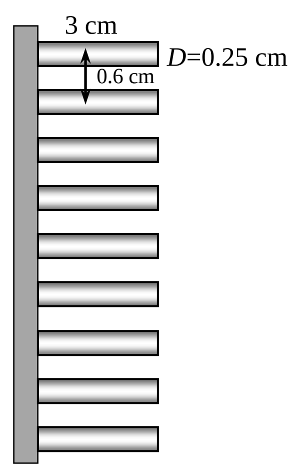
Properties The thermal conductivity of the aluminum plate and fins is given to be k = 237 W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(35\text{ W/m}^2.^\circ\text{C})}{(386\text{ W/m.}^\circ\text{C})(0.0025\text{ m})}} = 12.04\text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(12.04\text{ m}^{-1}\times 0.03\text{ m})}{12.04\text{ m}^{-1}\times 0.03\text{ m}} = 0.959$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are



$$n = \frac{1\text{ m}^2}{(0.006\text{ m})(0.006\text{ m})} = 27777$$

$$A_{\text{fin}} = 27777\left[\pi DL + \frac{\pi D^2}{4}\right] = 27777\left[\pi(0.0025)(0.03) + \frac{\pi(0.0025)^2}{4}\right] = 6.68\text{ m}^2$$

$$A_{\text{unfinned}} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left[\frac{\pi(0.0025)^2}{4}\right] = 0.86\text{ m}^2$$

$$\begin{aligned}\dot{Q}_{\text{finned}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)\\ &= 0.959(35\text{ W/m}^2.^\circ\text{C})(6.68\text{ m}^2)(100-30)^\circ\text{C}\\ &= 15{,}700\text{ W}\end{aligned}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_b - T_\infty) = (35\text{ W/m}^2\,^\text{o}\text{C})(0.86\text{ m}^2)(100-30)^\circ\text{C} = 2107\text{ W}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 15{,}700 + 2107 = 1.78\times 10^4\text{ W} = \mathbf{17.8\ W}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (1\text{ m})(1\text{ m}) = 1\text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (35\text{ W/m}^2.^\circ\text{C})(1\text{ m}^2)(100-30)^\circ\text{C} = 2450\text{ W}$$

Then the fin effectiveness becomes

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{17800}{2450} = \mathbf{7.27}$$

3-118

```
"GIVEN"
k_spoon=8.7 "[Btu/h-ft-F], parameter to be varied"
T_w=200 "[F]"
T_infinity=75 "[F]"
A_c=0.08/12*0.5/12 "[ft^2]"
"L=7 [in], parameter to be varied"
h=3 "[Btu/h-ft^2-F]"

"ANALYSIS"
p=2*(0.08/12+0.5/12)
a=sqrt((h*p)/(k_spoon*A_c))
(T_tip-T_infinity)/(T_w-T_infinity)=cosh(a*(L-x)*Convert(in, ft))/cosh(a*L*Convert(in, ft))
x=L "for tip temperature"
DELTAT=T_w-T_tip
```

k_{spoon} [Btu/h.ft.F]	ΔT [F]
5	124.9
16.58	122.6
28.16	117.8
39.74	112.5
51.32	107.1
62.89	102
74.47	97.21
86.05	92.78
97.63	88.69
109.2	84.91
120.8	81.42
132.4	78.19
143.9	75.19
155.5	72.41
167.1	69.82
178.7	67.4
190.3	65.14
201.8	63.02
213.4	61.04
225	59.17

k_{spoon} [Btu/h.ft.F]	ΔT [F]
5	122.4
5.5	123.4
6	124
6.5	124.3
7	124.6
7.5	124.7
8	124.8
8.5	124.9
9	124.9
9.5	125
10	125
10.5	125
11	125
11.5	125
12	125

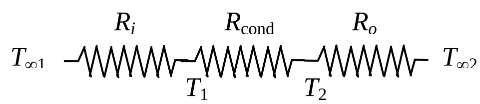
3-119 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges exposed to cold ambient air. The average outer surface temperature of the pipe, the fin efficiency, the rate of heat transfer from the flanges, and the equivalent pipe length of the flange for heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the flanges (fins) varies in one direction only (normal to the pipe). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the cast iron is given to be k = 52 W/m·°C.

Analysis (*a*) We treat the flanges as fins. The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.092\text{ m})(6\text{ m}) = 1.73\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.1\text{ m})(6\text{ m}) = 1.88\text{ m}^2$$



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(180\text{ W/m}^2.°\text{C})(1.73\text{ m}^2)} = 0.0032°\text{C/W}$$
$$R_{cond} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(5/4.6)}{2\pi(52\text{ W/m.°C})(6\text{ m})} = 0.00004°\text{C/W}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(1.88\text{ m}^2)} = 0.0213°\text{C/W}$$
$$R_{total} = R_i + R_{cond} + R_o = 0.0032 + 0.00004 + 0.0213 = 0.0245°\text{C/W}$$

The rate of heat transfer and average outer surface temperature of the pipe are

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(200-12)°\text{C}}{0.0245°\text{C}} = 7673\text{ W}$$
$$\dot{Q} = \frac{T_2 - T_{\infty2}}{R_o} \longrightarrow T_2 = T_{\infty2} + \dot{Q}R_o = 12\,°\text{C} + (7673\text{ W})(0.0213°\text{C/W}) = \mathbf{174.8°C}$$

(*b*) The fin efficiency can be determined from Fig. 3-70 to be

$$\left.\begin{aligned} &\frac{r_2 + \frac{t}{2}}{r_1} = \frac{0.1 + \frac{0.02}{2}}{0.05} = 2.23 \\ &\xi = \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.05\text{ m} + \frac{0.02}{2}\text{ m}\right)\sqrt{\frac{25\text{ W/m}^2\,°\text{C}}{(52\text{ W/m}°\text{C})(0.02\text{ m})}} = 0.29 \end{aligned}\right\}\eta_{fin} = 0.88$$

$$A_{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi[(0.1\text{ m})^2 - (0.05\text{ m})^2] + 2\pi(0.1\text{ m})(0.02\text{ m}) = 0.0597\text{ m}^2$$

The heat transfer rate from the flanges is

$$\begin{aligned}\dot{Q}_{finned} &= \eta_{fin}\dot{Q}_{fin,max} = \eta_{fin} h A_{fin}(T_b - T_\infty) \\ &= 0.88(25\text{ W/m}^2.°\text{C})(0.0597\text{ m}^2)(174.7-12)°\text{C} = \mathbf{214\ W}\end{aligned}$$

(*c*) A 6-m long section of the steam pipe is losing heat at a rate of 7673 W or 7673/6 = 1279 W per m length. Then for heat transfer purposes the flange section is equivalent to

$$\text{Equivalent legth} = \frac{214\text{ W}}{1279\text{ W/m}} = 0.167\text{ m} = \mathbf{16.7\,cm}$$

Therefore, the flange acts like a fin and increases the heat transfer by 16.7/2 = 8.35 times.