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سایت آموزش مهندسی مکانیک

Chapter 4

TRANSIENT HEAT CONDUCTION

Lumped System Analysis

4-1C In heat transfer analysis, some bodies are observed to behave like a "lump" whose entire body temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only. Heat transfer analysis which utilizes this idealization is known as the lumped system analysis. It is applicable when the Biot number (the ratio of conduction resistance within the body to convection resistance at the surface of the body) is less than or equal to 0.1.

4-2C The lumped system analysis is more likely to be applicable for the body cooled naturally since the Biot number is proportional to the convection heat transfer coefficient, which is proportional to the air velocity. Therefore, the Biot number is more likely to be less than 0.1 for the case of natural convection.

4-3C The lumped system analysis is more likely to be applicable for the body allowed to cool in the air since the Biot number is proportional to the convection heat transfer coefficient, which is larger in water than it is in air because of the larger thermal conductivity of water. Therefore, the Biot number is more likely to be less than 0.1 for the case of the solid cooled in the air

4-4C The temperature drop of the potato during the second minute will be less than 4°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

4-5C The temperature rise of the potato during the second minute will be less than 5°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

4-6C Biot number represents the ratio of conduction resistance within the body to convection resistance at the surface of the body. The Biot number is more likely to be larger for poorly conducting solids since such bodies have larger resistances against heat conduction.

4-7C The heat transfer is proportional to the surface area. Two half pieces of the roast have a much larger surface area than the single piece and thus a higher rate of heat transfer. As a result, the two half pieces will cook much faster than the single large piece.

4-8C The cylinder will cool faster than the sphere since heat transfer rate is proportional to the surface area, and the sphere has the smallest area for a given volume.

4-9C The lumped system analysis is more likely to be applicable in air than in water since the convection heat transfer coefficient and thus the Biot number is much smaller in air.

4-10C The lumped system analysis is more likely to be applicable for a golden apple than for an actual apple since the thermal conductivity is much larger and thus the Biot number is much smaller for gold.

4-11C The lumped system analysis is more likely to be applicable to slender bodies than the well-rounded bodies since the characteristic length (ratio of volume to surface area) and thus the Biot number is much smaller for slender bodies.

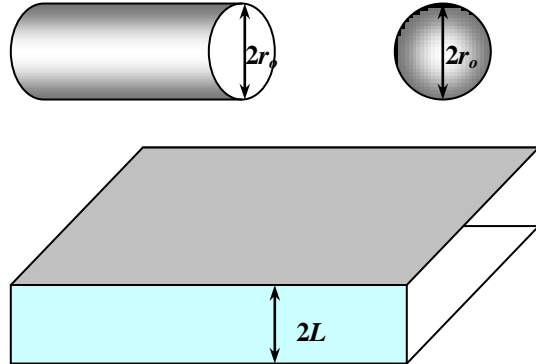
4-12 Relations are to be obtained for the characteristic lengths of a large plane wall of thickness $2L$, a very long cylinder of radius r_o and a sphere of radius r_o

Analysis Relations for the characteristic lengths of a large plane wall of thickness $2L$, a very long cylinder of radius r_o and a sphere of radius r_o are

$$L_{c,wall} = \frac{V}{A_{surface}} = \frac{2LA}{2A} = L$$

$$L_{c,cylinder} = \frac{V}{A_{surface}} = \frac{\pi r_o^2 h}{2\pi r_o h} = \frac{r_o}{2}$$

$$L_{c,sphere} = \frac{V}{A_{surface}} = \frac{4\pi r_o^3 / 3}{4\pi r_o^2} = \frac{r_o}{3}$$

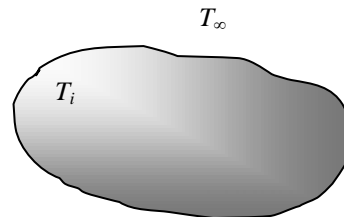


4-13 A relation for the time period for a lumped system to reach the average temperature $(T_i + T_\infty) / 2$ is to be obtained.

Analysis The relation for time period for a lumped system to reach the average temperature $(T_i + T_\infty) / 2$ can be determined as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{\frac{T_i + T_\infty}{2} - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T_i - T_\infty}{2(T_i - T_\infty)} = e^{-bt} \longrightarrow \frac{1}{2} = e^{-bt}$$

$$-bt = -\ln 2 \longrightarrow t = \frac{\ln 2}{b} = \frac{0.693}{b}$$



4-14 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

Assumptions **1** The junction is spherical in shape with a diameter of $D = 0.0012$ m. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is $Bi < 0.1$ so that the lumped system analysis is applicable (this assumption will be verified).

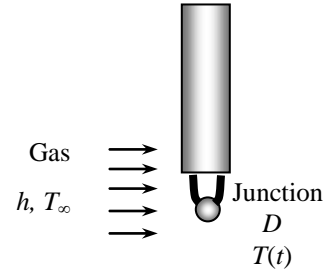
Properties The properties of the junction are given to be $k = 35$ W/m \cdot °C, $\rho = 8500$ kg/m 3 , and $C_p = 320$ J/kg \cdot °C.

Analysis The characteristic length of the junction and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3 / 6}{\pi D^2} = \frac{D}{6} = \frac{0.0012 \text{ m}}{6} = 0.0002 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(65 \text{ W/m}^2 \cdot \text{°C})(0.0002 \text{ m})}{(35 \text{ W/m} \cdot \text{°C})} = 0.00037 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the thermocouple to read 99% of the initial temperature difference is determined from



$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{65 \text{ W/m}^2 \cdot \text{°C}}{(8500 \text{ kg/m}^3)(320 \text{ J/kg} \cdot \text{°C})(0.0002 \text{ m})} = 0.1195 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.1195 \text{ s}^{-1})t} \longrightarrow t = \mathbf{38.5 \text{ s}}$$

4-15E A number of brass balls are to be quenched in a water bath at a specified rate. The temperature of the balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

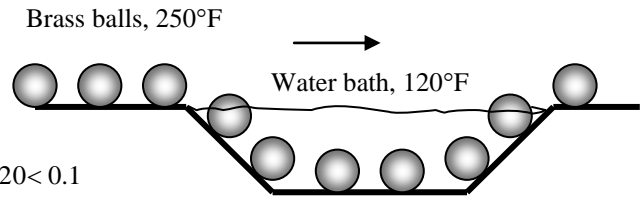
Assumptions **1** The balls are spherical in shape with a radius of $r_0 = 1$ in. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is $Bi < 0.1$ so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the brass balls are given to be $k = 64.1$ Btu/h.ft.°F, $\rho = 532$ lbm/ft³, and $C_p = 0.092$ Btu/lbm.°F.

Analysis (a) The characteristic length and the Biot number for the brass balls are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3 / 6}{\pi D^2} = \frac{D}{6} = \frac{2 / 12 \text{ ft}}{6} = 0.02778 \text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42 \text{ Btu/h.ft}^2 \cdot \text{°F})(0.02778 \text{ ft})}{(64.1 \text{ Btu/h.ft.°F})} = 0.01820 < 0.1$$



The lumped system analysis is applicable since $Bi < 0.1$. Then the temperature of the balls after quenching becomes

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{42 \text{ Btu/h.ft}^2 \cdot \text{°F}}{(532 \text{ lbm/ft}^3)(0.092 \text{ Btu/lbm.°F})(0.02778 \text{ ft})} = 30.9 \text{ h}^{-1} = 0.00858 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 120}{250 - 120} = e^{-(0.00858 \text{ s}^{-1})(120 \text{ s})} \longrightarrow T(t) = \mathbf{166^\circ\text{F}}$$

(b) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (532 \text{ lbm/ft}^3) \frac{\pi (2 / 12 \text{ ft})^3}{6} = 1.290 \text{ lbm}$$

$$Q = m C_p [T_i - T(t)] = (1.29 \text{ lbm})(0.092 \text{ Btu/lbm.°F})(250 - 166)^\circ\text{F} = 9.97 \text{ Btu}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{ball} Q_{ball} = (120 \text{ balls/min}) \times (9.97 \text{ Btu}) = \mathbf{1196 \text{ Btu/min}}$$

Therefore, heat must be removed from the water at a rate of 1196 Btu/min in order to keep its temperature constant at 120°F.

4-16E A number of aluminum balls are to be quenched in a water bath at a specified rate. The temperature of balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

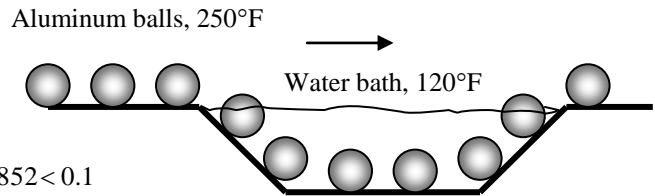
Assumptions 1 The balls are spherical in shape with a radius of $r_0 = 1$ in. 2 The thermal properties of the balls are constant. 3 The heat transfer coefficient is constant and uniform over the entire surface. 4 The Biot number is $Bi < 0.1$ so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the aluminum balls are $k = 137$ Btu/h.ft.°F, $\rho = 168$ lbm/ft³, and $C_p = 0.216$ Btu/lbm.°F (Table A-3E).

Analysis (a) The characteristic length and the Biot number for the aluminum balls are

$$L_c = \frac{V}{A} = \frac{\pi D^3 / 6}{\pi D^2} = \frac{D}{6} = \frac{2 / 12 \text{ ft}}{6} = 0.02778 \text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42 \text{ Btu/h.ft}^2 \cdot \text{°F})(0.02778 \text{ ft})}{(137 \text{ Btu/h.ft.°F})} = 0.00852 < 0.1$$



The lumped system analysis is applicable since $Bi < 0.1$. Then the temperature of the balls after quenching becomes

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{42 \text{ Btu/h.ft}^2 \cdot \text{°F}}{(168 \text{ lbm/ft}^3)(0.216 \text{ Btu/lbm.°F})(0.02778 \text{ ft})} = 41.66 \text{ h}^{-1} = 0.01157 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 120}{250 - 120} = e^{-(0.01157 \text{ s}^{-1})(120 \text{ s})} \longrightarrow T(t) = \mathbf{152^\circ\text{F}}$$

(b) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (168 \text{ lbm/ft}^3) \frac{\pi (2 / 12 \text{ ft})^3}{6} = 0.4072 \text{ lbm}$$

$$Q = m C_p [T_i - T(t)] = (0.4072 \text{ lbm})(0.216 \text{ Btu/lbm.°F})(250 - 152)^\circ\text{F} = 8.62 \text{ Btu}$$

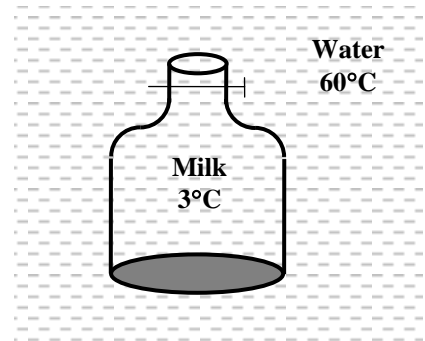
Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{ball} Q_{ball} = (120 \text{ balls/min}) \times (8.62 \text{ Btu}) = \mathbf{1034 \text{ Btu/min}}$$

Therefore, heat must be removed from the water at a rate of 1034 Btu/min in order to keep its temperature constant at 120°F.

4-17 Milk in a thin-walled glass container is to be warmed up by placing it into a large pan filled with hot water. The warming time of the milk is to be determined.

Assumptions **1** The glass container is cylindrical in shape with a radius of $r_0 = 3$ cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.



Properties The thermal conductivity, density, and specific heat of the milk at 20°C are $k = 0.607$ W/m.°C, $\rho = 998$ kg/m³, and $C_p = 4.182$ kJ/kg.°C (Table A-9).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.03\text{ m})^2(0.07\text{ m})}{2\pi(0.03\text{ m})(0.07\text{ m}) + 2\pi(0.03\text{ m})^2} = 0.01050\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(120\text{ W/m}^2\cdot\text{°C})(0.01050\text{ m})}{(0.607\text{ W/m}\cdot\text{°C})} = 2.076 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

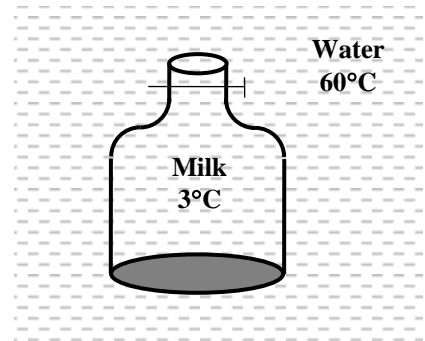
$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{120\text{ W/m}^2\cdot\text{°C}}{(998\text{ kg/m}^3)(4182\text{ J/kg}\cdot\text{°C})(0.01050\text{ m})} = 0.002738\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38 - 60}{3 - 60} = e^{-(0.002738\text{ s}^{-1})t} \longrightarrow t = 348\text{ s} = 5.8\text{ min}$$

Therefore, it will take about 6 minutes to warm the milk from 3 to 38°C.

4-18 A thin-walled glass containing milk is placed into a large pan filled with hot water to warm up the milk. The warming time of the milk is to be determined.

Assumptions **1** The glass container is cylindrical in shape with a radius of $r_0 = 3$ cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.



Properties The thermal conductivity, density, and specific heat of the milk at 20°C are $k = 0.607$ W/m.°C, $\rho = 998$ kg/m³, and $C_p = 4.182$ kJ/kg.°C (Table A-9).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.03\text{ m})^2(0.07\text{ m})}{2\pi(0.03\text{ m})(0.07\text{ m}) + 2\pi(0.03\text{ m})^2} = 0.01050\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(240\text{ W/m}^2\cdot\text{°C})(0.0105\text{ m})}{(0.607\text{ W/m}\cdot\text{°C})} = 4.15 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

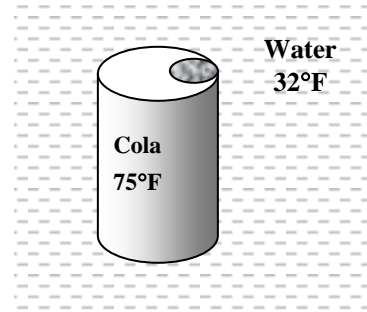
$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{240\text{ W/m}^2\cdot\text{°C}}{(998\text{ kg/m}^3)(4182\text{ J/kg}\cdot\text{°C})(0.0105\text{ m})} = 0.005477\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38 - 60}{3 - 60} = e^{-(0.005477\text{ s}^{-1})t} \longrightarrow t = \mathbf{174\text{ s} = 2.9\text{ min}}$$

Therefore, it will take about 3 minutes to warm the milk from 3 to 38°C.

4-19E A person shakes a can of drink in a iced water to cool it. The cooling time of the drink is to be determined.

Assumptions **1** The can containing the drink is cylindrical in shape with a radius of $r_0 = 1.25$ in. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.



Properties The density and specific heat of water at room temperature are $\rho = 62.22$ lbm/ft³, and $C_p = 0.999$ Btu/lbm.°F (Table A-9E).

Analysis Application of lumped system analysis in this case gives

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(1.25/12 \text{ ft})^2 (5/12 \text{ ft})}{2\pi(1.25/12 \text{ ft})(5/12 \text{ ft}) + 2\pi(1.25/12 \text{ ft})^2} = 0.04167 \text{ ft}$$

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{30 \text{ Btu/h}\cdot\text{ft}^2\cdot^\circ\text{F}}{(62.22 \text{ lbm/ft}^3)(0.999 \text{ Btu/lbm}\cdot^\circ\text{F})(0.04167 \text{ ft})} = 11.583 \text{ h}^{-1} = 0.00322 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{45 - 32}{80 - 32} = e^{-(0.00322 \text{ s}^{-1})t} \longrightarrow t = \mathbf{406 \text{ s}}$$

Therefore, it will take 7 minutes and 46 seconds to cool the canned drink to 45°F.

4-20 An iron whose base plate is made of an aluminum alloy is turned on. The time for the plate temperature to reach 140°C and whether it is realistic to assume the plate temperature to be uniform at all times are to be determined.

Assumptions **1** 85 percent of the heat generated in the resistance wires is transferred to the plate. **2** The thermal properties of the plate are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The density, specific heat, and thermal diffusivity of the aluminum alloy plate are given to be $\rho = 2770 \text{ kg/m}^3$, $C_p = 875 \text{ kJ/kg}\cdot^\circ\text{C}$, and $\alpha = 7.3 \times 10^{-5} \text{ m}^2/\text{s}$. The thermal conductivity of the plate can be determined from $\alpha = k/(\rho C_p) = 177 \text{ W/m}\cdot^\circ\text{C}$ (or it can be read from Table A-3).

Analysis The mass of the iron's base plate is

$$m = \rho V = \rho LA = (2770 \text{ kg/m}^3)(0.005 \text{ m})(0.03 \text{ m}^2) = 0.4155 \text{ kg}$$

Noting that only 85 percent of the heat generated is transferred to the plate, the rate of heat transfer to the iron's base plate is

$$\dot{Q}_{\text{in}} = 0.85 \times 1000 \text{ W} = 850 \text{ W}$$

The temperature of the plate, and thus the rate of heat transfer from the plate, changes during the process. Using the average plate temperature, the average rate of heat loss from the plate is determined from

$$\dot{Q}_{\text{loss}} = hA(T_{\text{plate,ave}} - T_\infty) = (12 \text{ W/m}^2 \cdot ^\circ\text{C})(0.03 \text{ m}^2) \left(\frac{140 + 22}{2} - 22 \right) ^\circ\text{C} = 21.2 \text{ W}$$

Energy balance on the plate can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{plate}} \rightarrow \dot{Q}_{\text{in}} \Delta t - \dot{Q}_{\text{out}} \Delta t = \Delta E_{\text{plate}} = mC_p \Delta T_{\text{plate}}$$

Solving for Δt and substituting,

$$\Delta t = \frac{mC_p \Delta T_{\text{plate}}}{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}} = \frac{(0.4155 \text{ kg})(875 \text{ J/kg}\cdot^\circ\text{C})(140 - 22)^\circ\text{C}}{(850 - 21.2) \text{ J/s}} = \mathbf{51.8 \text{ s}}$$

which is the time required for the plate temperature to reach 140°C. To determine whether it is realistic to assume the plate temperature to be uniform at all times, we need to calculate the Biot number,

$$L_c = \frac{V}{A_s} = \frac{LA}{A} = L = 0.005 \text{ m}$$

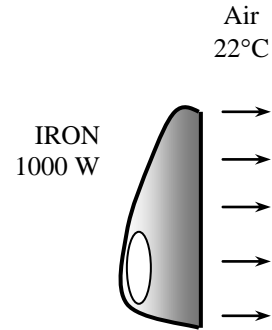
$$Bi = \frac{hL_c}{k} = \frac{(12 \text{ W/m}^2 \cdot ^\circ\text{C})(0.005 \text{ m})}{(177.0 \text{ W/m}\cdot^\circ\text{C})} = 0.00034 < 0.1$$

It is realistic to assume uniform temperature for the plate since $Bi < 0.1$.

Discussion This problem can also be solved by obtaining the differential equation from an energy balance on the plate for a differential time interval, and solving the differential equation. It gives

$$T(t) = T_\infty + \frac{\dot{Q}_{\text{in}}}{hA} \left(1 - \exp\left(-\frac{hA}{mC_p} t\right) \right)$$

Substituting the known quantities and solving for t again gives 51.8 s.



4-21 "PROBLEM 4-21"

"GIVEN"

$E_{\dot{}}=1000 \text{ [W]}$

$L=0.005 \text{ [m]}$

$A=0.03 \text{ [m}^2\text{]}$

$T_{\infty}=22 \text{ [C]}$

$T_i=T_{\infty}$

$h=12 \text{ [W/m}^2\text{-C], parameter to be varied}$

$f_{\text{heat}}=0.85$

$T_f=140 \text{ [C], parameter to be varied}$

"PROPERTIES"

$\rho=2770 \text{ [kg/m}^3\text{]}$

$C_p=875 \text{ [J/kg-C]}$

$\alpha=7.3E-5 \text{ [m}^2\text{/s]}$

"ANALYSIS"

$V=L \cdot A$

$m=\rho \cdot V$

$Q_{\dot{}}_{\text{in}}=f_{\text{heat}} \cdot E_{\dot{}}$

$Q_{\dot{}}_{\text{out}}=h \cdot A \cdot (T_{\text{ave}} - T_{\infty})$

$T_{\text{ave}}=1/2 \cdot (T_i + T_f)$

$(Q_{\dot{}}_{\text{in}} - Q_{\dot{}}_{\text{out}}) \cdot \text{time} = m \cdot C_p \cdot (T_f - T_i) \text{ "energy balance on the plate"}$

$h \text{ [W/m}^2\text{-C]}$	time [s]
5	51
7	51.22
9	51.43
11	51.65
13	51.88
15	52.1
17	52.32
19	52.55
21	52.78
23	53.01
25	53.24

$T_f \text{ [C]}$	time [s]
30	3.428
40	7.728
50	12.05
60	16.39
70	20.74
80	25.12
90	29.51
100	33.92
110	38.35
120	42.8
130	47.28
140	51.76
150	56.27
160	60.8
170	65.35
180	69.92
190	74.51

200	79.12
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4-22 Ball bearings leaving the oven at a uniform temperature of 900°C are exposed to air for a while before they are dropped into the water for quenching. The time they can stand in the air before their temperature falls below 850°C is to be determined.

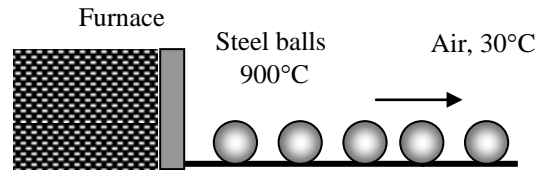
Assumptions **1** The bearings are spherical in shape with a radius of $r_0 = 0.6$ cm. **2** The thermal properties of the bearings are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is $Bi < 0.1$ so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the bearings are given to be $k = 15.1$ W/m·°C, $\rho = 8085$ kg/m³, and $C_p = 0.480$ kJ/kg·°F.

Analysis The characteristic length of the steel ball bearings and Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3 / 6}{\pi D^2} = \frac{D}{6} = \frac{0.012 \text{ m}}{6} = 0.002 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(125 \text{ W/m}^2 \cdot \text{°C})(0.002 \text{ m})}{(15.1 \text{ W/m} \cdot \text{°C})} = 0.0166 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the allowable time is determined to be

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{125 \text{ W/m}^2 \cdot \text{°C}}{(8085 \text{ kg/m}^3)(480 \text{ J/kg} \cdot \text{°C})(0.002 \text{ m})} = 0.01610 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{850 - 30}{900 - 30} = e^{-(0.0161 \text{ s}^{-1})t} \longrightarrow t = \mathbf{3.68 \text{ s}}$$

The result indicates that the ball bearing can stay in the air about 4 s before being dropped into the water.

4-23 A number of carbon steel balls are to be annealed by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The time of annealing and the total rate of heat transfer from the balls to the ambient air are to be determined.

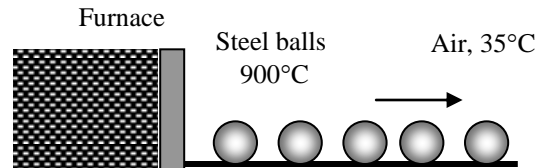
Assumptions 1 The balls are spherical in shape with a radius of $r_0 = 4$ mm. 2 The thermal properties of the balls are constant. 3 The heat transfer coefficient is constant and uniform over the entire surface. 4 The Biot number is $Bi < 0.1$ so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be $k = 54$ W/m.°C, $\rho = 7833$ kg/m³, and $C_p = 0.465$ kJ/kg.°C.

Analysis The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3 / 6}{\pi D^2} = \frac{D}{6} = \frac{0.008 \text{ m}}{6} = 0.0013 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(75 \text{ W/m}^2 \cdot \text{°C})(0.0013 \text{ m})}{(54 \text{ W/m} \cdot \text{°C})} = 0.0018 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the time for the annealing process is determined to be

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{75 \text{ W/m}^2 \cdot \text{°C}}{(7833 \text{ kg/m}^3)(465 \text{ J/kg} \cdot \text{°C})(0.0013 \text{ m})} = 0.01584 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100 - 35}{900 - 35} = e^{-(0.01584 \text{ s}^{-1})t} \longrightarrow t = \mathbf{163 \text{ s} = 2.7 \text{ min}}$$

The amount of heat transfer from a single ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (7833 \text{ kg/m}^3) \frac{\pi (0.008 \text{ m})^3}{6} = 0.0021 \text{ kg}$$

$$Q = mC_p [T_f - T_i] = (0.0021 \text{ kg})(465 \text{ J/kg} \cdot \text{°C})(900 - 100) \text{°C} = 781 \text{ J} = 0.781 \text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

$$\dot{Q} = \dot{n}_{\text{ball}} Q = (2500 \text{ balls/h}) \times (0.781 \text{ kJ/ball}) = 1,953 \text{ kJ/h} = \mathbf{543 \text{ W}}$$

4-24

!"PROBLEM 4-24"

"GIVEN"

D=0.008 "[m]"

"T_i=900 [C], parameter to be varied"

T_f=100 "[C]"

T_{infinity}=35 "[C]"

h=75 "[W/m²-C]"

n_{dot_ball}=2500 "[1/h]"

"PROPERTIES"

rho=7833 "[kg/m³]"

k=54 "[W/m-C]"

C_p=465 "[J/kg-C]"

alpha=1.474E-6 "[m²/s]"

"ANALYSIS"

A=pi*D²

V=pi*D³/6

L_c=V/A

Bi=(h*L_c)/k "if Bi < 0.1, the lumped sytem analysis is applicable"

b=(h*A)/(rho*C_p*V)

(T_f-T_{infinity})/(T_i-T_{infinity})=exp(-b*time)

m=rho*V

Q=m*C_p*(T_i-T_f)

Q_{dot}=n_{dot_ball}*Q*Convert(J/h, W)

T _i [C]	time [s]	Q [W]
500	127.4	271.2
550	134	305.1
600	140	339
650	145.5	372.9
700	150.6	406.9
750	155.3	440.8
800	159.6	474.7
850	163.7	508.6
900	167.6	542.5
950	171.2	576.4
1000	174.7	610.3

4-25 An electronic device is on for 5 minutes, and off for several hours. The temperature of the device at the end of the 5-min operating period is to be determined for the cases of operation with and without a heat sink.

Assumptions **1** The device and the heat sink are isothermal. **2** The thermal properties of the device and of the sink are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The specific heat of the device is given to be $C_p = 850 \text{ J/kg}\cdot^\circ\text{C}$. The specific heat of the aluminum sink is $903 \text{ J/kg}\cdot^\circ\text{C}$ (Table A-19), but can be taken to be $850 \text{ J/kg}\cdot^\circ\text{C}$ for simplicity in analysis.

Analysis (a) Approximate solution

This problem can be solved approximately by using an average temperature for the device when evaluating the heat loss. An energy balance on the device can be expressed as

$$E_{\text{in}} - E_{\text{out}} + E_{\text{generation}} = \Delta E_{\text{device}} \longrightarrow -\dot{Q}_{\text{out}}\Delta t + \dot{E}_{\text{generation}}\Delta t = mC_p\Delta T_{\text{device}}$$

or,
$$\dot{E}_{\text{generation}}\Delta t - hA_s\left(\frac{T + T_\infty}{2} - T_\infty\right)\Delta t = mC_p(T - T_\infty)$$

Substituting the given values,

$$(30 \text{ J/s})(5 \times 60 \text{ s}) - (12 \text{ W/m}^2 \cdot ^\circ\text{C})(0.0005 \text{ m}^2)\left(\frac{T - 25}{2}\right)^\circ\text{C}(5 \times 60 \text{ s}) = (0.02 \text{ kg})(850 \text{ J/kg}\cdot^\circ\text{C})(T - 25)^\circ\text{C}$$

which gives $T = 527.8^\circ\text{C}$

If the device were attached to an aluminum heat sink, the temperature of the device would be

$$(30 \text{ J/s})(5 \times 60 \text{ s}) - (12 \text{ W/m}^2 \cdot ^\circ\text{C})(0.0085 \text{ m}^2)\left(\frac{T - 25}{2}\right)^\circ\text{C}(5 \times 60 \text{ s}) = (0.20 + 0.02) \text{ kg} \times (850 \text{ J/kg}\cdot^\circ\text{C})(T - 25)^\circ\text{C}$$

which gives $T = 69.5^\circ\text{C}$

Note that the temperature of the electronic device drops considerably as a result of attaching it to a heat sink.

(b) Exact solution

This problem can be solved exactly by obtaining the differential equation from an energy balance on the device for a differential time interval dt . We will get

$$\frac{d(T - T_\infty)}{dt} + \frac{hA_s}{mC_p}(T - T_\infty) = \frac{\dot{E}_{\text{generation}}}{mC_p}$$

It can be solved to give

$$T(t) = T_\infty + \frac{\dot{E}_{\text{generation}}}{hA_s} \left(1 - \exp\left(-\frac{hA_s}{mC_p}t\right) \right)$$

Substituting the known quantities and solving for t gives 527.3°C for the first case and 69.4°C for the second case, which are practically identical to the results obtained from the approximate analysis.

Electronic
device
30 W

