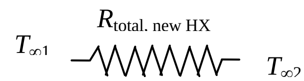
Review Problems

3-152E Steam is produced in copper tubes by heat transferred from another fluid condensing outside the tubes at a high temperature. The rate of heat transfer per foot length of the tube when a 0.01 in thick layer of limestone is formed on the inner surface of the tube is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

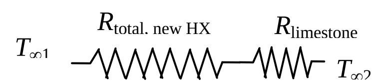
Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tubes and k = 1.7 Btu/h·ft·°F for limestone.

Analysis The total thermal resistance of the new heat exchanger is



$$\dot{Q}_{\text{new}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = \frac{T_{\infty 1} - T_{\infty 2}}{\dot{Q}_{\text{new}}} = \frac{(350-250)^\circ\text{F}}{2\times 10^4 \text{ Btu/h}} = 0.005 \text{ h.}^\circ\text{F/Btu}$$

After 0.01 in thick layer of limestone forms, the new value of thermal resistance and heat transfer rate are determined to be



$$R_{\text{limestone,i}} = \frac{\ln(r_1/r_i)}{2\pi kL} = \frac{\ln(0.5/0.49)}{2\pi(1.7 \text{ Btu/h.ft.}^\circ\text{F})(1 \text{ ft})} = 0.00189 \text{ h}^\circ\text{F/Btu}$$

$$R_{\text{total,w/lime}} = R_{\text{total,new}} + R_{\text{limestone,i}} = 0.005 + 0.00189 = 0.00689 \text{ h}^\circ\text{F/Btu}$$

$$\dot{Q}_{\text{w/lime}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,w/lime}}} = \frac{(350-250)^\circ\text{F}}{0.00689 \text{h}^\circ\text{F/Btu}} = \mathbf{1.45 \times 10^4 \text{ Btu/h}} \quad \text{(a decline of 27\%)}$$

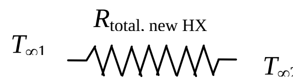
Discussion Note that the limestone layer will change the inner surface area of the pipe and thus the internal convection resistance slightly, but this effect should be negligible.

3-153E Steam is produced in copper tubes by heat transferred from another fluid condensing outside the tubes at a high temperature. The rate of heat transfer per foot length of the tube when a 0.01 in thick layer of limestone is formed on the inner and outer surfaces of the tube is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

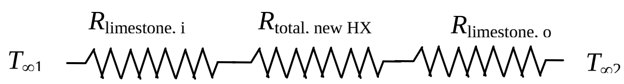
Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tubes and k = 1.7 Btu/h·ft·°F for limestone.

Analysis The total thermal resistance of the new heat exchanger is



$$\dot{Q}_{\text{new}} = \frac{T_{\infty1} - T_{\infty2}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = \frac{T_{\infty1} - T_{\infty2}}{\dot{Q}_{\text{new}}} = \frac{(350-250)^\circ\text{F}}{2\times10^4 \text{ Btu/h}} = 0.005 \text{ h.}^\circ\text{F/Btu}$$

After 0.01 in thick layer of limestone forms, the new value of thermal resistance and heat transfer rate are determined to be



$$R_{\text{limestone,i}} = \frac{\ln(r_1/r_i)}{2\pi kL} = \frac{\ln(0.5/0.49)}{2\pi(1.7 \text{ Btu/h.ft.}^\circ\text{F})(1 \text{ ft})} = 0.00189 \text{ h.}^\circ\text{F/Btu}$$

$$R_{\text{limestone,i}} = \frac{\ln(r_o/r_2)}{2\pi kL} = \frac{\ln(0.66/0.65)}{2\pi(1.7 \text{ Btu/h.ft.}^\circ\text{F})(1 \text{ ft})} = 0.00143 \text{ h.}^\circ\text{F/Btu}$$

$$R_{\text{total,w/lime}} = R_{\text{total,new}} + R_{\text{limestone,i}} + R_{\text{limestone,o}} = 0.005 + 0.00189 + 0.00143 = 0.00832 \text{ h.}^\circ\text{F/Btu}$$

$$\dot{Q}_{\text{w/lime}} = \frac{T_{\infty1} - T_{\infty2}}{R_{\text{total,w/lime}}} = \frac{(350-250)^\circ\text{F}}{0.00832\text{h}^\circ\text{F/Btu}} = \mathbf{1.20\times10^4 \text{ Btu/h}} \quad \text{(a decline of 40\%)}$$

Discussion Note that the limestone layer will change the inner surface area of the pipe and thus the internal convection resistance slightly, but this effect should be negligible.

3-154 A cylindrical tank filled with liquid propane at 1 atm is exposed to convection and radiation. The time it will take for the propane to evaporate completely as a result of the heat gain from the surroundings for the cases of no insulation and 7.5-cm thick glass wool insulation are to be determined.

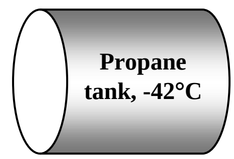
Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the propane inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid propane at 1 atm are given to be 425 kJ/kg and 581 kg/m^3, respectively. The thermal conductivity of glass wool insulation is given to be k = 0.038 W/m·°C.

Analysis (a) If the tank is not insulated, the heat transfer rate is determined to be

$$A_{\text{tank}} = \pi DL + 2\pi(\pi D^2/4) = \pi(1.2\text{ m})(6\text{ m}) + 2\pi(1.2\text{ m})^2/4 = 24.88\text{ m}^2$$

$$\dot{Q} = hA_{\text{tank}}(T_{\infty 1} - T_{\infty 2}) = (25\text{ W/m}^2.°\text{C})(24.88\text{ m}^2)[30-(-42)]°\text{C} = 44{,}787\text{ W}$$



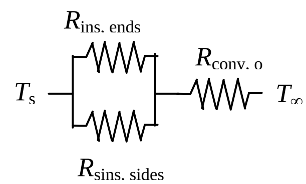
The volume of the tank and the mass of the propane are

$$V = \pi r^2 L = \pi(0.6\text{ m})^2(6\text{ m}) = 6.786\text{ m}^3$$

$$m = \rho V = (581\text{ kg/m}^3)(6.786\text{ m}^3) = 3942.6\text{ kg}$$

The rate of vaporization of propane is

$$\dot{Q} = \dot{m}h_{fg} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{44.787\text{ kJ/s}}{425\text{ kJ/kg}} = 0.1054\text{ kg/s}$$



Then the time period for the propane tank to empty becomes

$$\Delta t = \frac{m}{\dot{m}} = \frac{3942.6\text{ kg}}{0.1054\text{ kg/s}} = 37{,}413\text{ s} = \mathbf{10.4\ hours}$$

(b) We now repeat calculations for the case of insulated tank with 7.5-cm thick insulation.

$$A_o = \pi DL + 2\pi(\pi D^2/4) = \pi(1.35\text{ m})(6\text{ m}) + 2\pi(1.35\text{ m})^2/4 = 28.31\text{ m}^2$$

$$R_{\text{conv,o}} = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(28.31\text{ m}^2)} = 0.001413°\text{C/W}$$

$$R_{\text{insulation,side}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(67.5/60)}{2\pi(0.038\text{ W/m.°C})(6\text{ m})} = 0.08222°\text{C/W}$$

$$R_{\text{insulation,ends}} = 2\frac{L}{kA_{ave}} = \frac{2\times 0.075\text{ m}}{(0.038\text{ W/m.°C})[\pi(1.275\text{ m})^2/4]} = 3.0917°\text{C/W}$$

Noting that the insulation on the side surface and the end surfaces are in parallel, the equivalent resistance for the insulation is determined to be

$$R_{\text{insulation}} = \left(\frac{1}{R_{\text{insulation,side}}} + \frac{1}{R_{\text{insulation,ends}}}\right)^{-1} = \left(\frac{1}{0.08222°\text{C/W}} + \frac{1}{3.0917°\text{C/W}}\right)^{-1} = 0.08009°\text{C/W}$$

Then the total thermal resistance and the heat transfer rate become

$$R_{\text{total}} = R_{\text{conv,o}} + R_{\text{insulation}} = 0.001413 + 0.08009 = 0.081503°\text{C/W}$$

$$\dot{Q} = \frac{T_\infty - T_s}{R_{\text{total}}} = \frac{[30-(-42)]°\text{C}}{0.081503°\text{C/W}} = 883.4\text{ W}$$

Then the time period for the propane tank to empty becomes

$$\dot{Q} = \dot{m}h_{fg} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.8834\text{ kJ/s}}{425\text{ kJ/kg}} = 0.002079\text{ kg/s}$$

$$\Delta t = \frac{m}{\dot{m}} = \frac{3942.6\text{ kg}}{0.002079\text{ kg/s}} = 1{,}896{,}762\text{ s} = 526.9\text{ hours} = \mathbf{21.95\ days}$$

3-155 Hot water is flowing through a 3-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement, and it experiences a 3°C-temperature drop. The combined convection and radiation heat transfer coefficient at the outer surface of the pipe is to be determined.

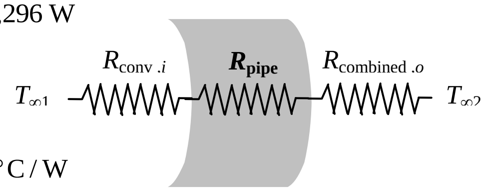
Assumptions **1** Heat transfer is steady since there is no indication of any significant change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no significant variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity of cast iron is given to be k = 52 W/m·°C.

Analysis Using water properties at room temperature, the mass flow rate of water and rate of heat transfer from the water are determined to be

$$\dot{m} = \rho\dot{V}_c = \rho V A_c = (1000 \text{ kg/m}^3)(1.5 \text{ m/s})\ \pi(0.03)^2/4 \text{ m}^2 = 1.06 \text{ kg/s}$$

$$\dot{Q} = \dot{m}C_p\Delta T = (1.06 \text{ kg/s})(4180 \text{ J/kg.°C})(70-67)\text{°C} = 13{,}296 \text{ W}$$



The thermal resistances for convection in the pipe and the pipe itself are

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.75/1.5)}{2\pi(52 \text{ W/m.°C})(15 \text{ m})} = 0.000031 \text{ °C/W}$$

$$R_{conv,i} = \frac{1}{h_i A_i} = \frac{1}{(400 \text{ W/m}^2\text{.°C})[\pi(0.03)(15)]\text{m}^2} = 0.001768 \text{ °C/W}$$

Using arithmetic mean temperature (70+67)/2 = 68.5°C for water, the heat transfer can be expressed as

$$\dot{Q} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{total}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{conv,i} + R_{pipe} + R_{combined,o}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{conv,i} + R_{pipe} + \dfrac{1}{h_{combined} A_o}}$$

Substituting,

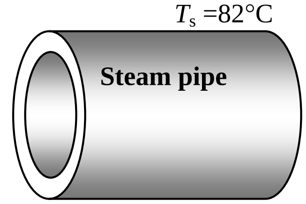
$$13{,}296 \text{ W} = \frac{(68.5-15)\text{°C}}{(0.000031 \text{ °C/W}) + (0.001768 \text{ °C/W}) + \dfrac{1}{h_{combined}[\pi(0.035)(15)]\text{m}^2}}$$

Solving for the combined heat transfer coefficient gives

$$h_{combined} = \mathbf{272.5 \ W/m^2.°C}$$

3-156 An 10-m long section of a steam pipe exposed to the ambient is to be insulated to reduce the heat loss through that section of the pipe by 90 percent. The amount of heat loss from the steam in 10 h and the amount of saved per year by insulating the steam pipe.

Assumptions **1** Heat transfer through the pipe is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficients accounts for the radiation effects. **5** The temperatures of the pipe surface and the surroundings are representative of annual average during operating hours. **6** The plant operates 110 days a year.



Analysis The rate of heat transfer for the uninsulated case is

$$A_o = \pi D_o L = \pi(0.12\,\text{m})(10\,\text{m}) = 3.77\,\text{m}^2$$

$$\dot{Q} = hA_o(T_s - T_{air}) = (25\,\text{W/m}^2.°\text{C})(3.77\,\text{m}^2)(82-8)°\text{C} = 6974\,\text{W}$$

The amount of heat loss during a 10-hour period is

$$Q = \dot{Q}\Delta t = (6.974\,\text{kJ/s})(10\times 3600\text{s}) = \mathbf{2.511\times 10^5\ kJ}\ \ (\text{per day})$$

The steam generator has an efficiency of 80%, and steam heating is used for 110 days a year. Then the amount of natural gas consumed per year and its cost are

$$\text{Fuel used} = \frac{2.511\times 10^5\ \text{kJ}}{0.80}\left(\frac{1\,\text{therm}}{105{,}500\text{kJ}}\right)(110\,\text{days/yr}) = 327.2\ \text{therms/yr}$$

$$\begin{aligned}\text{Cost of fuel} &= (\text{Amount of fuel})(\text{Unit cost of fuel})\\ &= (327.2\ \text{therms/yr})(\$0.60/\text{therm}) = \$196.3/\text{yr}\end{aligned}$$

Then the money saved by reducing the heat loss by 90% by insulation becomes

$$\text{Money saved} = 0.9\times(\text{Cost of fuel}) = 0.9\times\$196.3/\text{yr} = \mathbf{\$177}$$

3-157 A multilayer circuit board dissipating 27 W of heat consists of 4 layers of copper and 3 layers of epoxy glass sandwiched together. The circuit board is attached to a heat sink from both ends maintained at 35°C. The magnitude and location of the maximum temperature that occurs in the board is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional. **3** Thermal conductivities are constant. **4** Heat is generated uniformly in the epoxy layers of the board. **5** Heat transfer from the top and bottom surfaces of the board is negligible. **6** The thermal contact resistances at the copper-epoxy interfaces are negligible.

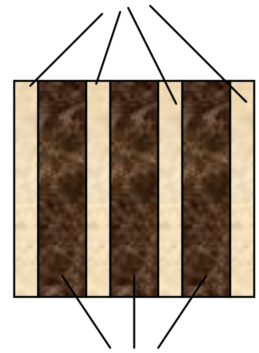
Properties The thermal conductivities are given to be k = 386 W/m·°C for copper layers and k = 0.26 W/m·°C for epoxy glass boards.

Analysis The effective conductivity of the multilayer circuit board is first determined to be

$$(kt)_{\text{copper}} = 4[(386 \text{ W/m.}^\circ\text{C})(0.0002 \text{ m})] = 0.3088 \text{ W/}^\circ\text{C}$$

$$(kt)_{\text{epoxy}} = 3[(0.26 \text{ W/m.}^\circ\text{C})(0.0015 \text{ m})] = 0.00117 \text{ W/}^\circ\text{C}$$

$$k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}} = \frac{(0.3088 + 0.00117)\text{W/}^\circ\text{C}}{[4(0.0002) + 3(0.0015)\text{m}} = 58.48 \text{ W/m.}^\circ\text{C}$$



The maximum temperature will occur at the midplane of the board that is the farthest to the heat sink. Its value is

$$A = 0.18[4(0.0002) + 3(0.0015)] = 0.000954 \text{ m}^2$$

$$\dot{Q} = \frac{k_{\text{eff}} A}{L}(T_1 - T_2)$$

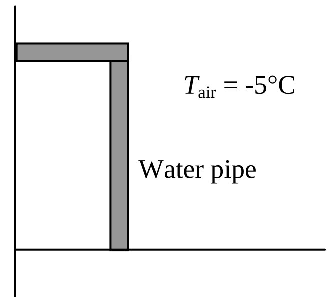
$$T_{\max} = T_1 = T_2 + \frac{\dot{Q}L}{k_{\text{eff}} A} = 35^\circ\text{C} + \frac{(27/2 \text{ W})(0.18/2 \text{ m})}{(58.48 \text{ W/m.}^\circ\text{C})(0.000954 \text{ m}^2)} = \mathbf{56.8^\circ C}$$

3-158 The plumbing system of a house involves some section of a plastic pipe exposed to the ambient air. The pipe is initially filled with stationary water at 0°C. It is to be determined if the water in the pipe will completely freeze during a cold night.

Assumptions **1** Heat transfer is transient, but can be treated as steady since the water temperature remains constant during freezing. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties of water are constant. **4** The water in the pipe is stationary, and its initial temperature is 0°C. **5** The convection resistance inside the pipe is negligible so that the inner surface temperature of the pipe is 0°C.

Properties The thermal conductivity of the pipe is given to be k = 0.16 W/m·°C. The density and latent heat of fusion of water at 0°C are ρ = 1000 kg/m^3 and h_{if} = 333.7 kJ/kg (Table A-9).

Analysis We assume the inner surface of the pipe to be at 0°C at all times. The thermal resistances involved and the rate of heat transfer are



$$R_{pipe} = \frac{\ln(r_2 / r_1)}{2\pi kL} = \frac{\ln(1.2/1)}{2\pi(0.16 \text{ W/m.°C})(0.5 \text{ m})} = 0.3627 \text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(40 \text{ W/m}^2.\text{°C})[\pi(0.024 \text{ m})(0.5 \text{ m})]} = 0.6631 \text{ °C/W}$$

$$R_{total} = R_{pipe} + R_{conv,o} = 0.3627 + 0.6631 = 1.0258 \text{ °C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[0-(-5)]\text{°C}}{1.0258 \text{ °C/W}} = 4.87 \text{ W}$$

The total amount of heat lost by the water during a 14-h period that night is

$$Q = \dot{Q}\Delta t = (4.87 \text{ J/s})(14 \times 3600 \text{ s}) = 245.65 \text{ kJ}$$

The amount of heat required to freeze the water in the pipe completely is

$$m = \rho V = \rho \pi r^2 L = (1000 \text{ kg/m}^3)\pi(0.01 \text{ m})^2(0.5 \text{ m}) = 0.157 \text{ kg}$$
$$Q = mh_{fg} = (0.157 \text{ kg})(333.7 \text{ kJ/kg}) = 52.4 \text{ kJ}$$

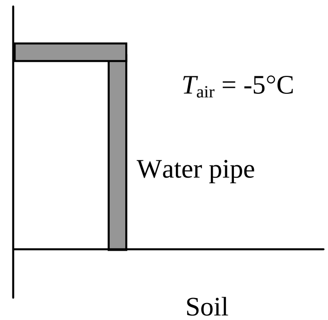
The water in the pipe will **freeze completely** that night since the amount heat loss is greater than the amount it takes to freeze the water completely $(245.65 > 52.4)$.

3-159 The plumbing system of a house involves some section of a plastic pipe exposed to the ambient air. The pipe is initially filled with stationary water at 0°C. It is to be determined if the water in the pipe will completely freeze during a cold night.

Assumptions **1** Heat transfer is transient, but can be treated as steady since the water temperature remains constant during freezing. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties of water are constant. **4** The water in the pipe is stationary, and its initial temperature is 0°C. **5** The convection resistance inside the pipe is negligible so that the inner surface temperature of the pipe is 0°C.

Properties The thermal conductivity of the pipe is given to be k = 0.16 W/m·°C. The density and latent heat of fusion of water at 0°C are ρ = 1000 kg/m^3 and h_{if} = 333.7 kJ/kg (Table A-9).

Analysis We assume the inner surface of the pipe to be at 0°C at all times. The thermal resistances involved and the rate of heat transfer are



$$R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.2/1)}{2\pi(0.16 \text{ W/m.°C})(0.5 \text{ m}^2)} = 0.3627 \text{ °C/W}$$

$$R_{\text{conv,o}} = \frac{1}{h_o A} = \frac{1}{(10 \text{ W/m}^2.\text{°C})[\pi(0.024 \text{ m})(0.5 \text{ m})]} = 2.6526 \text{ °C/W}$$

$$R_{\text{total}} = R_{\text{pipe}} + R_{\text{conv,o}} = 0.3627 + 2.6526 = 3.0153 \text{ °C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[0-(-5)]\text{°C}}{3.0153 \text{ °C/W}} = 1.658 \text{ W}$$

$$Q = \dot{Q}\Delta t = (1.658 \text{ J/s})(14 \times 3600 \text{ s}) = 83.57 \text{ kJ}$$

The amount of heat required to freeze the water in the pipe completely is

$$m = \rho V = \rho \pi r^2 L = (1000 \text{ kg/m}^3)\pi(0.01 \text{ m})^2(0.5 \text{ m}) = 0.157 \text{ kg}$$

$$Q = mh_{fg} = (0.157 \text{ kg})(333.7 \text{ kJ/kg}) = 52.4 \text{ kJ}$$

The water in the pipe will **freeze completely** that night since the amount heat loss is greater than the amount it takes to freeze the water completely $(83.57 > 52.4)$.

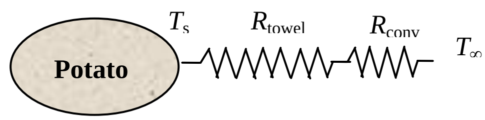
3-160E The surface temperature of a baked potato drops from 300°F to 200°F in 5 minutes in an environment at 70°F. The average heat transfer coefficient and the cooling time of the potato if it is wrapped completely in a towel are to be determined.

Assumptions **1**Thermal properties of potato are constant, and can be taken to be the properties of water. **2** The thermal contact resistance at the interface is negligible. **3** The heat transfer coefficients for wrapped and unwrapped potatoes are the same.

Properties The thermal conductivity of a thick towel is given to be k = 0.035 Btu/h·ft·°F. We take the properties of potato to be those of water at room temperature, ρ = 62.2 lbm/ft^3 and C_p = 0.998 Btu/lbm·°F.

Analysis This is a transient heat conduction problem, and the rate of heat transfer will decrease as the potato cools down and the temperature difference between the potato and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (300+200)/2 = 250°F for the potato during the process. The mass of the potato is

$$m = \rho V = \rho \frac{4}{3}\pi r^3$$
$$= (62.2\text{ lbm/ft}^3)\frac{4}{3}\pi(1.5/12\text{ ft})^3 = 0.5089\text{ lbm}$$



The amount of heat lost as the potato is cooled from 300 to 200°F is

$$Q = mC_p\Delta T = (0.5089\text{ lbm})(0.998\text{ Btu/lbm.°F})(300-200)\text{°F} = 50.8\text{ Btu}$$

The rate of heat transfer and the average heat transfer coefficient between the potato and its surroundings are

$$\dot{Q} = \frac{Q}{\Delta t} = \frac{50.8\text{ Btu}}{(5/60\text{ h})} = 609.4\text{ Btu/h}$$

$$\dot{Q} = hA_o(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_o(T_s - T_\infty)} = \frac{609.4\text{ Btu/h}}{\pi(3/12\text{ ft})^2(250-70)\text{°F}} = \mathbf{17.2\ Btu/h.ft^2.°F}$$

When the potato is wrapped in a towel, the thermal resistance and heat transfer rate are determined to be

$$R_{\text{towel}} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{[(1.5+0.12)/12]\text{ft} - (1.5/12)\text{ft}}{4\pi(0.035\text{ Btu/h.ft.°F})[(1.5+0.12)/12]\text{ft}(1.5/12)\text{ft}} = 1.3473\text{ h°F/Btu}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(17.2\text{ Btu/h.ft}^2\text{.°F})\pi(3.24/12)^2\text{ ft}^2} = 0.2539\text{ h.°F/Btu}$$

$$R_{\text{total}} = R_{\text{towel}} + R_{\text{conv}} = 1.3473 + 0.2539 = 1.6012\text{ h°F/Btu}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{(250-70)\text{°F}}{1.6012\text{ h°F/Btu}} = 112.4\text{ Btu/h} \qquad \Delta t = \frac{Q}{\dot{Q}} = \frac{50.8\text{ Btu}}{112.4\text{ Btu/h}} = 0.452\text{ h} = \mathbf{27.1\ min}$$

This result is conservative since the heat transfer coefficient will be lower in this case because of the smaller exposed surface temperature.

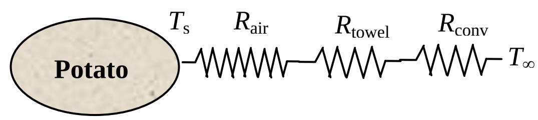
3-161E The surface temperature of a baked potato drops from 300°F to 200°F in 5 minutes in an environment at 70°F. The average heat transfer coefficient and the cooling time of the potato if it is loosely wrapped completely in a towel are to be determined.

Assumptions **1**Thermal properties of potato are constant, and can be taken to be the properties of water. **2** The heat transfer coefficients for wrapped and unwrapped potatoes are the same.

Properties The thermal conductivity of a thick towel is given to be k = 0.035 Btu/h·ft·°F. The thermal conductivity of air is given to be k = 0.015 Btu/h·ft·°F. We take the properties of potato to be those of water at room temperature, ρ = 62.2 lbm/ft^3 and C_p = 0.998 Btu/lbm·°F.

Analysis This is a transient heat conduction problem, and the rate of heat transfer will decrease as the potato cools down and the temperature difference between the potato and the surroundings decreases. However, we can solve this problem approximately by assuming a constant average temperature of (300+200)/2 = 250°F for the potato during the process. The mass of the potato is

$$m = \rho V = \rho \frac{4}{3}\pi r^3$$
$$= (62.2\ \text{lbm/ft}^3)\frac{4}{3}\pi(1.5/12\ \text{ft})^3 = 0.5089\ \text{lbm}$$



The amount of heat lost as the potato is cooled from 300 to 200°F is

$$Q = mC_p \Delta T = (0.5089\ \text{lbm})(0.998\ \text{Btu/lbm.°F})(300-200)\text{°F} = 50.8\ \text{Btu}$$

The rate of heat transfer and the average heat transfer coefficient between the potato and its surroundings are

$$\dot{Q} = \frac{Q}{\Delta t} = \frac{50.8\ \text{Btu}}{(5/60\ \text{h})} = 609.4\ \text{Btu/h}$$

$$\dot{Q} = hA_o(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_o(T_s - T_\infty)} = \frac{609.4\ \text{Btu/h}}{\pi(3/12\ \text{ft})^2(250-70)\text{°F}} = \mathbf{17.2\ Btu/h.ft^2.°F}$$

When the potato is wrapped in a towel, the thermal resistance and heat transfer rate are determined to be

$$R_{air} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{[(1.50+0.02)/12]\text{ft} - (1.50/12)\text{ft}}{4\pi(0.015\ \text{Btu/h.ft.°F})[(1.50+0.02)/12]\text{ft}(1.50/12)\text{ft}} = 0.5584\ \text{h.°F/Btu}$$

$$R_{towel} = \frac{r_3 - r_2}{4\pi k r_2 r_3} = \frac{[(1.52+0.12)/12]\text{ft} - (1.52/12)\text{ft}}{4\pi(0.035\ \text{Btu/h.ft.°F})[(1.52+0.12)/12]\text{ft}(1.52/12)\text{ft}} = 1.3134\ \text{h°F/Btu}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{(17.2\ \text{Btu/h.ft}^2\text{.°F})\pi(3.28/12)^2\ \text{ft}^2} = 0.2477\ \text{h.°F/Btu}$$

$$R_{total} = R_{air} + R_{towel} + R_{conv} = 0.5584 + 1.3134 + 0.2477 = 2.1195\ \text{h°F/Btu}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{total}} = \frac{(250-70)\text{°F}}{2.1195\ \text{h.°F/Btu}} = 84.9\ \text{Btu/h}$$

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{50.8\ \text{Btu}}{84.9\ \text{Btu/h}} = 0.598\ \text{h} = \mathbf{35.9 min}$$

This result is conservative since the heat transfer coefficient will be lower because of the smaller exposed surface temperature.

3-162 An ice chest made of 3-cm thick styrofoam is initially filled with 45 kg of ice at 0°C. The length of time it will take for the ice in the chest to melt completely is to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional. **3** Thermal conductivity is constant. **4** The inner surface temperature of the ice chest can be taken to be 0°C at all times. **5** Heat transfer from the base of the ice chest is negligible.

Properties The thermal conductivity of styrofoam is given to be k = 0.033 W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7 \text{ kJ/kg}$.

Analysis Disregarding any heat loss through the bottom of the ice chest, the total thermal resistance and the heat transfer rate are determined to be

$$A_i = 2(0.3-0.03)(0.4-0.06)+2(0.3-0.03)(0.5-0.06)+(0.4-0.06)(0.5-0.06) = 0.5708\,\text{m}^2$$

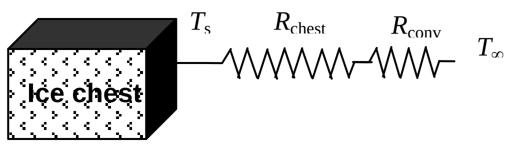
$$A_o = 2(0.3)(0.4)+2(0.3)(0.5)+(0.4)(0.5) = 0.74\,\text{m}^2$$

$$R_{\text{chest}} = \frac{L}{kA_i} = \frac{0.03\,\text{m}}{(0.033\,\text{W/m.°C})(0.5708\,\text{m}^2)} = 1.5927\,°\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{hA_o} = \frac{1}{(18\,\text{W/m}^2.°\text{C})(0.74\,\text{m}^2)} = 0.07508°\text{C/W}$$

$$R_{\text{total}} = R_{\text{chest}} + R_{\text{conv}} = 1.5927 + 0.07508 = 1.6678°\text{C/W}$$

$$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{(30-0)°\text{C}}{1.6678°\text{C/W}} = 20.99\,\text{W}$$



The total amount of heat necessary to melt the ice completely is

$$Q = mh_{if} = (45\text{ kg})(333.7\text{ kJ/kg}) = 15{,}016.5\text{ kJ}$$

Then the time period to transfer this much heat to the cooler to melt the ice completely becomes

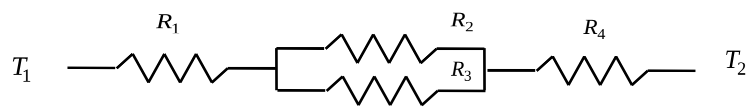
$$\Delta t = \frac{Q}{\dot{Q}} = \frac{15{,}016{,}500\,\text{J}}{20.99\,\text{kJ/s}} = 715{,}549\,\text{s} = 198.8\,\text{h} = \textbf{8.28 days}$$

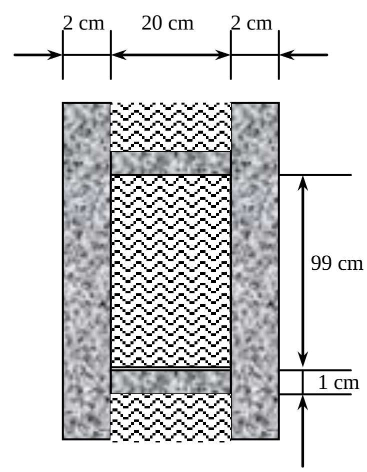
3-163 A wall is constructed of two large steel plates separated by 1-cm thick steel bars placed 99 cm apart. The remaining space between the steel plates is filled with fiberglass insulation. The rate of heat transfer through the wall is to be determined, and it is to be assessed if the steel bars between the plates can be ignored in heat transfer analysis since they occupy only 1 percent of the heat transfer surface area.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall can be approximated to be one-dimensional. **3** Thermal conductivities are constant. **4** The surfaces of the wall are maintained at constant temperatures.

Properties The thermal conductivities are given to be *k* = 15 W/m·°C for steel plates and *k* = 0.035 W/m·°C for fiberglass insulation.

Analysis We consider 1 m high and 1 m wide portion of the wall which is representative of entire wall. Thermal resistance network and individual resistances are





$$R_1 = R_4 = R_{\text{steel}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(15\ \text{W/m.°C})(1\ \text{m}^2)} = 0.00133\text{°C/W}$$

$$R_2 = R_{\text{steel}} = \frac{L}{kA} = \frac{0.2\ \text{m}}{(15\ \text{W/m.°C})(0.01\ \text{m}^2)} = 1.333\text{°C/W}$$

$$R_3 = R_{\text{insulation}} = \frac{L}{kA} = \frac{0.2\ \text{m}}{(0.035\ \text{W/m.°C})(0.99\ \text{m}^2)} = 5.772\ \text{°C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.333} + \frac{1}{5.772} \longrightarrow R_{eq} = 1.083\text{°C/W}$$

$$R_{\text{total}} = R_1 + R_{\text{eqv}} + R_4 = 0.00133 + 1.083 + 0.00133 = 1.0856\text{°C/W}$$

The rate of heat transfer per m^2 surface area of the wall is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{22\ \text{°C}}{1.0857\text{°C/W}} = 20.26\ \text{W}$$

The total rate of heat transfer through the entire wall is then determined to be

$$\dot{Q}_{\text{total}} = (4 \times 6)\dot{Q} = 24(20.26\ \text{W}) = \mathbf{486.3\ W}$$

If the steel bars were ignored since they constitute only 1% of the wall section, the R_{equiv} would simply be equal to the thermal resistance of the insulation, and the heat transfer rate in this case would be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_1 + R_{\text{insulation}} + R_4} = \frac{22\ \text{°C}}{(0.00133 + 5.772 + 0.00133)\text{°C/W}} = 3.81\text{W}$$

which is mush less than 20.26 W obtained earlier. Therefore, (20.26-3.81)/20.26 = 81.2% of the heat transfer occurs through the steel bars across the wall despite the negligible space that they occupy, and obviously their effect cannot be neglected. The connecting bars are serving as “thermal bridges.”