Heat Transfer In Common Configurations

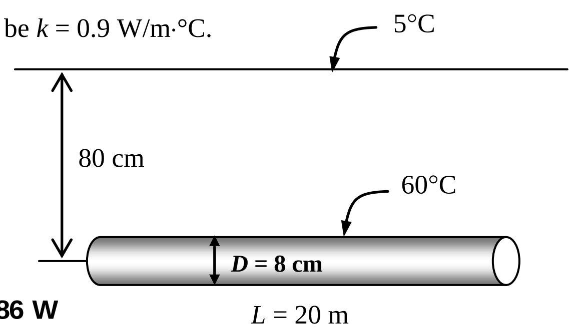
3-120C Under steady conditions, the rate of heat transfer between two surfaces is expressed as $\dot{Q} = Sk(T_1 - T_2)$ where S is the conduction shape factor. It is related to the thermal resistance by $S=1/(kR)$.

3-121C It provides an easy way of calculating the steady rate of heat transfer between two isothermal surfaces in common configurations.

3-122 The hot water pipe of a district heating system is buried in the soil. The rate of heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be k = 0.9 W/m·°C.



Analysis Since z>1.5D, the shape factor for this configuration is given in Table 3-5 to be

$$S = \frac{2\pi L}{\ln(4z/D)} = \frac{2\pi(20\,\text{m})}{\ln[4(0.8\,\text{m})/(0.08\,\text{m})]} = 34.07\,\text{m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (34.07\,\text{m})(0.9\,\text{W/m.}^\circ\text{C})(60-5)^\circ\text{C} = \mathbf{1686\ W}$$

3-123

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"!PROBLEM 3-123"

"GIVEN"
L=20 "[m]"
D=0.08 "[m]"
"z=0.80 [m], parameter to be varied"
T_1=60 "[C]"
T_2=5 "[C]"
k=0.9 "[W/m-C]"

"ANALYSIS"
S=(2*pi*L)/ln(4*z/D)
Q_dot=S*k*(T_1-T_2)
```

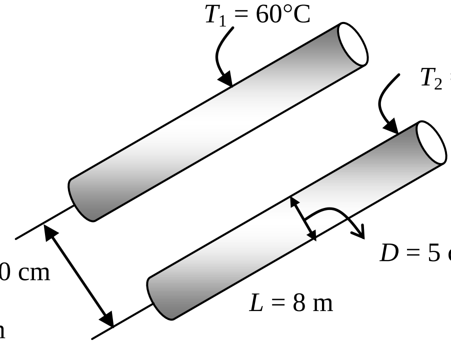
z [m]	Q [W]
0.2	2701
0.38	2113
0.56	1867
0.74	1723
0.92	1625
1.1	1552
1.28	1496
1.46	1450
1.64	1412
1.82	1379
2	1351

3-124 Hot and cold water pipes run parallel to each other in a thick concrete layer. The rate of heat transfer between the pipes is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-5 to be



$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^{\ 2} - D_2^{\ 2}}{2D_1 D_2}\right)}$$

$$= \frac{2\pi(8\text{ m})}{\cosh^{-1}\left(\dfrac{4(0.4\text{ m})^2 - (0.05\text{ m})^2 - (0.05\text{ m})^2}{2(0.05\text{ m})(0.05\text{ m})}\right)} = 9.078\text{ m}$$

Then the steady rate of heat transfer between the pipes becomes

$$\dot{Q} = Sk(T_1 - T_2) = (9.078\text{ m})(0.75\text{ W/m.°C})(60-15)\text{°C} = \mathbf{306\ W}$$

3-125

```
"!PROBLEM 3-125"
"GIVEN"
L=8 "[m]"
D_1=0.05 "[m]"
D_2=D_1
"z=0.40 [m], parameter to be varied"
T_1=60 "[C]"
T_2=15 "[C]"
k=0.75 "[W/m-C]"

"ANALYSIS"
S=(2*pi*L)/(arccosh((4*z^2-D_1^2-D_2^2)/(2*D_1*D_2)))
Q_dot=S*k*(T_1-T_2)
```

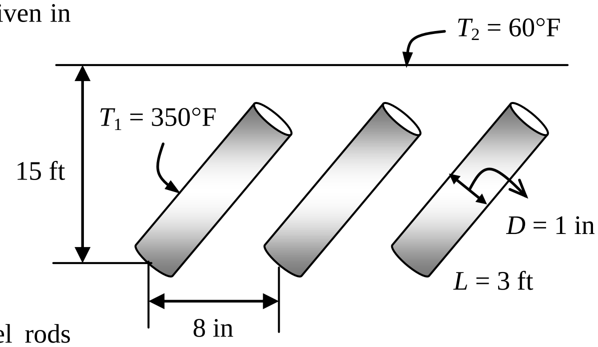
z [m]	Q [W]
0.1	644.1
0.2	411.1
0.3	342.3
0.4	306.4
0.5	283.4
0.6	267
0.7	254.7
0.8	244.8
0.9	236.8
1	230

3-126E A row of used uranium fuel rods are buried in the ground parallel to each other. The rate of heat transfer from the fuel rods to the atmosphere through the soil is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be k = 0.6 Btu/h·ft·°F.

Analysis The shape factor for this configuration is given in Table 3-5 to be



$$S_{\text{total}} = 4\times\frac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$$

$$= 4\times\frac{2\pi(3\text{ ft})}{\ln\left(\dfrac{2(8/12\text{ ft})}{\pi(1/12\text{ ft})}\sinh\dfrac{2\pi(15\text{ ft})}{(8/12\text{ ft})}\right)} = 0.5298$$

Then the steady rate of heat transfer from the fuel rods becomes

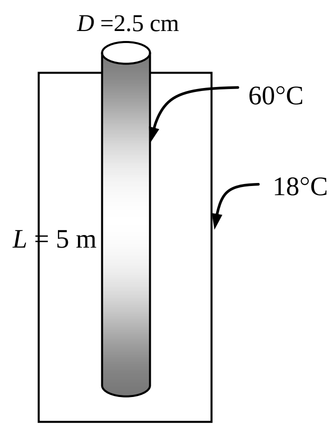
$$\dot{Q} = S_{\text{total}}k(T_1 - T_2) = (0.5298\text{ft})(0.6\text{ Btu/h.ft.°F})(350-60)°\text{C} = \mathbf{92.2 Btu/h}$$

3-127 Hot water flows through a 5-m long section of a thin walled hot water pipe that passes through the center of a 14-cm thick wall filled with fiberglass insulation. The rate of heat transfer from the pipe to the air in the rooms and the temperature drop of the hot water as it flows through the pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the fiberglass insulation is constant. **4** The pipe is at the same temperature as the hot water.

Properties The thermal conductivity of fiberglass insulation is given to be k = 0.035 W/m·°C.

Analysis (a)The shape factor for this configuration is given in Table 3-5 to be



$$S = \frac{2\pi L}{\ln\left(\dfrac{8z}{\pi D}\right)} = \frac{2\pi(5\text{ m})}{\ln\left[\dfrac{8(0.07\text{ m})}{\pi(0.025\text{ m})}\right]} = 16\text{ m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (16\text{ m})(0.035\text{ W/m.°C})(60-18)°\text{C} = \mathbf{23.5\ W}$$

(b) Using the water properties at the room temperature, the temperature drop of the hot water as it flows through this 5-m section of the wall becomes

$$\dot{Q} = \dot{m}C_p\Delta T$$

$$\Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{\dot{Q}}{\rho\dot{V}C_p} = \frac{\dot{Q}}{\rho VA_cC_p} = \frac{23.5\text{ J/s}}{(1000\text{ kg/m}^3)(0.6\text{ m/s})\left[\dfrac{\pi(0.025\text{ m})^2}{4}\right](4180\text{ J/kg.°C})} = \mathbf{0.02°C}$$

3-128 Hot water is flowing through a pipe that extends 2 m in the ambient air and continues in the ground before it enters the next building. The surface of the ground is covered with snow at 0°C. The total rate of heat loss from the hot water and the temperature drop of the hot water in the pipe are to be determined.

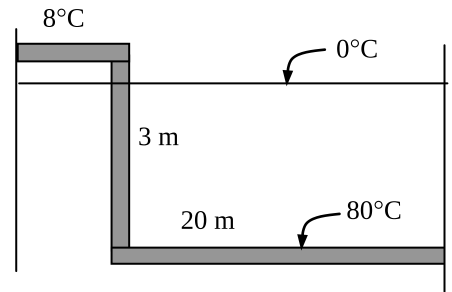
Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** The pipe is at the same temperature as the hot water.

Properties The thermal conductivity of the ground is given to be k = 1.5 W/m·°C.

Analysis (a) We assume that the surface temperature of the tube is equal to the temperature of the water. Then the heat loss from the part of the tube that is on the ground is

$$A_s = \pi DL = \pi(0.05\,\text{m})(2\,\text{m}) = 0.3142\,\text{m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$= (22\,\text{W/m}^2.°\text{C})(0.3142\,\text{m}^2)(80-8)°\text{C} = 498\,\text{W}$$



Considering the shape factor, the heat loss for vertical part of the tube can be determined from

$$S = \frac{2\pi L}{\ln\left(\frac{4L}{D}\right)} = \frac{2\pi(3\,\text{m})}{\ln\left[\frac{4(3\,\text{m})}{(0.05\,\text{m})}\right]} = 3.44\,\text{m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (3.44\text{ m})(1.5\text{ W/m.°C})(80-0)°\text{C} = 413\text{ W}$$

The shape factor, and the rate of heat loss on the horizontal part that is in the ground are

$$S = \frac{2\pi L}{\ln\left(\frac{4z}{D}\right)} = \frac{2\pi(20\,\text{m})}{\ln\left[\frac{4(3\,\text{m})}{(0.05\,\text{m})}\right]} = 22.9\,\text{m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (22.9\text{ m})(1.5\text{ W/m.°C})(80-0)°\text{C} = 2748\text{ W}$$

and the total rate of heat loss from the hot water becomes

$$\dot{Q}_{\text{total}} = 498 + 413 + 2748 = \mathbf{3659\,W}$$

(b) Using the water properties at the room temperature, the temperature drop of the hot water as it flows through this 25-m section of the wall becomes

$$\dot{Q} = \dot{m}C_p\Delta T$$

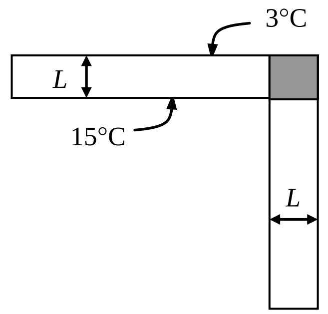
$$\Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{\dot{Q}}{(\rho\dot{V})C_p} = \frac{\dot{Q}}{(\rho VA_c)C_p} = \frac{3659\,\text{J/s}}{(1000\,\text{kg/m}^3)(1.5\,\text{m/s})\left[\frac{\pi(0.05\,\text{m})^2}{4}\right](4180\,\text{J/kg.°C})} = \mathbf{0.30°C}$$

3-129 The walls and the roof of the house are made of 20-cm thick concrete, and the inner and outer surfaces of the house are maintained at specified temperatures. The rate of heat loss from the house through its walls and the roof is to be determined, and the error involved in ignoring the edge and corner effects is to be assessed.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer at the edges and corners is two-or three-dimensional. **3** Thermal conductivity of the concrete is constant. **4** The edge effects of adjoining surfaces on heat transfer are to be considered.

Properties The thermal conductivity of the concrete is given to be k = 0.75 W/m·°C.

Analysis The rate of heat transfer excluding the edges and corners is first determined to be



$$A_{\text{total}} = (12-0.4)(12-0.4)+4(12-0.4)(6-0.2) = 403.7\ \text{m}^2$$

$$\dot{Q} = \frac{kA_{\text{total}}}{L}(T_1-T_2) = \frac{(0.75\ \text{W/m.°C})(403.7\ \text{m}^2)}{0.2\ \text{m}}(15-3)°\text{C} = 18{,}167\ \text{W}$$

The heat transfer rate through the edges can be determined using the shape factor relations in Table 3-5,

$$S_{\text{corners+edges}} = 4\times\text{corners}+4\times\text{edges} = 4\times0.15L+4\times0.54w$$
$$= 4\times0.15(0.2\ \text{m})+4\times0.54(12\text{m}) = 26.04\ \text{m}$$

$$\dot{Q}_{\text{corners+edges}} = S_{\text{corners+edges}}k(T_1-T_2) = (26.04\ \text{m})(0.75\ \text{W/m.°C})(15-3)°\text{C} = 234\ \text{W}$$

and $\dot{Q}_{\text{total}} = 18{,}167+234 = 1.840\times10^4\ \text{W} = \mathbf{18.4\ kW}$

Ignoring the edge effects of adjoining surfaces, the rate of heat transfer is determined from

$$A_{\text{total}} = (12)(12)+4(12)(6) = 432\ \text{m}^2$$

$$\dot{Q} = \frac{kA_{\text{total}}}{L}(T_1-T_2) = \frac{(0.75\ \text{W/m.°C})(432\ \text{m}^2)}{0.2\ \text{m}}(15-3)°\text{C} = 1.94\times10^4 = 19.4\ \text{kW}$$

The percentage error involved in ignoring the effects of the edges then becomes

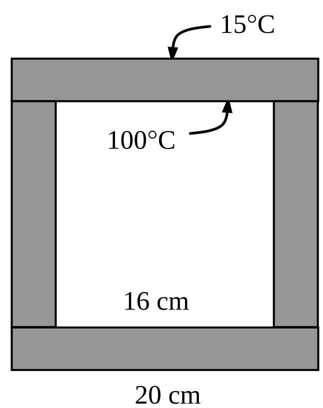
$$\%\text{error} = \frac{19.4-18.4}{18.4}\times100 = \mathbf{5.6\%}$$

3-130 The inner and outer surfaces of a long thick-walled concrete duct are maintained at specified temperatures. The rate of heat transfer through the walls of the duct is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-5 to be



$$\frac{a}{b} = \frac{16}{20} = 0.8 < 1.41 \longrightarrow S = \frac{2\pi L}{0.785\ln\left(\frac{a}{b}\right)} = \frac{2\pi(10\ \text{m})}{0.785\ln 0.8} = 358.7\ \text{m}$$

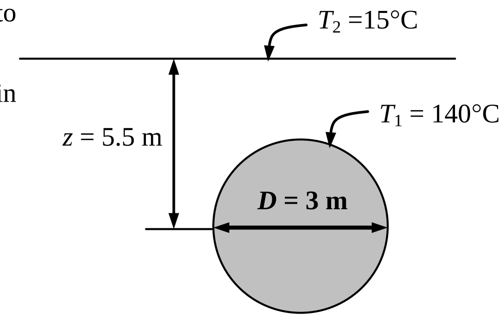
Then the steady rate of heat transfer through the walls of the duct becomes

$$\dot{Q} = Sk(T_1-T_2) = (358.7\ \text{m})(0.75\ \text{W/m.°C})(100-15)°\text{C} = 2.29\times10^4\ \text{W} = \mathbf{22.9\ kW}$$

3-131 A spherical tank containing some radioactive material is buried in the ground. The tank and the ground surface are maintained at specified temperatures. The rate of heat transfer from the tank is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant.

Properties The thermal conductivity of the ground is given to be k = 1.4 W/m·°C.



Analysis The shape factor for this configuration is given in Table 3-5 to be

$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(3\text{ m})}{1 - 0.25\dfrac{3\text{ m}}{5.5\text{ m}}} = 21.83\text{ m}$$

Then the steady rate of heat transfer from the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (21.83\,\text{m})(1.4\text{ W/m.°C})(140 - 15)\text{°C} = \mathbf{3820\ W}$$

3-132

```
"!PROBLEM 3-132"

"GIVEN"
"D=3 [m], parameter to be varied"
k=1.4 "[W/m-C]"
h=4 "[m]"
T_1=140 "[C]"
T_2=15 "[C]"

"ANALYSIS"
z=h+D/2
S=(2*pi*D)/(1-0.25*D/z)
Q_dot=S*k*(T_1-T_2)
```

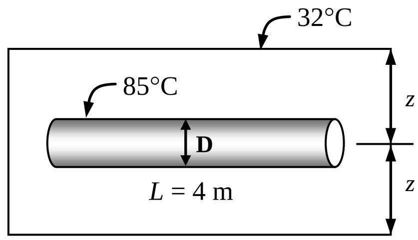
D [m]	Q [W]
0.5	566.4
1	1164
1.5	1791
2	2443
2.5	3120
3	3820
3.5	4539
4	5278
4.5	6034
5	6807

3-133 Hot water passes through a row of 8 parallel pipes placed vertically in the middle of a concrete wall whose surfaces are exposed to a medium at 20°C with a heat transfer coefficient of 8 W/m^2.°C. The rate of heat loss from the hot water, and the surface temperature of the wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-5 to be



$$S = \frac{2\pi L}{\ln\left(\frac{8z}{\pi D}\right)} = \frac{2\pi(4\text{ m})}{\ln\left(\frac{8(0.075\text{ m})}{\pi(0.03\text{ m})}\right)} = 13.58\text{ m}$$

Then rate of heat loss from the hot water in 8 parallel pipes becomes

$$\dot{Q} = 8Sk(T_1 - T_2) = 8(13.58\text{ m})(0.75\text{ W/m.°C})(85-32)\text{°C} = \mathbf{4318\ W}$$

The surface temperature of the wall can be determined from

$$A_s = 2(4\text{ m})(8\text{ m}) = 64\text{ m}^2 \quad \text{(from both sides)}$$

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 32\text{°C} + \frac{4318\text{ W}}{(12\text{ W/m}^2.\text{°C})(64\text{ m}^2)} = \mathbf{37.6°C}$$

Special Topic: Heat Transfer Through the Walls and Roofs

3-134C The *R*-value of a wall is the thermal resistance of the wall per unit surface area. It is the same as the unit thermal resistance of the wall. It is the inverse of the *U*-factor of the wall, $R = 1/U$.

3-135C The effective emissivity for a plane-parallel air space is the "equivalent" emissivity of one surface for use in the relation $\dot{Q}_{\text{rad}} = \varepsilon_{\text{effective}}\sigma A_s(T_2^4 - T_1^4)$ that results in the same rate of radiation heat transfer between the two surfaces across the air space. It is determined from

$$\frac{1}{\varepsilon_{\text{effective}}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1$$

where ε_1 and ε_2 are the emissivities of the surfaces of the air space. When the effective emissivity is known, the radiation heat transfer through the air space is determined from the $\dot{Q}_{\text{rad}}$ relation above.

3-136C The unit thermal resistances (*R*-value) of both 40-mm and 90-mm vertical air spaces are given to be the same, which implies that more than doubling the thickness of air space in a wall has no effect on heat transfer through the wall. This is not surprising since the convection currents that set in in the thicker air space offset any additional resistance due to a thicker air space.

3-137C Radiant barriers are highly reflective materials that minimize the radiation heat transfer between surfaces. Highly reflective materials such as aluminum foil or aluminum coated paper are suitable for use as radiant barriers. Yes, it is worthwhile to use radiant barriers in the attics of homes by covering at least one side of the attic (the roof or the ceiling side) since they reduce radiation heat transfer between the ceiling and the roof considerably.

3-138C The roof of a house whose attic space is ventilated effectively so that the air temperature in the attic is the same as the ambient air temperature at all times will still have an effect on heat transfer through the ceiling since the roof in this case will act as a radiation shield, and reduce heat transfer by radiation.

3-139 The R-value and the U-factor of a wood frame wall are to be determined.

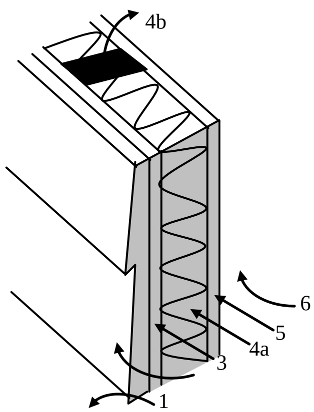
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-6.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the U-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{insulation} + (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available R-values from Table 3-6 and calculating others, the total R-values for each section is determined in the table below.



Construction	R-value, m^2.°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Fiberboard sheathing, 25 mm	0.23	0.23
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, R (in m^2.°C/W)	4.309	1.593
The U-factor of each section, $U = 1/R$, in W/m^2.°C	0.232	0.628
Area fraction of each section, f_{area}	0.80	0.20
Overall U-factor, $U = \Sigma f_{area,i} U_i = 0.80 \times 0.232 + 0.20 \times 0.628$	**0.311 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**3.213 m^2.°C/W**	

Therefore, the R-value and U-factor of the wall are R = 3.213 m^2.°C/W and U = 0.311 W/m^2.°C.

3-140 The change in the *R*-value of a wood frame wall due to replacing fiberwood sheathing in the wall by rigid foam sheathing is to be determined.

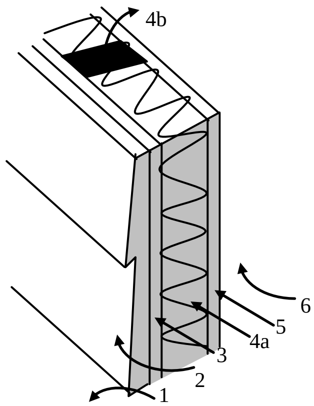
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-6.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{insulation}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 3-6 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, m^2.°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Fiberboard sheathing, 25 mm	0.23	0.23
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, *R* (in m^2.°C/W)	4.309	1.593
The *U*-factor of each section, $U = 1/R$, in W/m^2.°C	0.232	0.628
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}} = 0.80\times0.232+0.20\times0.628$	0.311 W/m^2.°C	
Overall unit thermal resistance, $R = 1/U$	3.213 m^2.°C/W	

Therefore, the *R*-value of the existing wall is $R = 3.213$ m^2.°C/W.

Noting that the *R*-values of the wood fiberboard and the rigid foam insulation are 0.23 m^2.°C/W and 0.98 m^2.°C/W, respectively, and the added and removed thermal resistances are in series, the overall *R*-value of the wall after modification becomes

$$R_{\text{new}} = R_{\text{old}} - R_{\text{removed}} + R_{\text{added}} = 3.213 - 0.23 + 0.98 = 3.963 \text{ m}^2.^\circ\text{C/W}$$

Then the change in the *R*-value becomes

$$\%\text{Change} = \frac{\Delta R - \text{value}}{R - \text{value, old}} = \frac{3.963 - 3.213}{3.963} = 0.189 \quad (\text{or } \mathbf{18.9\%})$$

3-141E The *R*-value and the *U*-factor of a masonry cavity wall are to be determined.

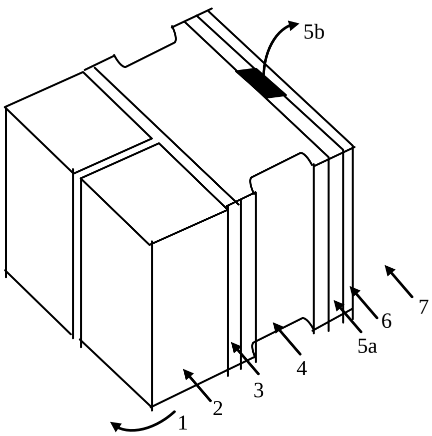
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-6.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{air\ space} + (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.80 for air space and 0.20 for the ferrings and similar structures. Using the available *R*-values from Table 3-6 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, h.ft^2.°F/Btu Between furring	At furring
1. Outside surface, 15 mph wind	0.17	0.17
2. Face brick, 4 in	0.43	0.43
3. Cement mortar, 0.5 in	0.10	0.10
4. Concrete block, 4-in	1.51	1.51
5a. Air space, 3/4-in, nonreflective	2.91	--
5b. Nominal 1 × 3 vertical ferring	--	0.94
6. Gypsum wallboard, 0.5 in	0.45	0.45
7. Inside surface, still air	0.68	0.68

Total unit thermal resistance of each section, *R*	6.25	4.28
The *U*-factor of each section, $U = 1/R$, in Btu/h.ft^2.°F	0.160	0.234
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{area,i} U_i = 0.80\times0.160+0.20\times0.234$	**0.175 Btu/h.ft^2.°F**	
Overall unit thermal resistance, $R = 1/U$	**5.72 h.ft^2.°F/Btu**	

Therefore, the overall unit thermal resistance of the wall is $R = 5.72$ h.ft^2.°F/Btu and the overall U-factor is $U = 0.118$ Btu/h.ft^2.°F. These values account for the effects of the vertical ferring.

3-142 The winter *R*-value and the *U*-factor of a flat ceiling with an air space are to be determined for the cases of air space with reflective and nonreflective surfaces.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

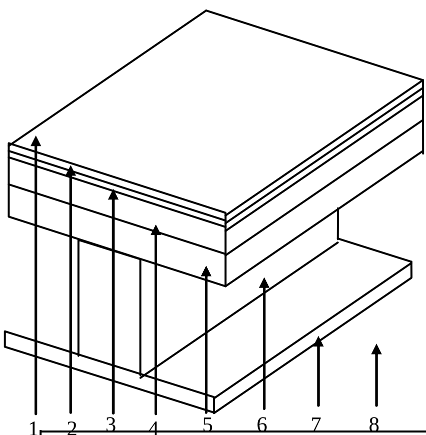
Properties The R-values are given in Table 3-6 for different materials, and in Table 3-9 for air layers.

Analysis The schematic of the ceiling as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.82 for air space and 0.18 for stud section since the headers which constitute a small part of the wall are to be treated as studs.

(*a*) Nonreflective surfaces, $\varepsilon_1 = \varepsilon_2 = 0.9$ and thus $\varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.9 + 1/0.9 - 1} = 0.82$.



Construction	*R*-value, m^2.°C/W Between studs	At studs
1. Still air above ceiling	0.12	0.044
2. Linoleum (R = 0.009 m^2.°C/W)	0.009	0.14
3. Felt (R = 0.011 m^2.°C/W)	0.011	0.23
4. Plywood, 13 mm	0.11	
5. Wood subfloor (R = 0.166 m^2.°C/W)	0.166	
6a. Air space, 90 mm, nonreflective	0.16	---
6b. Wood stud, 38 mm by 90 mm	---	0.63
7. Gypsum wallboard, 13 mm	0.079	0.079
8. Still air below ceiling	0.12	0.12
Total unit thermal resistance of each section, *R* (in m^2.°C/W)	0.775	1.243
The *U*-factor of each section, $U = 1/R$, in W/m^2.°C	1.290	0.805
Area fraction of each section, f_{area}	0.82	0.18
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.82×1.290+0.18×0.805	**1.203 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**0.831 m^2.°C/W**	

(*b*) One-reflective surface, $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9 \rightarrow \varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.9 - 1} = 0.05$

In this case we replace item 6a from 0.16 to 0.47 m^2.°C/W. It gives *R* = 1.085 m^2.°C/W and *U* = 0.922 W/ m^2.°C for the air space. Then,

Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.82×1.085+0.18×0.805	**1.035 W/m^2.°C**
Overall unit thermal resistance, $R = 1/U$	**0.967 m^2.°C/W**

(*c*) Two-reflective surface, $\varepsilon_1 = \varepsilon_2 = 0.05 \rightarrow \varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.05 - 1} = 0.03$

In this case we replace item 6a from 0.16 to 0.49 m^2.°C/W. It gives *R* = 1.105 m^2.°C/W and *U* = 0.905 W/ m^2.°C for the air space. Then,

Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.82×1.105+0.18×0.805	**1.051 W/m^2.°C**
Overall unit thermal resistance, $R = 1/U$	**0.951 m^2.°C/W**

3-143 The winter *R*-value and the *U*-factor of a masonry cavity wall are to be determined.

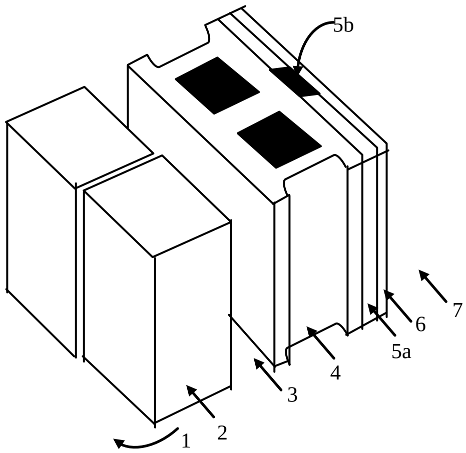
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-6.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{air\ space} + (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. Using the available *R*-values from Tables 3-6 and 3-9 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, $m^2.°C/W$	
	Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, nonreflective	0.16	0.16
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, nonreflective	0.17	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12
Total unit thermal resistance of each section, *R*	0.949	1.719
The *U*-factor of each section, $U = 1/R$, in $W/m^2.°C$	1.054	0.582
Area fraction of each section, f_{area}	0.84	0.16
Overall *U*-factor, $U = \Sigma f_{area,i}U_i = 0.84\times1.054+0.16\times0.582$	**0.978 W/m²·°C**	
Overall unit thermal resistance, $R = 1/U$	**1.02 m²·°C/W**	

Therefore, the overall unit thermal resistance of the wall is $R = 1.02\ m^2.°C/W$ and the overall U-factor is $U = 0.978\ W/m^2.°C$. These values account for the effects of the vertical ferring.

3-144 The winter *R*-value and the *U*-factor of a masonry cavity wall with a reflective surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-6. The *R*-values of air spaces are given in Table 3-9.

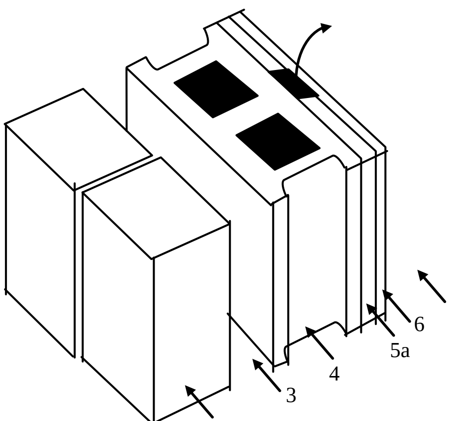
Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. For an air space with one-reflective surface, we have $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9$, and thus

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.05 + 1/0.9 - 1} = 0.05$$

Using the available *R*-values from Tables 3-6 and 3-9 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.



Construction	*R*-value, m²·°C/W Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, reflective with ε = 0.05	0.45	0.45
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, reflective with ε =0.05	0.49	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, *R*	1.559	2.009
The *U*-factor of each section, *U* = 1/*R*, in W/m²·°C	0.641	0.498
Area fraction of each section, f_{area}	0.84	0.16
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_i$ = 0.84×0.641+0.16×0.498	**0.618 W/m²·°C**	
Overall unit thermal resistance, *R* = 1/*U*	**1.62 m²·°C/W**	

Therefore, the overall unit thermal resistance of the wall is *R* = 1.62 m²·°C/W and the overall U-factor is *U* = 0.618 W/m²·°C. These values account for the effects of the vertical ferring.

Discussion The change in the *U*-value as a result of adding reflective surfaces is

$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value, nonreflective}} = \frac{0.978 - 0.618}{0.978} = 0.368$$

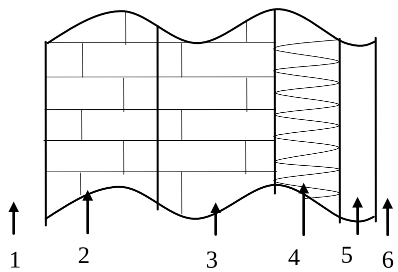
Therefore, the rate of heat transfer through the wall will decrease by 36.8% as a result of adding a reflective surface.

3-145 The winter *R*-value and the *U*-factor of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-6.

Analysis Using the available *R*-values from Tables 3-6, the total *R*-value of the wall is determined in the table below.



Construction	*R*-value, m^2.°C/W
1. Outside surface, 24 km/h	0.030
2. Face brick, 100 mm	0.075
3. Common brick, 100 mm	0.12
4. Urethane foam insulation, 25-mm	0.98
5. Gypsum wallboard, 13 mm	0.079
6. Inside surface, still air	0.12

Total unit thermal resistance of each section, *R*	**1.404 m^2.°C/W**
The *U*-factor of each section, *U* = 1/*R*	**0.712 W/m^2.°C**

Therefore, the overall unit thermal resistance of the wall is R = 1.404 m^2.°C/W and the overall U-factor is U = 0.712 W/m^2.°C.

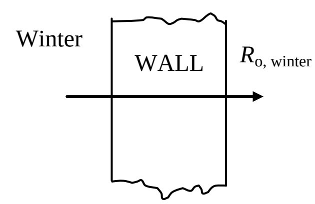
3-146 The *U*-value of a wall under winter design conditions is given. The *U*-value of the wall under summer design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (12 km/h winds) and winter (24 km/h winds) conditions are given in Table 3-6 to be $R_{\text{o, summer}}$ = 0.044 m^2.°C/W and $R_{\text{o, winter}}$ = 0.030 m^2.°C/W.

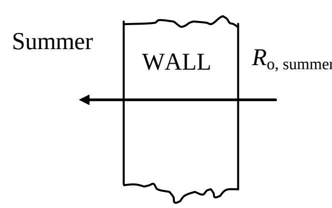
Analysis The *R*-value of the existing wall is

$$R_{\text{winter}} = 1/U_{\text{winter}} = 1/1.55 = 0.645 \text{ m}^2\cdot°\text{C/W}$$



Noting that the added and removed thermal resistances are in series, the overall *R*-value of the wall under summer conditions becomes

$$\begin{aligned} R_{\text{summer}} &= R_{\text{winter}} - R_{\text{o,winter}} + R_{\text{o,summer}} \\ &= 0.645 - 0.030 + 0.044 \\ &= 0.659 \text{ m}^2\cdot°\text{C/W} \end{aligned}$$



Then the summer *U*-value of the wall becomes

$$R_{\text{summer}} = 1/U_{\text{summer}} = 1/0.659 = \mathbf{1.52\ m^2\cdot°C/W}$$

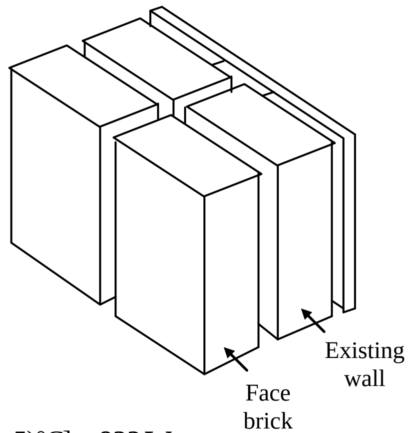
3-147 The *U*-value of a wall is given. A layer of face brick is added to the outside of a wall, leaving a 20-mm air space between the wall and the bricks. The new *U*-value of the wall and the rate of heat transfer through the wall is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The *U*-value of a wall is given to be U = 2.25 W/m^2.°C. The R - values of 100-mm face brick and a 20-mm air space between the wall and the bricks various layers are 0.075 and 0.170 m^2.°C/W, respectively.

Analysis The *R*-value of the existing wall for the winter conditions is

$$R_{\text{existing wall}} = 1/U_{\text{existing wall}} = 1/2.25 = 0.444\ \text{m}^2\cdot°\text{C/W}$$



Noting that the added thermal resistances are in series, the overall *R*-value of the wall becomes

$$R_{\text{modified wall}} = R_{\text{existing wall}} + R_{\text{brick}} + R_{\text{air layer}}$$
$$= 0.44 + 0.075 + 0.170 = 0.689\,\text{m}^2\cdot°\text{C/W}$$

Then the *U*-value of the wall after modification becomes

$$R_{\text{modified wall}} = 1/U_{\text{modified wall}} = 1/0.689 = \mathbf{1.45\ m^2\cdot°C/W}$$

The rate of heat transfer through the modified wall is

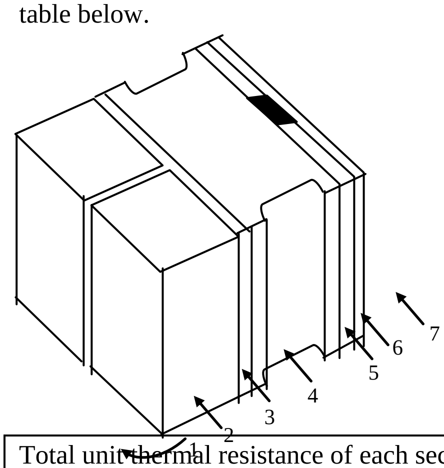
$$\dot{Q}_{\text{wall}} = (UA)_{\text{wall}}(T_i - T_o) = (1.45\,\text{W/m}^2\cdot°\text{C})(3\times 7\,\text{m}^2)[22-(-5)°\text{C}] = \mathbf{822\,W}$$

3-148 The summer and winter *R*-values of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant. **4** The air cavity does not have any reflecting surfaces.

Properties The R-values of different materials are given in Table 3-6.

Analysis Using the available *R*-values from Tables 3-6, the total *R*-value of the wall is determined in the table below.



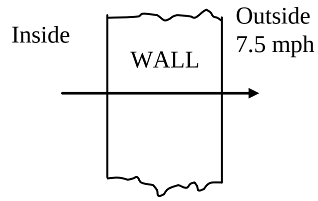
Construction	*R*-value, m^2.°C/W Summer	Winter
1a. Outside surface, 24 km/h (winter)	---	0.030
1b. Outside surface, 12 km/h (summer)	0.044	---
2. Face brick, 100 mm	0.075	0.075
3. Cement mortar, 13 mm	0.018	0.018
4. Concrete block, lightweight, 100 mm	0.27	0.27
5. Air space, nonreflecting, 40-mm	0.16	0.16
5. Plaster board, 20 mm	0.122	0.122
6. Inside surface, still air	0.12	0.12
Total unit thermal resistance of each section (the *R*-value) , m^2.°C/W	**0.809**	**0.795**

Therefore, the overall unit thermal resistance of the wall is R = 0.809 m^2.°C/W in summer and R = 0.795 m^2.°C/W in winter.

3-149E The *U*-value of a wall for 7.5 mph winds outside are given. The *U*-value of the wall for the case of 15 mph winds outside is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (7.5 mph winds) and winter (15 mph winds) conditions are given in Table 3-6 to be



$$R_{\text{o, 7.5 mph}} = R_{\text{o, summer}} = 0.25 \text{ h.ft}^2.°\text{F/Btu}$$

and $$R_{\text{o, 15 mph}} = R_{\text{o, winter}} = 0.17 \text{ h.ft}^2.°\text{F/Btu}$$

Analysis The R-value of the wall at 7.5 mph winds (summer) is

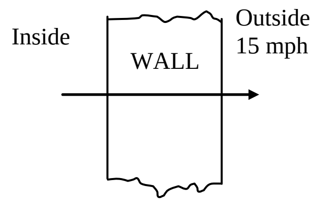
$$R_{\text{wall, 7.5 mph}} = 1/U_{\text{wall, 7.5 mph}} = 1/0.09 = 11.11 \text{ h.ft}^2\cdot°\text{F/Btu}$$

Noting that the added and removed thermal resistances are in series, the overall R-value of the wall at 15 mph (winter) conditions is obtained by replacing the summer value of outer convection resistance by the winter value,

$$R_{\text{wall, 15 mph}} = R_{\text{wall, 7.5 mph}} - R_{\text{o, 7.5 mph}} + R_{\text{o, 15 mph}} = 11.11 - 0.25 + 0.17 = 11.03 \text{ h.ft}^2\cdot°\text{F/Btu}$$

Then the U-value of the wall at 15 mph winds becomes

$$R_{\text{wall, 15 mph}} = 1/U_{\text{wal, 15 mph}} = 1/11.03 = \mathbf{0.0907 \text{ h.ft}^2\cdot°\text{F/Btu}}$$



Discussion Note that the effect of doubling the wind velocity on the U-value of the wall is less than 1 percent since

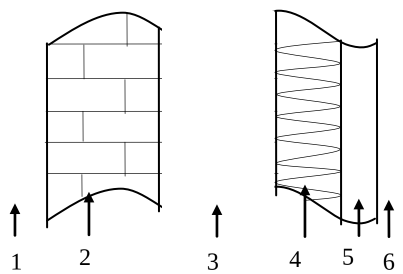
$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value}} = \frac{0.0907 - 0.09}{0.09} = 0.0078 \quad \text{(or 0.78\%)}$$

3-150 Two homes are identical, except that their walls are constructed differently. The house that is more energy efficient is to be determined.

Assumptions **1** The homes are identical, except that their walls are constructed differently. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-6.

Analysis Using the available *R*-values from Tables 3-6, the total *R*-value of the masonry wall is determined in the table below.



Construction	*R*-value, $m^2.°C/W$
1. Outside surface, 24 km/h (winter)	0.030
2. Concrete block, light weight, 200 mm	2×0.27=0.54
3. Air space, nonreflecting, 20 mm	0.17
5. Plasterboard, 20 mm	0.12
6. Inside surface, still air	0.12
Total unit thermal resistance (the *R*-value)	**0.98 $m^2.°C/W$**

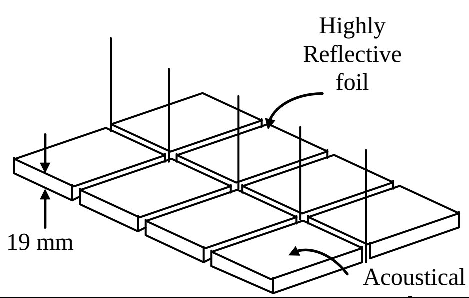
which is less than 2.4 $m^2.°C/W$. Therefore, the standard *R*-2.4 $m^2.°C/W$ wall is better insulated and thus it is more energy efficient.

3-151 A ceiling consists of a layer of reflective acoustical tiles. The *R*-value of the ceiling is to be determined for winter conditions.

Assumptions **1** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Tables 3-6 and 3-7.

Analysis Using the available *R*-values, the total *R*-value of the ceiling is determined in the table below.



Construction	*R*-value, $m^2.°C/W$
1. Still air, reflective horizontal surface facing up	R = 1/h = 1/4.32 = 0.23
2. Acoustic tile, 19 mm	0.32
3. Still air, horizontal surface, facing down	R = 1/h = 1/9.26 = 0.11
Total unit thermal resistance (the *R*-value)	**0.66 $m^2.°C/W$**

Therefore, the R-value of the hanging ceiling is 0.66 $m^2.°C/W$.