Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres

4-26C A cylinder whose diameter is small relative to its length can be treated as an infinitely long cylinder. When the diameter and length of the cylinder are comparable, it is not proper to treat the cylinder as being infinitely long. It is also not proper to use this model when finding the temperatures near the bottom or top surfaces of a cylinder since heat transfer at those locations can be two-dimensional.

4-27C Yes. A plane wall whose one side is insulated is equivalent to a plane wall that is twice as thick and is exposed to convection from both sides. The midplane in the latter case will behave like an insulated surface because of thermal symmetry.

4-28C The solution for determination of the one-dimensional transient temperature distribution involves many variables that make the graphical representation of the results impractical. In order to reduce the number of parameters, some variables are grouped into dimensionless quantities.

4-29C The Fourier number is a measure of heat conducted through a body relative to the heat stored. Thus a large value of Fourier number indicates faster propagation of heat through body. Since Fourier number is proportional to time, doubling the time will also double the Fourier number.

4-30C This case can be handled by setting the heat transfer coefficient h to infinity ∞ since the temperature of the surrounding medium in this case becomes equivalent to the surface temperature.

4-31C The maximum possible amount of heat transfer will occur when the temperature of the body reaches the temperature of the medium, and can be determined from $Q_{\max} = mC_p(T_\infty - T_i)$.

4-32C When the Biot number is less than 0.1, the temperature of the sphere will be nearly uniform at all times. Therefore, it is more convenient to use the lumped system analysis in this case.

4-33 A student calculates the total heat transfer from a spherical copper ball. It is to be determined whether his/her result is reasonable.

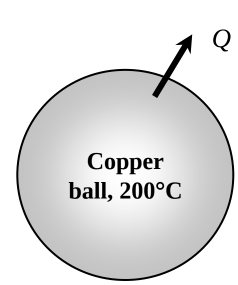
Assumptions The thermal properties of the copper ball are constant at room temperature.

Properties The density and specific heat of the copper ball are ρ = 8933 kg/m^3, and C_p = 0.385 kJ/kg.°C (Table A-3).

Analysis The mass of the copper ball and the maximum amount of heat transfer from the copper ball are

$$m = \rho V = \rho\left(\frac{\pi D^3}{6}\right) = (8933\,\text{kg/m}^3)\left[\frac{\pi(0.15\,\text{m})^3}{6}\right] = 15.79\,\text{kg}$$

$$Q_{\max} = mC_p[T_i - T_\infty] = (15.79\,\text{kg})(0.385\,\text{kJ/kg.°C})(200-25)\text{°C} = 1064\,\text{kJ}$$



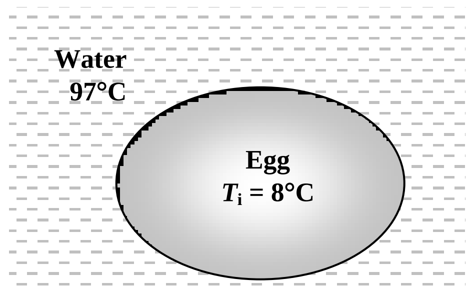
Discussion The student's result of 4520 kJ is **not reasonable** since it is greater than the maximum possible amount of heat transfer.

4-34 An egg is dropped into boiling water. The cooking time of the egg is to be determined. √

Assumptions **1** The egg is spherical in shape with a radius of r_0 = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs are given to be k = 0.6 W/m.°C and $\alpha = 0.14\times10^{-6}$ m^2/s.

Analysis The Biot number for this process is



$$Bi = \frac{hr_o}{k} = \frac{(1400 \text{ W/m}^2.°\text{C})(0.0275 \text{ m})}{(0.6 \text{ W/m}.°\text{C})} = 64.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 3.0877 \text{ and } A_1 = 1.9969$$

Then the Fourier number becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-97}{8-97} = (1.9969)e^{-(3.0877)^2 \tau} \longrightarrow \tau = 0.198 \approx 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the time required for the temperature of the center of the egg to reach 70°C is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.198)(0.0275\text{m})^2}{(0.14\times10^{-6} \text{ m}^2/\text{s})} = 1068\text{s} = \mathbf{17.8 min}$$

4-35

```
"!PROBLEM 4-35"
"GIVEN"
D=0.055 "[m]"
T_i=8 "[C]"
"T_o=70 [C], parameter to be varied"
T_infinity=97 "[C]"
h=1400 "[W/m^2-C]"

"PROPERTIES"
k=0.6 "[W/m-C]"
alpha=0.14E-6 "[m^2/s]"

"ANALYSIS"
Bi=(h*r_o)/k
r_o=D/2
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1=1.9969
A_1=3.0863
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
time=(tau*r_o^2)/alpha*Convert(s, min)
```

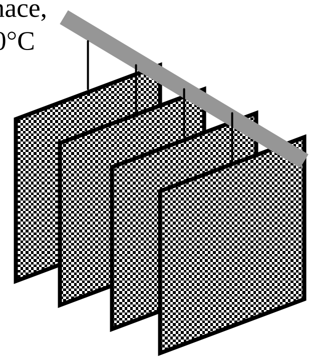
T_o [C]	time [min]
50	39.86
55	42.4
60	45.26
65	48.54
70	52.38
75	57
80	62.82
85	70.68
90	82.85
95	111.1

4-36 Large brass plates are heated in an oven. The surface temperature of the plates leaving the oven is to be determined.

Assumptions **1** Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the plate are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of brass at room temperature are given to be k = 110 W/m.°C, $\alpha = 33.9\times10^{-6}$ m²/s

Analysis The Biot number for this process is



$$Bi = \frac{hL}{k} = \frac{(80 \text{ W/m}^2.°\text{C})(0.015 \text{ m})}{(110 \text{ W/m}.°\text{C})} = 0.0109$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 0.1039 \quad \text{and} \quad A_1 = 1.0018$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9\times10^{-6} \text{ m}^2/\text{s})(10 \text{ min}\times 60 \text{ s/min})}{(0.015 \text{ m})^2} = 90.4 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the plates becomes

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0018)e^{-(0.1039)^2(90.4)}\cos(0.1039) = 0.378$$

$$\frac{T(L,t)-700}{25-700} = 0.378 \longrightarrow T(L,t) = \mathbf{445°C}$$

Discussion This problem can be solved easily using the lumped system analysis since Bi < 0.1, and thus the lumped system analysis is applicable. It gives

$$\alpha = \frac{k}{\rho C_p} \rightarrow \rho C_p = \frac{k}{\alpha} = \frac{110 \text{ W/m}\cdot°\text{C}}{33.9\times10^{-6} \text{ m}^2/s} = 3.245\times10^6 \text{ W}\cdot\text{s/m}^3\cdot°\text{C}$$

$$b = \frac{hA}{\rho V C_p} = \frac{hA}{\rho(LA)C_p} = \frac{h}{\rho L C_p} = \frac{h}{L(k/\alpha)} = \frac{80 \text{ W/m}^2\cdot°\text{C}}{(0.015 \text{ m})(3.245\times10^6 \text{ W}\cdot\text{s/m}^3\cdot°\text{C})} = 0.001644\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt} = 700°\text{C} + (25\text{-}700°\text{C})e^{-(0.001644\text{ s}^{-1})(600\text{ s})} = \mathbf{448°C}$$

which is almost identical to the result obtained above.

4-37 "!PROBLEM 4-37"

```
"GIVEN"
L=0.03/2 "[m]"
T_i=25 "[C]"
T_infinity=700 "[C], parameter to be varied"
time=10 "[min], parameter to be varied"
h=80 "[W/m^2-C]"

"PROPERTIES"
k=110 "[W/m-C]"
alpha=33.9E-6 "[m^2/s]"

"ANALYSIS"
Bi=(h*L)/k
"From Table 4-1, corresponding to this Bi number, we read"
lambda_1=0.1039
A_1=1.0018
tau=(alpha*time*Convert(min, s))/L^2
(T_L-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*Cos(lambda_1*L/L)
```

T_∞ [C]	T_L [C]
500	321.6
525	337.2
550	352.9
575	368.5
600	384.1
625	399.7
650	415.3
675	430.9
700	446.5
725	462.1
750	477.8
775	493.4
800	509
825	524.6
850	540.2
875	555.8
900	571.4

time [min]	T_L [C]
2	146.7
4	244.8
6	325.5
8	391.9
10	446.5
12	491.5
14	528.5
16	558.9
18	583.9
20	604.5
22	621.4
24	635.4
26	646.8
28	656.2

30	664

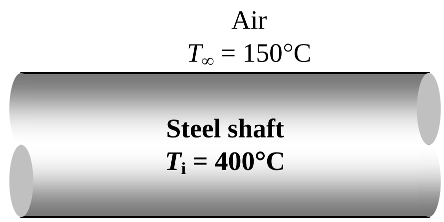
4-38 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be $k = 14.9$ W/m.°C, $\rho = 7900$ kg/m^3, $C_p = 477$ J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60\,\text{W/m}^2.°\text{C})(0.175\,\text{m})}{(14.9\,\text{W/m.°C})} = 0.705$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.0935 \quad \text{and} \quad A_1 = 1.1558$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(20\times60\,\text{s})}{(0.175\,\text{m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.1558)e^{-(1.0935)^2(0.1548)} = 0.9605$$

$$\frac{T_0 - 150}{400 - 150} = 0.9605 \longrightarrow T_0 = \mathbf{390°C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho V = \rho\pi r_o^2 L = (7900\,\text{kg/m}^3)[\pi(0.175\,\text{m})^2(1\,\text{m})] = 760.1\,\text{kg}$$

$$Q_{\max} = mC_p[T_\infty - T_i] = (760.1\,\text{kg})(0.477\,\text{kJ/kg.°C})(400-150)°\text{C} = 90{,}638\,\text{kJ}$$

Once the constant $J_1 = 0.4689$ is determined from Table 4-2 corresponding to the constant $\lambda_1 = 1.0935$, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{390-150}{400-150}\right)\frac{0.4689}{1.0935} = 0.177$$

$$Q = 0.177(90{,}638\,\text{kJ}) = \mathbf{16{,}015\,kJ}$$

4-39

```
"!PROBLEM 4-39"

"GIVEN"
r_o=0.35/2 "[m]"
T_i=400 "[C]"
T_infinity=150 "[C]"
h=60 "[W/m^2-C]"
"time=20 [min], parameter to be varied"

"PROPERTIES"
k=14.9 "[W/m-C]"
rho=7900 "[kg/m^3]"
C_p=477 "[J/kg-C]"
alpha=3.95E-6 "[m^2/s]"

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1=1.0935
A_1=1.1558
J_1=0.4709 "From Table 4-2, corresponding to lambda_1"
tau=(alpha*time*Convert(min, s))/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)

L=1 "[m], 1 m length of the cylinder is considered"
V=pi*r_o^2*L
m=rho*V
Q_max=m*C_p*(T_i-T_infinity)*Convert(J, kJ)
Q/Q_max=1-2*(T_o-T_infinity)/(T_i-T_infinity)*J_1/lambda_1
```

time [min]	T_o [C]	Q [kJ]
5	425.9	4491
10	413.4	8386
15	401.5	12105
20	390.1	15656
25	379.3	19046
30	368.9	22283
35	359	25374
40	349.6	28325
45	340.5	31142
50	331.9	33832
55	323.7	36401
60	315.8	38853

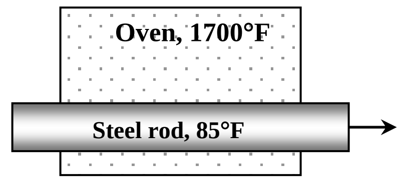
4-40E Long cylindrical steel rods are heat-treated in an oven. Their centerline temperature when they leave the oven is to be determined.

Assumptions **1** Heat conduction in the rods is one-dimensional since the rods are long and they have thermal symmetry about the center line. **2** The thermal properties of the rod are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of AISI stainless steel rods are given to be k = 7.74 Btu/h.ft.°F, α = 0.135 ft^2/h.

Analysis The time the steel rods stays in the oven can be determined from

$$t = \frac{\text{length}}{\text{velocity}} = \frac{30 \text{ ft}}{10 \text{ ft / min}} = 3 \text{ min} = 180 \text{ s}$$



The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(20 \text{ Btu/h.ft}^2.°\text{F})(2/12 \text{ ft})}{(7.74 \text{ Btu/h.ft.°F})} = 0.4307$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 0.8784 \quad \text{and} \quad A_1 = 1.0995$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.135 \text{ ft}^2 \text{ / h})(3/60 \text{ h})}{(2/12 \text{ ft})^2} = 0.243$$

Then the temperature at the center of the rods becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0995)e^{-(0.8784)^2(0.243)} = 0.912$$

$$\frac{T_0 - 1700}{85 - 1700} = 0.912 \longrightarrow T_0 = \mathbf{228°F}$$

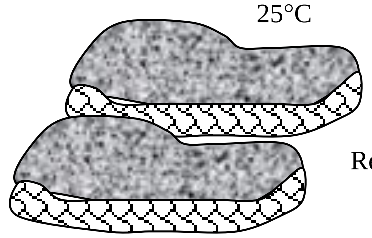
4-41 Steaks are cooled by passing them through a refrigeration room. The time of cooling is to be determined.

Assumptions **1** Heat conduction in the steaks is one-dimensional since the steaks are large relative to their thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the steaks are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of steaks are given to be k = 0.45 W/m.°C and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hL}{k} = \frac{(9\text{ W/m}^2.°\text{C})(0.01\text{ m})}{(0.45\text{ W/m.°C})} = 0.200$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311$$

The Fourier number is

$$\frac{T(L,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 L / L)$$

$$\frac{2-(-11)}{25-(-11)} = (1.0311)e^{-(0.4328)^2 \tau} \cos(0.4328) \longrightarrow \tau = 5.601 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the length of time for the steaks to be kept in the refrigerator is determined to be

$$t = \frac{\tau L^2}{\alpha} = \frac{(5.601)(0.01\text{ m})^2}{(0.91\times10^{-7}\text{ m}^2\text{/s})} = 6155\text{ s} = \mathbf{102.6\,min}$$

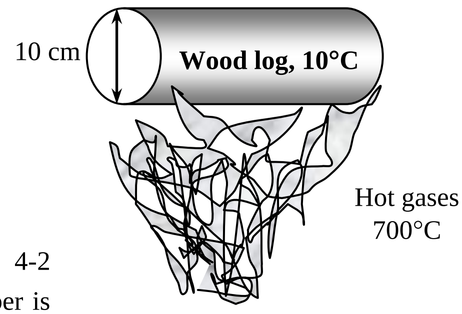
4-42 A long cylindrical wood log is exposed to hot gases in a fireplace. The time for the ignition of the wood is to be determined.

Assumptions **1** Heat conduction in the wood is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the wood are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of wood are given to be $k = 0.17$ W/m.°C, $\alpha = 1.28\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(13.6 \text{ W/m}^2.°\text{C})(0.05 \text{ m})}{(0.17 \text{ W/m.°C})} = 4.00$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.9081 \text{ and } A_1 = 1.4698$$

Once the constant J_0 is determined from Table 4-2 corresponding to the constant λ_1=1.9081, the Fourier number is determined to be

$$\frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o / r_o)$$

$$\frac{420-500}{10-500} = (1.4698)e^{-(1.9081)^2\tau}(0.2771) \longrightarrow \tau = 0.251$$

which is above the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can be used. Then the length of time before the log ignites is

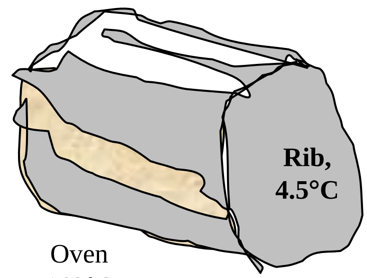
$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.251)(0.05 \text{ m})^2}{(1.28\times10^{-7} \text{ m}^2\text{/s})} = 4904 \text{ s} = \mathbf{81.7 \text{ min}}$$

4-43 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is rare done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be k = 0.45 W/m.°C, ρ = 1200 kg/m^3, C_p = 4.1 kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s.

Analysis (*a*) The radius of the roast is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.002667\text{m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.002667\text{m}^3)}{4\pi}} = 0.08603\text{m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2\text{/s})(2\times3600+45\times60)\text{s}}{(0.08603\text{m})^2} = 0.1217$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{60-163}{4.5-163} = 0.65 = A_1 e^{-\lambda_1^2(0.1217)}$$

It is determined from Table 4-1 by trial and error that this equation is satisfied when Bi = 30, which corresponds to $\lambda_1 = 3.0372$ and $A_1 = 1.9898$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(30)}{(0.08603\text{m})} = \mathbf{156.9\ W/m^2.°C}$$

This value seems to be larger than expected for problems of this kind. This is probably due to the Fourier number being less than 0.2.

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9898)e^{-(3.0372)^2(0.1217)}\frac{\sin(3.0372\text{ rad})}{3.0372}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.0222 \longrightarrow T(r_o,t) = \mathbf{159.5°C}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mC_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163 - 4.5)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.65)\frac{\sin(3.0372) - (3.0372)\cos(3.0372)}{(3.0372)^3} = 0.783$$

$$Q = 0.783Q_{\max} = (0.783)(2080\text{ kJ}) = \mathbf{1629\ kJ}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{71-163}{4.5-163} = (1.9898)e^{-(3.0372)^2\tau} \longrightarrow \tau = 0.1336$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1336)(0.08603\text{m})^2}{(0.91\times10^{-7}\text{ m}^2/\text{s})} = 10{,}866\text{ s} = 181\text{ min} \cong \mathbf{3\ hr}$$

This result is close to the listed value of 3 hours and 20 minutes. The difference between the two results is due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

4-44 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is well-done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

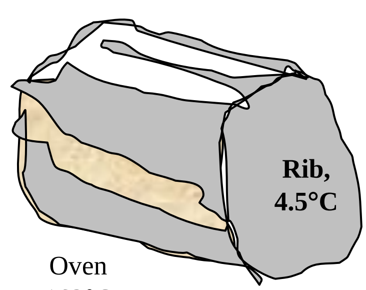
Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be k = 0.45 W/m.°C, ρ = 1200 kg/m^3, C_p = 4.1 kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis (*a*) The radius of the rib is determined to be

$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.00267\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.00267\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$



The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(4\times3600 + 15\times60)\text{s}}{(0.08603\text{m})^2} = 0.1881$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution formulation can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{77-163}{4.5-163} = 0.543 = A_1 e^{-\lambda_1^2 (0.1881)}$$

It is determined from Table 4-1 by trial and error that this equation is satisfied when Bi = 4.3, which corresponds to $\lambda_1 = 2.4900$ and $A_1 = 1.7402$. Then the heat transfer coefficient can be determined from.

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\,\text{W/m.°C})(4.3)}{(0.08603\text{m})} = \mathbf{22.5\,W/m^2.°C}$$

(b) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.7402)e^{-(2.49)^2 (0.1881)} \frac{\sin(2.49)}{2.49}$$

$$\frac{T(r_o,t) - 163}{4.5-163} = 0.132 \longrightarrow T(r_o,t) = \mathbf{142.1°C}$$

(c) The maximum possible heat transfer is

$$Q_{\max} = mC_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\,\text{kJ/kg.°C})(163-4.5)\text{°C} = 2080\,\text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph} \frac{\sin(\lambda_1) - \lambda_1 \cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.543) \frac{\sin(2.49) - (2.49)\cos(2.49)}{(2.49)^3} = 0.727$$

$$Q = 0.727 Q_{\max} = (0.727)(2080\,\text{kJ}) = \mathbf{1512\,kJ}$$

(d) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{71-163}{4.5-163} = (1.7402)e^{-(2.49)^2 \tau} \longrightarrow \tau = 0.177$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.177)(0.08603\text{m})^2}{(0.91\times10^{-7}\text{ m}^2/\text{s})} = 14{,}403\text{s} = 240.0\text{ min} = \mathbf{4\,hr}$$

This result is close to the listed value of 4 hours and 15 minutes. The difference between the two results is probably due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

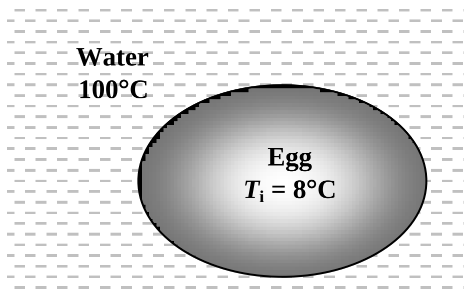
Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

4-45 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of r_0 = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be k = 0.607 W/m.°C, $\alpha = k / \rho C_p$ = 0.146×10^{-6} m^2/s (Table A-9).

Analysis The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(800\,\text{W/m}^2.°\text{C})(0.0275\,\text{m})}{(0.607\,\text{W/m.°C})} = 36.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 3.0533 \text{ and } A_1 = 1.9925$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{60-100}{8-100} = (1.9925)e^{-(3.0533)^2 \tau} \longrightarrow \tau = 0.1633$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1633)(0.0275\,\text{m})^2}{(0.146\times 10^{-6}\text{ m}^2/\text{s})} = 846\,\text{s} = \mathbf{14.1\,min}$$

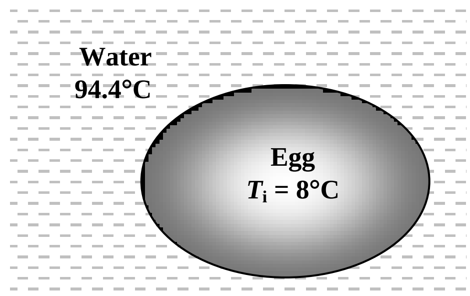
4-46 An egg is cooked in boiling water. The cooking time of the egg is to be determined for a location at 1610-m elevation.

Assumptions **1** The egg is spherical in shape with a radius of r_0 = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg and heat transfer coefficient are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be $k = 0.607$ W/m.°C, $\alpha = k/\rho C_p = 0.146\times10^{-6}$ m^2/s (Table A-9).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(800\,\text{W/m}^2.°\text{C})(0.0275\,\text{m})}{(0.607\,\text{W/m}.°\text{C})} = 36.2$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 3.0533 \text{ and } A_1 = 1.9925$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{60-94.4}{8-94.4} = (1.9925)e^{-(3.0533)^2 \tau} \longrightarrow \tau = 0.1727$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1727)(0.0275\,\text{m})^2}{(0.146\times10^{-6}\ \text{m}^2/\text{s})} = 895\,\text{s} = \mathbf{14.9\,min}$$