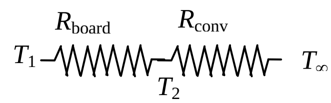
3-164 A circuit board houses electronic components on one side, dissipating a total of 15 W through the backside of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 20 aluminum fins of rectangular profile on the backside.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the backside of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be k = 12 W/m·°C for the circuit board, k = 237 W/m·°C for the aluminum plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (*a*) The thermal resistance of the board and the convection resistance on the backside of the board are



$$R_{board} = \frac{L}{kA} = \frac{0.002\text{ m}}{(12\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.011\text{ °C/W}$$

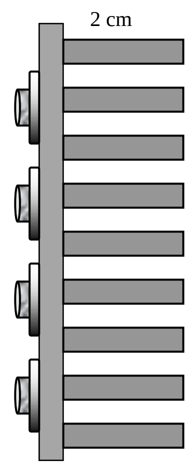
$$R_{conv} = \frac{1}{hA} = \frac{1}{(45\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 1.481\text{ °C/W}$$

$$R_{total} = R_{board} + R_{conv} = 0.011 + 1.481 = 1.492\text{ °C/W}$$

Then surface temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 37\text{°C} + (15\text{ W})(1.492\text{ °C/W}) = \mathbf{59.4°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{board}} \longrightarrow T_2 = T_1 - \dot{Q}R_{board} = 59.4\text{°C} - (15\text{ W})(0.011\text{ °C/W}) = \mathbf{59.2°C}$$



(*b*) Noting that the cross-sectional areas of the fins are constant, the efficiency of these rectangular fins is determined to be

$$a = \sqrt{\frac{hp}{kA_c}} \cong \sqrt{\frac{h(2w)}{k(tw)}} = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(45\text{ W/m}^2\text{.°C})}{(237\text{ W/m.°C})(0.002\text{ m})}} = 13.78\text{ m}^{-1}$$

$$\eta_{fin} = \frac{\tanh aL}{aL} = \frac{\tanh(13.78\text{ m}^{-1} \times 0.02\text{ m})}{13.78\text{ m}^{-1} \times 0.02\text{ m}} = 0.975$$

The finned and unfinned surface areas are

$$A_{finned} = (20)2w\left(L + \frac{t}{2}\right) = (20)2(0.15)\left(0.02 + \frac{0.002}{2}\right) = 0.126\text{ m}^2$$

$$A_{unfinned} = (0.1)(0.15) - 20(0.002)(0.15) = 0.0090\text{ m}^2$$

Then,

$$\dot{Q}_{finned} = \eta_{fin}\dot{Q}_{fin,max} = \eta_{fin} hA_{fin}(T_{base} - T_\infty)$$

$$\dot{Q}_{unfinned} = hA_{unfinned}(T_{base} - T_\infty)$$

$$\dot{Q}_{total} = \dot{Q}_{unfinned} + \dot{Q}_{finned} = h(T_{base} - T_\infty)(\eta_{fin} A_{fin} + A_{unfinned})$$

Substituting, the base temperature of the finned surfaces is determined to be

$$T_{base} = T_\infty + \frac{\dot{Q}_{total}}{h(\eta_{fin} A_{fin} + A_{unfinned})}$$

$$= 37\text{°C} + \frac{15\text{ W}}{(45\text{ W/m}^2\text{.°C})[(0.975)(0.126\text{ m}^2) + (0.0090\text{ m}^2)]} = \mathbf{39.5°C}$$

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

$$R_{\text{aluminum}} = \frac{L}{kA} = \frac{0.001 \text{ m}}{(237 \text{ W/m.}^\circ\text{C})(0.1 \text{ m})(0.15 \text{ m})} = 0.00028\ ^\circ\text{C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.00015 \text{ m}}{(1.8 \text{ W/m.}^\circ\text{C})(0.1 \text{ m})(0.15 \text{ m})} = 0.00555\ ^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_1 - T_{\text{base}}}{R_{\text{aluminum}} + R_{\text{epoxy}} + R_{\text{board}}} = \frac{(T_1 - 39.5)^\circ\text{C}}{(0.00028 + 0.00555 + 0.011)\ ^\circ\text{C/W}}$$

$$\longrightarrow T_1 = 39.5^\circ\text{C} + (15 \text{ W})(0.0168^\circ\text{C/W}) = \mathbf{39.8^\circ C}$$

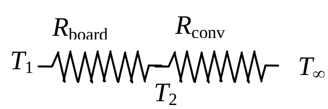
$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 39.8^\circ\text{C} - (15 \text{ W})(0.011^\circ\text{C/W}) = \mathbf{39.6^\circ C}$$

3-165 A circuit board houses electronic components on one side, dissipating a total of 15 W through the backside of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 20 copper fins of rectangular profile on the backside.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the backside of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be k = 12 W/m·°C for the circuit board, k = 386 W/m·°C for the copper plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (*a*) The thermal resistance of the board and the convection resistance on the backside of the board are



$$R_{\text{board}} = \frac{L}{kA} = \frac{0.002 \text{ m}}{(12 \text{ W/m.}^\circ\text{C})(0.1 \text{ m})(0.15 \text{ m})} = 0.011\ ^\circ\text{C/W}$$

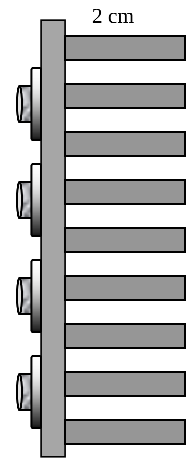
$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(45 \text{ W/m.}^\circ\text{C})(0.1 \text{ m})(0.15 \text{ m})} = 1.481\ ^\circ\text{C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.011 + 1.481 = 1.492\ ^\circ\text{C/W}$$

Then surface temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 37^\circ\text{C} + (15 \text{ W})(1.492\ ^\circ\text{C/W}) = \mathbf{59.4^\circ C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 59.4^\circ\text{C} - (15 \text{ W})(0.011\ ^\circ\text{C/W}) = \mathbf{59.2^\circ C}$$



(*b*) Noting that the cross-sectional areas of the fins are constant, the efficiency of these rectangular fins is determined to be

$$a=\sqrt{\frac{hp}{kA_c}}\cong\sqrt{\frac{h(2w)}{k(tw)}}=\sqrt{\frac{2h}{kt}}=\sqrt{\frac{2(45\text{ W/m}^2.^\circ\text{C})}{(386\text{ W/m.}^\circ\text{C})(0.002\text{ m})}}=10.80\text{ m}^{-1}$$

$$\eta_{\text{fin}}=\frac{\tanh aL}{aL}=\frac{\tanh(10.80\text{ m}^{-1}\times 0.02\text{ m})}{10.80\text{ m}^{-1}\times 0.02\text{ m}}=0.985$$

The finned and unfinned surface areas are

$$A_{\text{finned}}=(20)2w\left(L+\frac{t}{2}\right)=(20)2(0.15)\left(0.02+\frac{0.002}{2}\right)=0.126\text{m}^2$$

$$A_{\text{unfinned}}=(0.1)(0.15)-20(0.002)(0.15)=0.0090\text{m}^2$$

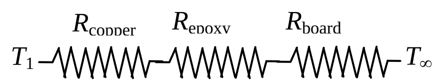
Then,

$$\dot{Q}_{\text{finned}}=\eta_{\text{fin}}\dot{Q}_{\text{fin,max}}=\eta_{\text{fin}}hA_{\text{fin}}(T_{\text{base}}-T_\infty)$$

$$\dot{Q}_{\text{unfinned}}=hA_{\text{unfinned}}(T_{\text{base}}-T_\infty)$$

$$\dot{Q}_{\text{total}}=\dot{Q}_{\text{unfinned}}+\dot{Q}_{\text{finned}}=h(T_{\text{base}}-T_\infty)(\eta_{\text{fin}}A_{\text{fin}}+A_{\text{unfinned}})$$

Substituting, the base temperature of the finned surfaces determine to be



$$T_{\text{base}}=T_\infty+\frac{\dot{Q}_{\text{total}}}{h(\eta_{\text{fin}}A_{\text{fin}}+A_{\text{unfinned}})}$$

$$=37^\circ\text{C}+\frac{15\text{ W}}{(45\text{ W/m}^2.^\circ\text{C})[(0.985)(0.126\text{m}^2)+(0.0090\text{m}^2)]}=\mathbf{39.5^\circ C}$$

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

$$R_{\text{copper}}=\frac{L}{kA}=\frac{0.001\text{ m}}{(386\text{ W/m.}^\circ\text{C})(0.1\text{ m})(0.15\text{ m})}=0.00017\ ^\circ\text{C/W}$$

$$R_{\text{epoxy}}=\frac{L}{kA}=\frac{0.00015\text{ m}}{(1.8\text{ W/m.}^\circ\text{C})(0.1\text{ m})(0.15\text{ m})}=0.00555\ ^\circ\text{C/W}$$

$$\dot{Q}=\frac{T_1-T_{base}}{R_{\text{copper}}+R_{\text{epoxy}}+R_{\text{board}}}=\frac{(T_1-39.5)^\circ\text{C}}{(0.00017+0.00555+0.011)\ ^\circ\text{C/W}}$$

$$\longrightarrow T_1=39.5^\circ\text{C}+(15\text{ W})(0.0167^\circ\text{C/W})=\mathbf{39.8^\circ C}$$

$$\dot{Q}=\frac{T_1-T_2}{R_{\text{board}}}\longrightarrow T_2=T_1-\dot{Q}R_{\text{board}}=39.8^\circ\text{C}-(15\text{ W})(0.011^\circ\text{C/W})=\mathbf{39.6^\circ C}$$

3-166 Steam passes through a row of 10 parallel pipes placed horizontally in a concrete floor exposed to room air at 25°C with a heat transfer coefficient of 12 W/m^2.°C. If the surface temperature of the concrete floor is not to exceed 40°C, the minimum burial depth of the steam pipes below the floor surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis In steady operation, the rate of heat loss from the steam through the concrete floor by conduction must be equal to the rate of heat transfer from the concrete floor to the room by combined convection and radiation, which is determined to be

$$\dot{Q} = hA_s(T_s - T_\infty) = (12\ \text{W/m}^2.°\text{C})[(10\ \text{m})(5\ \text{m})](40-25)°\text{C} = 9000\ \text{W}$$

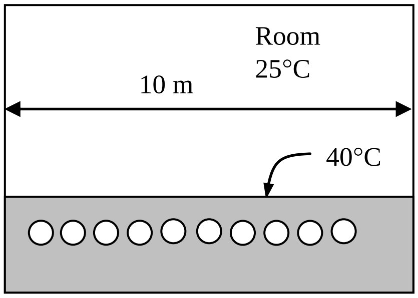
Then the depth the steam pipes should be buried can be determined with the aid of shape factor for this configuration from Table 3-5 to be

$$\dot{Q} = nSk(T_1 - T_2) \longrightarrow S = \frac{\dot{Q}}{nk(T_1 - T_2)} = \frac{9000\ \text{W}}{10(0.75\ \text{W/m.°C})(150-40)°\text{C}} = 10.91\ \text{m}\ \ \text{(per pipe)}$$

$$w = \frac{a}{n} = \frac{10\ \text{m}}{10} = 1\ \text{m (center-to-center distance of pipes)}$$

$$S = \frac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$$

$$10.91\ \text{m} = \frac{2\pi(5\ \text{m})}{\ln\left[\dfrac{2(1\text{m})}{\pi(0.06\text{m})}\sinh\dfrac{2\pi z}{(1\ \text{m})}\right]} \longrightarrow z = 0.205\ \text{m} = \textbf{20.5 cm}$$



3-167 Two persons are wearing different clothes made of different materials with different surface areas. The fractions of heat lost from each person's body by respiration are to be determined.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is accounted for in the heat transfer coefficient. **5** The human body is assumed to be cylindrical in shape for heat transfer purposes.

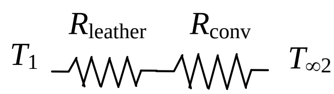
Properties The thermal conductivities of the leather and synthetic fabric are given to be k = 0.159 W/m·°C and k = 0.13 W/m·°C, respectively.

Analysis The surface area of each body is first determined from

$$A_1 = \pi DL/2 = \pi(0.25\,\text{m})(1.7\,\text{m})/2 = 0.6675\,\text{m}^2$$
$$A_2 = 2A_1 = 2\times 0.6675 = 1.335\,\text{m}^2$$

The sensible heat lost from the first person's body is

$$R_{\text{leather}} = \frac{L}{kA} = \frac{0.001\,\text{m}}{(0.159\,\text{W/m.°C})(0.6675\,\text{m}^2)} = 0.00942\text{°C/W}$$



$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(15\,\text{W/m}^2.\text{°C})(0.6675\,\text{m}^2)} = 0.09988\text{°C/W}$$
$$R_{\text{total}} = R_{\text{leather}} + R_{\text{conv}} = 0.00942 + 0.09988 = 0.10930\text{°C/W}$$

The total sensible heat transfer is the sum of heat transferred through the clothes and the skin

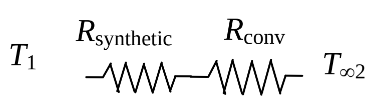
$$\dot{Q}_{\text{clothes}} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(32-30)\text{°C}}{0.10930\text{°C/W}} = 18.3\,\text{W}$$
$$\dot{Q}_{\text{skin}} = \frac{T_1 - T_{\infty 2}}{R_{\text{conv}}} = \frac{(32-30)\text{°C}}{0.09988\text{°C/W}} = 20.0\,\text{W}$$
$$\dot{Q}_{\text{sensible}} = \dot{Q}_{\text{clothes}} + \dot{Q}_{\text{skin}} = 18.3 + 20 = 38.3\,\text{W}$$

Then the fraction of heat lost by respiration becomes

$$f = \frac{\dot{Q}_{\text{respiration}}}{\dot{Q}_{\text{total}}} = \frac{\dot{Q}_{\text{total}} - \dot{Q}_{\text{sensible}}}{\dot{Q}_{\text{total}}} = \frac{60-38.3}{60} = \mathbf{0.362}$$

Repeating similar calculations for the second person's body

$$R_{\text{synthetic}} = \frac{L}{kA} = \frac{0.001\,\text{m}}{(0.13\,\text{W/m.°C})(1.335\,\text{m}^2)} = 0.00576\text{°C/W}$$
$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(15\,\text{W/m}^2.\text{°C})(1.335\,\text{m}^2)} = 0.04994\text{°C/W}$$



$$R_{\text{total}} = R_{\text{leather}} + R_{\text{conv}} = 0.00576 + 0.04994 = 0.05570\text{°C/W}$$

$$\dot{Q}_{\text{sensible}} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(32-30)\text{°C}}{0.05570\text{°C/W}} = 35.9\,\text{W}$$

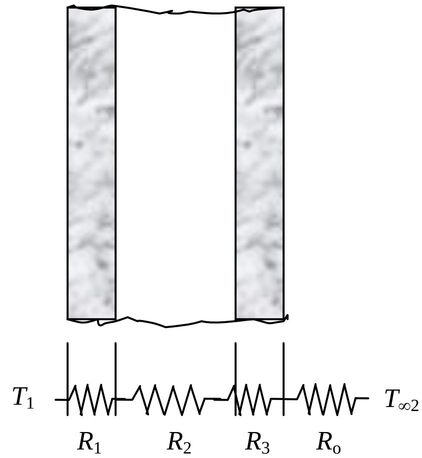
$$f = \frac{\dot{Q}_{\text{respiration}}}{\dot{Q}_{\text{total}}} = \frac{\dot{Q}_{\text{total}} - \dot{Q}_{\text{sensible}}}{\dot{Q}_{\text{total}}} = \frac{60-35.9}{60} = \mathbf{0.402}$$

3-168 A wall constructed of three layers is considered. The rate of hat transfer through the wall and temperature drops across the plaster, brick, covering, and surface-ambient air are to be determined.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is accounted for in the heat transfer coefficient.

Properties The thermal conductivities of the plaster, brick, and covering are given to be k = 0.72 W/m·°C, k = 0.36 W/m·°C, k = 1.40 W/m·°C, respectively.

Analysis The surface area of the wall and the individual resistances are



$$A = (6\text{ m}) \times (2.8\text{ m}) = 16.8\text{ m}^2$$

$$R_1 = R_{\text{plaster}} = \frac{L_1}{k_1 A} = \frac{0.01\text{ m}}{(0.36\text{ W/m.°C})(16.8\text{ m}^2)} = 0.00165\text{ °C/W}$$

$$R_2 = R_{\text{brick}} = \frac{L_2}{k_2 A} = \frac{0.20\text{ m}}{(0.72\text{ W/m.°C})(16.8\text{ m}^2)} = 0.01653\text{ °C/W}$$

$$R_3 = R_{\text{covering}} = \frac{L_3}{k_3 A} = \frac{0.02\text{ m}}{(1.4\text{ W/m.°C})(16.8\text{ m}^2)} = 0.00085\text{ °C/W}$$

$$R_o = R_{\text{conv,2}} = \frac{1}{h_2 A} = \frac{1}{(17\text{ W/m}^2\text{.°C})(16.8\text{ m}^2)} = 0.00350\text{ °C/W}$$

$$R_{\text{total}} = R_1 + R_2 + R_3 + R_{\text{conv,2}} = 0.00165 + 0.01653 + 0.00085 + 0.00350 = 0.02253\text{ °C/W}$$

The steady rate of heat transfer through the wall then becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(23-8)\text{°C}}{0.02253\text{°C/W}} = \mathbf{665.8\ W}$$

The temperature drops are

$$\Delta T_{\text{plaster}} = \dot{Q} R_{\text{plaster}} = (665.8\text{ W})(0.00165\text{ °C/W}) = \mathbf{1.1\ °C}$$

$$\Delta T_{\text{brick}} = \dot{Q} R_{\text{brick}} = (665.8\text{ W})(0.01653\text{ °C/W}) = \mathbf{11.0\ °C}$$

$$\Delta T_{\text{covering}} = \dot{Q} R_{\text{covering}} = (665.8\text{ W})(0.00085\text{ °C/W}) = \mathbf{0.6\ °C}$$

$$\Delta T_{\text{conv}} = \dot{Q} R_{\text{conv}} = (665.8\text{ W})(0.00350\text{ °C/W}) = \mathbf{2.3\ °C}$$

3-169 An insulation is to be added to a wall to decrease the heat loss by 85%. The thickness of insulation and the outer surface temperature of the wall are to be determined for two different insulating materials.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is accounted for in the heat transfer coefficient.

Properties The thermal conductivities of the plaster, brick, covering, polyurethane foam, and glass fiber are given to be 0.72 W/m·°C, 0.36 W/m·°C, 1.40 W/m·°C, 0.025 W/m·°C, 0.036 W/m·°C, respectively.

Analysis The surface area of the wall and the individual resistances are

$$A = (6\text{ m})\times(2.8\text{ m}) = 16.8\text{ m}^2$$

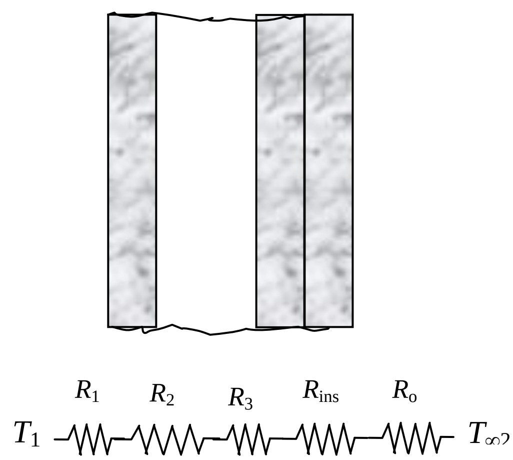
$$R_1 = R_{\text{plaster}} = \frac{L_1}{k_1 A} = \frac{0.01\text{ m}}{(0.36\text{ W/m.°C})(16.8\text{ m}^2)} = 0.00165\text{°C/W}$$

$$R_2 = R_{\text{brick}} = \frac{L_2}{k_2 A} = \frac{0.20\text{ m}}{(0.72\text{ W/m.°C})(16.8\text{ m}^2)} = 0.01653\text{°C/W}$$

$$R_3 = R_{\text{covering}} = \frac{L_3}{k_3 A} = \frac{0.02\text{ m}}{(1.4\text{ W/m.°C})(16.8\text{ m}^2)} = 0.00085\text{°C/W}$$

$$R_o = R_{\text{conv,2}} = \frac{1}{h_2 A} = \frac{1}{(17\text{ W/m}^2.\text{°C})(16.8\text{ m}^2)} = 0.00350\text{°C/W}$$

$$\begin{aligned} R_{\text{total, no ins}} &= R_1 + R_2 + R_3 + R_{\text{conv,2}} \\ &= 0.00165 + 0.01653 + 0.00085 + 0.00350 \\ &= 0.02253\text{°C/W} \end{aligned}$$



The rate of heat loss without the insulation is

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total, no ins}}} = \frac{(23-8)\text{°C}}{0.02253\text{ °C/W}} = 666\text{ W}$$

(*a*) The rate of heat transfer after insulation is

$$\dot{Q}_{\text{ins}} = 0.15\dot{Q}_{\text{no ins}} = 0.15\times 666 = 99.9\text{ W}$$

The total thermal resistance with the foam insulation is

$$\begin{aligned} R_{\text{total}} &= R_1 + R_2 + R_3 + R_{\text{foam}} + R_{\text{conv,2}} \\ &= 0.02253\text{°C/W} + \frac{L_4}{(0.025\text{ W/m.°C})(16.8\text{ m}^2)} = 0.02253\text{°C/W} + \frac{L_4}{(0.42\text{ W.m/°C})} \end{aligned}$$

The thickness of insulation is determined from

$$\dot{Q}_{\text{ins}} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow 99.9\text{ W} = \frac{(23-8)\text{°C}}{0.02253\text{°C/W} + \dfrac{L_4}{(0.42\text{ W.m/°C})}} \longrightarrow L_4 = \mathbf{0.054\,m} = \mathbf{5.4\,cm}$$

The outer surface temperature of the wall is determined from

$$\dot{Q}_{\text{ins}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv}}} \longrightarrow 99.9\text{ W} = \frac{(T_2 - 8)\text{°C}}{0.00350\text{°C/W}} \longrightarrow T_2 = \mathbf{8.3°C}$$

(*b*) The total thermal resistance with the fiberglass insulation is

$$R_{total} = R_1 + R_2 + R_3 + R_{fiberglass} + R_{conv,2}$$

$$= 0.02253\ °C/W + \frac{L_4}{(0.036\ W/m.°C)(16.8\ m^2)} = 0.02253\ °C/W + \frac{L_4}{(0.6048\ W.m/°C)}$$

The thickness of insulation is determined from

$$\dot{Q}_{ins} = \frac{T_1 - T_{\infty 2}}{R_{total}} \longrightarrow 99.9\ W = \frac{(23-8)°C}{0.02253°C/W + \frac{L_4}{(0.6048\ W.m/°C}} \longrightarrow L_4 = \mathbf{0.077\ m} = \mathbf{7.7\ cm}$$

The outer surface temperature of the wall is determined from

$$\dot{Q}_{ins} = \frac{T_2 - T_{\infty 2}}{R_{conv}} \longrightarrow 99.9 = \frac{(T_2 - 8)°C}{0.00350°C/W} \longrightarrow T_2 = \mathbf{8.3°C}$$

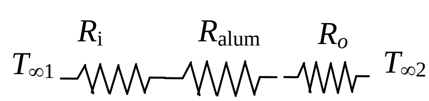
Discussion The outer surface temperature is same for both cases since the rate of heat transfer does not change.

3-170 Cold conditioned air is flowing inside a duct of square cross-section. The maximum length of the duct for a specified temperature increase in the duct is to be determined.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Steady one-dimensional heat conduction relations can be used due to small thickness of the duct wall. **5** When calculating the conduction thermal resistance of aluminum, the average of inner and outer surface areas will be used.

Properties The thermal conductivity of aluminum is given to be 237 W/m·°C. The specific heat of air at the given temperature is C_p = 1006 J/kg·°C (Table A-15).

Analysis The inner and the outer surface areas of the duct per unit length and the individual thermal resistances are



$$A_1 = 4a_1 L = 4(0.22\ m)(1\ m) = 0.88\ m^2$$

$$A_2 = 4a_2 L = 4(0.25\ m)(1\ m) = 1.0\ m^2$$

$$R_i = \frac{1}{h_1 A} = \frac{1}{(75\ W/m^2.°C)(0.88\ m^2)} = 0.01515°C/W$$

$$R_{alum} = \frac{L}{kA} = \frac{0.015\ m}{(237\ W/m.°C)[(0.88+1)/2]m^2} = 0.00007°C/W$$

$$R_o = \frac{1}{h_2 A} = \frac{1}{(8\ W/m^2.°C)(1.0\ m^2)} = 0.12500°C/W$$

$$R_{total} = R_i + R_{alum} + R_o = 0.01515 + 0.00007 + 0.12500 = 0.14022°C/W$$

The rate of heat loss from the air inside the duct is

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(33-12)°C}{0.14022°C/W} = 149.8\ W$$

For a temperature rise of 1°C, the air inside the duct should gain heat at a rate of

$$\dot{Q}_{total} = \dot{m} C_p \Delta T = (0.8\ kg/s)(1006 J/kg.°C)(1°C) = 804\ W$$

Then the maximum length of the duct becomes

$$L = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{804\ W}{149.8\ W} = \mathbf{5.37\ m}$$

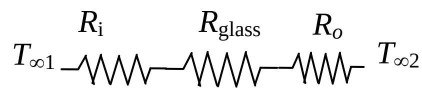
3-171 Heat transfer through a window is considered. The percent error involved in the calculation of heat gain through the window assuming the window consist of glass only is to be determined.

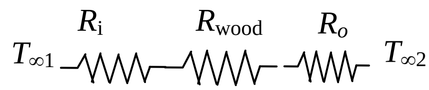
Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Radiation is accounted for in heat transfer coefficients.

Properties The thermal conductivities are given to be 0.7 W/m·°C for glass and 0.12 W/m·°C for pine wood.

Analysis The surface areas of the glass and the wood and the individual thermal resistances are

$$A_{\text{glass}} = 0.85(1.5\text{ m})(2\text{ m}) = 2.55\text{ m}^2 \qquad A_{\text{wood}} = 0.15(1.5\text{ m})(2\text{ m}) = 0.45\text{ m}^2$$





$$R_{\text{i,glass}} = \frac{1}{h_1 A_{\text{glass}}} = \frac{1}{(7\text{ W/m}^2.°\text{C})(2.55\text{ m}^2)} = 0.05602°\text{C/W}$$

$$R_{\text{i,wood}} = \frac{1}{h_1 A_{\text{wood}}} = \frac{1}{(7\text{ W/m}^2.°\text{C})(0.45\text{ m}^2)} = 0.31746°\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{k_{\text{glass}} A_{\text{glass}}} = \frac{0.003\text{ m}}{(0.7\text{ W/m.°C})(2.55\text{ m}^2)} = 0.00168°\text{C/W}$$

$$R_{\text{wood}} = \frac{L_{\text{wood}}}{k_{\text{wood}} A_{\text{wood}}} = \frac{0.05\text{ m}}{(0.12\text{ W/m.°C})(0.45\text{ m}^2)} = 0.92593°\text{C/W}$$

$$R_{\text{o,glass}} = \frac{1}{h_2 A_{\text{glass}}} = \frac{1}{(13\text{ W/m}^2.°\text{C})(2.55\text{ m}^2)} = 0.03017°\text{C/W}$$

$$R_{\text{o,wood}} = \frac{1}{h_2 A_{\text{wood}}} = \frac{1}{(13\text{ W/m}^2.°\text{C})(0.45\text{ m}^2)} = 0.17094°\text{C/W}$$

$$R_{\text{total, glass}} = R_{\text{i,glass}} + R_{\text{glass}} + R_{\text{o,glass}} = 0.05602 + 0.00168 + 0.03017 = 0.08787°\text{C/W}$$

$$R_{\text{total,wood}} = R_{\text{i,wood}} + R_{\text{wood}} + R_{\text{o,wood}} = 0.31746 + 0.92593 + 0.17094 = 1.41433°\text{C/W}$$

The rate of heat gain through the glass and the wood and their total are

$$\dot{Q}_{\text{glass}} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{\text{total,glass}}} = \frac{(40-24)°\text{C}}{0.08787°\text{C/W}} = 182.1\text{ W} \qquad \dot{Q}_{\text{wood}} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{\text{total,wood}}} = \frac{(40-24)°\text{C}}{1.41433°\text{C/W}} = 11.3\text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{glass}} + \dot{Q}_{\text{wood}} = 182.1 + 11.3 = 193.4\text{ W}$$

If the window consists of glass only the heat gain through the window is

$$A_{\text{glass}} = (1.5\text{ m})(2\text{ m}) = 3.0\text{ m}^2$$

$$R_{\text{i,glass}} = \frac{1}{h_1 A_{\text{glass}}} = \frac{1}{(7\text{ W/m}^2.°\text{C})(3.0\text{ m}^2)} = 0.04762°\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{k_{\text{glass}} A_{\text{glass}}} = \frac{0.003\text{ m}}{(0.7\text{ W/m.°C})(3.0\text{ m}^2)} = 0.00143°\text{C/W}$$

$$R_{\text{o,glass}} = \frac{1}{h_2 A_{\text{glass}}} = \frac{1}{(13\text{ W/m}^2.°\text{C})(3.0\text{ m}^2)} = 0.02564°\text{C/W}$$

$$R_{\text{total, glass}} = R_{\text{i,glass}} + R_{\text{glass}} + R_{\text{o,glass}} = 0.04762 + 0.00143 + 0.02564 = 0.07469°\text{C/W}$$

$$\dot{Q}_{\text{glass}} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{\text{total,glass}}} = \frac{(40-24)°\text{C}}{0.07469°\text{C/W}} = 214.2\text{ W}$$

Then the percentage error involved in heat gain through the window assuming the window consist of glass only becomes

$$\%\text{ Error} = \frac{\dot{Q}_{\text{glass only}} - \dot{Q}_{\text{with wood}}}{\dot{Q}_{\text{with wood}}} = \frac{214.2 - 193.4}{193.4} \times 100 = \mathbf{10.8\%}$$

3-172 Steam is flowing inside a steel pipe. The thickness of the insulation needed to reduce the heat loss by 95 percent and the thickness of the insulation needed to reduce outer surface temperature to 40°C are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

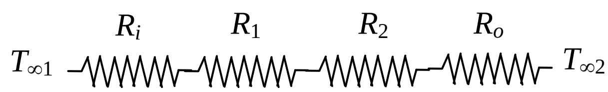
Properties The thermal conductivities are given to be k = 61 W/m·°C for steel and k = 0.038 W/m·°C for insulation.

Analysis (a) Considering a unit length of the pipe, the inner and the outer surface areas of the pipe and the insulation are

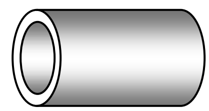
$$A_1 = \pi D_i L = \pi(0.10\,\text{m})(1\,\text{m}) = 0.3142\,\text{m}^2$$

$$A_2 = \pi D_o L = \pi(0.12\,\text{m})(1\,\text{m}) = 0.3770\,\text{m}^2$$

$$A_3 = \pi D_3 L = \pi D_3 (1\,\text{m}) = 3.1416 D_3\ \text{m}^2$$



The individual thermal resistances are



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(105\,\text{W/m}^2.°\text{C})(0.3142\,\text{m}^2)} = 0.03031°\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(6/5)}{2\pi(61\,\text{W/m.°C})(1\,\text{m})} = 0.00048°\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(D_3/0.12)}{2\pi(0.038\,\text{W/m.°C})(1\,\text{m})} = \frac{\ln(D_3/0.12)}{0.23876}\ °\text{C/W}$$

$$R_{\text{o,steel}} = \frac{1}{h_o A_o} = \frac{1}{(14\,\text{W/m}^2.°\text{C})(0.3770\,\text{m}^2)} = 0.18947°\text{C/W}$$

$$R_{\text{o,insulation}} = \frac{1}{h_o A_o} = \frac{1}{(14\,\text{W/m}^2.°\text{C})(3.1416 D_3\ \text{m}^2)} = \frac{0.02274}{D_3}\ °\text{C/W}$$

$$R_{\text{total, no insulation}} = R_i + R_1 + R_{\text{o,steel}} = 0.03031 + 0.00048 + 0.18947 = 0.22026°\text{C/W}$$

$$\begin{aligned} R_{\text{total, insulation}} &= R_i + R_1 + R_2 + R_{\text{o,insulation}} \\ &= 0.03031 + 0.00048 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3} \\ &= 0.03079 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}\ °\text{C/W} \end{aligned}$$

Then the steady rate of heat loss from the steam per meter pipe length for the case of no insulation becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(235-20)°\text{C}}{0.22026°\text{C/W}} = 976.1\text{W}$$

The thickness of the insulation needed in order to save 95 percent of this heat loss can be determined from

$$\dot{Q}_{\text{insulation}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,insulation}}} \longrightarrow (0.05 \times 976.1)\ \text{W} = \frac{(235-20)°\text{C}}{\left(0.03079 + \dfrac{\ln(D_3/0.12)}{0.23876} + \dfrac{0.02274}{D_3}\right)°\text{C/W}}$$

whose solution is $D_3 = 0.3355\,\text{m} \longrightarrow \text{thickness} = \dfrac{D_3 - D_2}{2} = \dfrac{33.55 - 12}{2} = \mathbf{10.78\,cm}$

(*b*) The thickness of the insulation needed that would maintain the outer surface of the insulation at a maximum temperature of 40°C can be determined from

$$\dot{Q}_{\text{insulation}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,insulation}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{o, insulation}}}$$

$$\rightarrow \frac{(235-20)°\text{C}}{\left(0.03079 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}\right)°\text{C/W}} = \frac{(40-20)°\text{C}}{\frac{0.02274}{D_3}\ °\text{C/W}}$$

whose solution is

$$D_3 = 0.1644\,\text{m} \longrightarrow \text{thickness} = \frac{D_3 - D_2}{2} = \frac{16.44 - 12}{2} = \mathbf{2.22\,cm}$$

3-173 A 6-m-diameter spherical tank filled with liquefied natural gas (LNG) at -160°C is exposed to ambient air. The time for the LNG temperature to rise to -150°C is to be determined.

Assumptions **1** Heat transfer can be considered to be steady since the specified thermal conditions at the boundaries do not change with time significantly. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Radiation is accounted for in the combined heat transfer coefficient. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the LNG inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

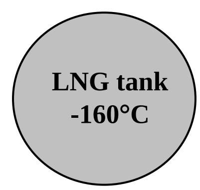
Properties The density and specific heat of LNG are given to be 425 kg/m^3 and 3.475 kJ/kg·°C, respectively. The thermal conductivity of super insulation is given to be k = 0.00008 W/m·°C.

Analysis The inner and outer surface areas of the insulated tank and the volume of the LNG are

$$A_1 = \pi D_1^2 = \pi(6\,\text{m})^2 = 113.1\,\text{m}^2$$

$$A_2 = \pi D_2^2 = \pi(6.10\,\text{m})^2 = 116.9\,\text{m}^2$$

$$V_1 = \pi D_1^3/6 = \pi(6\,\text{m})^3/6 = 113.1\,\text{m}^3$$



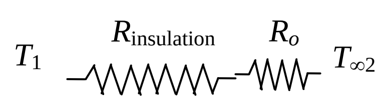
The rate of heat transfer to the LNG is

$$R_{\text{insulation}} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(3.05-3.0)\,\text{m}}{4\pi(0.00008\,\text{W/m.°C})(3.0\,\text{m})(3.05\,\text{m})} = 5.43562\,°\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(22\,\text{W/m}^2.°\text{C})(116.9\,\text{m}^2)} = 0.00039\,°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.00039 + 5.43562 = 5.43601\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 2} - T_1}{R_{\text{total}}} = \frac{[18-(-160)]°\text{C}}{5.43601\,°\text{C/W}} = 32.74\,\text{W}$$



The amount of heat transfer to increase the LNG temperature from -160°C to -150°C is

$$m = \rho V_1 = (425\,\text{kg/m}^3)(113.1\text{m}^3) = 48{,}067.5\ \text{kg}$$

$$Q = mC\Delta T = (48{,}067.5\,\text{kg})(3.475\,\text{kJ/kg.°C})[(-150) - (-160)°\text{C}] = 1{,}670{,}346\,\text{kJ}$$

Assuming that heat will be lost from the LNG at an average rate of 32.74 W, the time period for the LNG temperature to rise to -150°C becomes

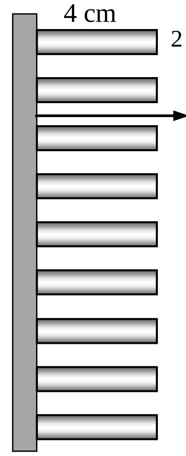
$$\Delta t = \frac{Q}{\dot{Q}} = \frac{1{,}670{,}346\,\text{kJ}}{0.03274\ \text{kW}} = 51{,}018{,}498\,\text{s} = 14{,}174\,\text{h} = \mathbf{590.5\,days}$$

3-174 A hot plate is to be cooled by attaching aluminum fins of square cross section on one side. The number of fins needed to triple the rate of heat transfer is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum fins is given to be k = 237 W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the square cross-section fins can be determined to be



$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{4ha}{ka^2}} = \sqrt{\frac{4(20\text{ W/m}^2.°\text{C})(0.002\text{ m})}{(237\text{ W/m.°C})(0.002\text{ m})^2}} = 12.99\text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(12.99\text{ m}^{-1} \times 0.04\text{ m})}{12.99\text{ m}^{-1} \times 0.04\text{ m}} = 0.919$$

The finned and unfinned surface areas, and heat transfer rates from these areas are

$$A_{\text{fin}} = n_{\text{fin}} \times 4 \times (0.002\text{ m})(0.04\text{m}) = 0.00032 n_{\text{fin}}\ \text{ m}^2$$
$$A_{\text{unfinned}} = (0.15\text{ m})(0.20\text{m}) - n_{\text{fin}}(0.002\text{ m})(0.002\text{m})$$
$$= 0.03 - 0.000004 n_{\text{fin}}\ \text{ m}^2$$
$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}} \dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} h A_{\text{fin}} (T_b - T_\infty)$$
$$= 0.919(20\text{ W/m}^2.°\text{C})(0.00032 n_{\text{fin}}\text{ m}^2)(85-25)°\text{C}$$
$$= 0.35328 n_{\text{fin}}\ \text{W}$$
$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_b - T_\infty) = (20\text{ W/m}^2.°\text{C})(0.03 - 0.000004 n_{\text{fin}}\ \text{ m}^2)(85-25)°\text{C}$$
$$= 36 - 0.0048 n_{\text{fin}}\ \text{W}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 0.35328 n_{\text{fin}} + 36 - 0.0048 n_{\text{fin}}\ \text{W}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (0.15\text{ m})(0.20\text{ m}) = 0.03\text{ m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (20\text{ W/m}^2.°\text{C})(0.03\text{ m}^2)(85-25)°\text{C} = 36\text{ W}$$

The number of fins can be determined from the overall fin effectiveness equation

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} \longrightarrow 3 = \frac{0.35328 n_{\text{fin}} + 36 - 0.0048 n_{\text{fin}}}{36} \longrightarrow n_{\text{fin}} = \mathbf{207}$$

3-175

```
"!PROBLEM 3-175"

"GIVEN"
A_surface=0.15*0.20 "[m^2]"
T_b=85 "[C]"
k=237 "[W/m-C]"
side=0.002  "[m]"
L=0.04 "[m]"
T_infinity=25 "[C]"
h=20 "[W/m^2-C]"
"epsilon_fin=3 parameter to be varied"

"ANALYSIS"
A_c=side^2
p=4*side
a=sqrt((h*p)/(k*A_c))
eta_fin=tanh(a*L)/(a*L)
A_fin=n_fin*4*side*L
A_unfinned=A_surface-n_fin*side^2
Q_dot_finned=eta_fin*h*A_fin*(T_b-T_infinity)
Q_dot_unfinned=h*A_unfinned*(T_b-T_infinity)
Q_dot_total_fin=Q_dot_finned+Q_dot_unfinned
Q_dot_nofin=h*A_surface*(T_b-T_infinity)
epsilon_fin=Q_dot_total_fin/Q_dot_nofin
```

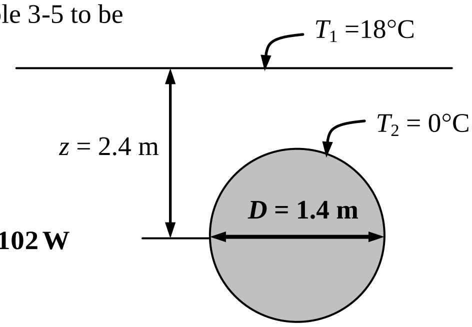
ε_{fin}	n_{fin}
1.5	51.72
1.75	77.59
2	103.4
2.25	129.3
2.5	155.2
2.75	181
3	206.9
3.25	232.8
3.5	258.6
3.75	284.5
4	310.3
4.25	336.2
4.5	362.1
4.75	387.9
5	413.8

3-176 A spherical tank containing iced water is buried underground. The rate of heat transfer to the tank is to be determined for the insulated and uninsulated ground surface cases.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant. **4** The tank surface is assumed to be at the same temperature as the iced water because of negligible resistance through the steel.

Properties The thermal conductivity of the concrete is given to be k = 0.55 W/m·°C.

ğAnalysis The shape factor for this configuration is given in Table 3-5 to be



$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(1.4\,\text{m})}{1 - 0.25\dfrac{1.4\,\text{m}}{2.4\,\text{m}}} = 10.30\,\text{m}$$

Then the steady rate of heat transfer from the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (10.30\,\text{m})(0.55\,\text{W/m.°C})(18 - 0)\text{°C} = \mathbf{102\,W}$$

If the ground surface is insulated,

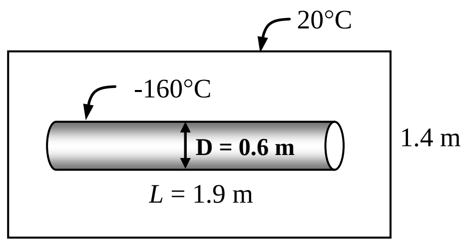
$$S = \frac{2\pi D}{1 + 0.25\dfrac{D}{z}} = \frac{2\pi(1.4\,\text{m})}{1 + 0.25\dfrac{1.4\,\text{m}}{2.4\,\text{m}}} = 7.68\,\text{m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (7.68\,\text{m})(0.55\,\text{W/m.°C})(18 - 0)\text{°C} = \mathbf{76\,W}$$

3-177 A cylindrical tank containing liquefied natural gas (LNG) is placed at the center of a square solid bar. The rate of heat transfer to the tank and the LNG temperature at the end of a one-month period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the bar is constant. **4** The tank surface is at the same temperature as the iced water.

Properties The thermal conductivity of the bar is given to be k = 0.0006 W/m·°C. The density and the specific heat of LNG are given to be 425 kg/m^3 and 3.475 kJ/kg·°C, respectively,



Analysis The shape factor for this configuration is given in Table 3-5 to be

$$S = \frac{2\pi L}{\ln\left(\frac{1.08w}{D}\right)} = \frac{2\pi(1.9\text{ m})}{\ln\left(1.08\frac{1.4\text{ m}}{0.6\text{ m}}\right)} = 12.92\text{ m}$$

Then the steady rate of heat transfer to the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (12.92\text{ m})(0.0006\text{ W/m.°C})[20-(-160)]\text{°C} = \mathbf{1.395\ W}$$

The mass of LNG is

$$m = \rho V = \rho\pi\frac{D^3}{6} = (425\text{ kg/m}^3)\pi\frac{(0.6\text{ m})^3}{6} = 48.07\text{ kg}$$

The amount heat transfer to the tank for a one-month period is

$$Q = \dot{Q}\Delta t = (1.395\text{ W})(30\times 24\times 3600\text{s}) = 3{,}615{,}840\text{ J}$$

Then the temperature of LNG at the end of the month becomes

$$Q = mC_p(T_1 - T_2)$$
$$3{,}615{,}840\text{ J} = (48.07\text{kg})(3475\text{J/kg.°C})[(-160) - T_2]\text{°C}$$
$$T_2 = \mathbf{-138.4°C}$$

3-178 ··· 3-184 Design and Essay Problems
