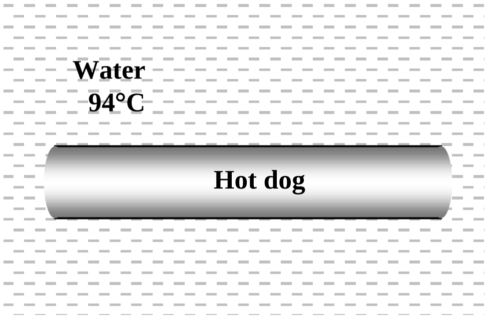
4-47 A hot dog is dropped into boiling water, and temperature measurements are taken at certain time intervals. The thermal diffusivity and thermal conductivity of the hot dog and the convection heat transfer coefficient are to be determined.

Assumptions **1** Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of hot dog available are given to be $\rho = 980$ kg/m^3 and $C_p = 3900$ J/kg.°C.

Analysis (*a*) From Fig. 4-14b we have

$$\left.\begin{aligned} \frac{T-T_\infty}{T_o-T_\infty} &= \frac{88-94}{59-94} = 0.17 \\ \frac{r}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 0.15$$



The Fourier number is determined from Fig. 4-14a to be

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.15 \\ \frac{T_o-T_\infty}{T_i-T_\infty} &= \frac{59-94}{20-94} = 0.47 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^{\,2}} = 0.20$$

The thermal diffusivity of the hot dog is determined to be

$$\frac{\alpha t}{r_o^{\,2}} = 0.20 \longrightarrow \alpha = \frac{0.2 r_o^{\,2}}{t} = \frac{(0.2)(0.011\text{m})^2}{120\,\text{s}} = \mathbf{2.017 \times 10^{-7}\ m^2/s}$$

(*b*) The thermal conductivity of the hot dog is determined from

$$k = \alpha\rho C_p = (2.017\times10^{-7}\ \text{m}^2/\text{s})(980\,\text{kg/m}^3)(3900\text{J/kg.°C}) = \mathbf{0.771\ \ W/m.\,°C}$$

(*c*) From part (*a*) we have $\frac{1}{Bi} = \frac{k}{hr_o} = 0.15$. Then,

$$\frac{k}{h} = 0.15 r_0 = (0.15)(0.011\text{m}) = 0.00165\text{m}$$

Therefore, the heat transfer coefficient is

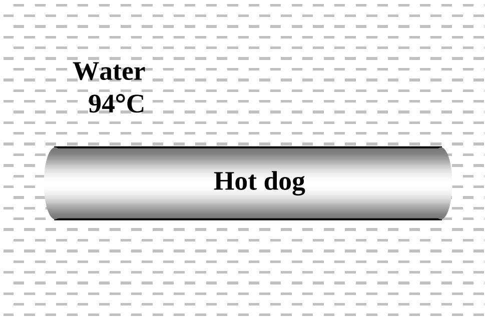
$$\frac{k}{h} = 0.00165 \longrightarrow h = \frac{0.771\text{W/m.°C}}{0.00165\text{m}} = \mathbf{467\ \ W/m^2.°C}$$

4-48 Using the data and the answers given in Prob. 4-43, the center and the surface temperatures of the hot dog 4 min after the start of the cooking and the amount of heat transferred to the hot dog are to be determined.

Assumptions **1** Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of hot dog and the convection heat transfer coefficient are given or obtained in P4-47 to be k = 0.771 W/m.°C, ρ = 980 kg/m^3, C_p = 3900 J/kg.°C, $\alpha = 2.017\times10^{-7}$ m^2/s, and h = 467 W/m^2.°C.

Analysis The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(467\,\text{W/m}^2.°\text{C})(0.011\text{m})}{(0.771\text{W/m.°C})} = 6.66$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 2.0785 \text{ and } A_1 = 1.5357$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(2.017\times10^{-7}\text{ m}^2/\text{s})(4\,\text{min}\times60\,\text{s/min})}{(0.011\text{m})^2} = 0.4001 > 0.2$$

Then the temperature at the center of the hot dog is determined to be

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.5357)e^{-(2.0785)^2(0.4001)} = 0.2727$$

$$\frac{T_0 - 94}{20-94} = 0.2727 \longrightarrow T_0 = \mathbf{73.8°C}$$

From Table 4-2 we read J_0=0.2194 corresponding to the constant λ_1=2.0785. Then the temperature at the surface of the hot dog becomes

$$\frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o / r_o) = (1.5357)e^{-(2.0785)^2(0.4001)}(0.2194) = 0.05982$$

$$\frac{T(r_o,t) - 94}{20-94} = 0.05982 \longrightarrow T(r_o,t) = \mathbf{89.6°C}$$

The maximum possible amount of heat transfer is

$$m = \rho V = \rho\pi r_o{}^2 L = (980\,\text{kg/m}^3)\left[\pi(0.011\text{m})^{2\cdot}(0.125\,\text{m})\right] = 0.04657\text{kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (0.04657\text{kg})(3900\text{J/kg.°C})(94-20)°\text{C} = 13{,}440\text{J}$$

From Table 4-2 we read J_1= 0.5760 corresponding to the constant λ_1=2.0785. Then the actual heat transfer becomes

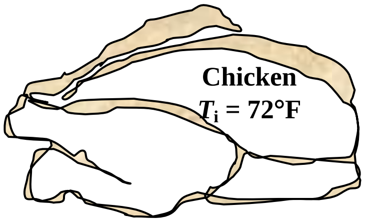
$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.2727)\frac{0.5760}{2.0785} = 0.8489 \longrightarrow Q = 0.8489(13{,}440\,\text{kJ}) = \mathbf{11{,}409kJ}$$

4-49E Whole chickens are to be cooled in the racks of a large refrigerator. Heat transfer coefficient that will enable to meet temperature constraints of the chickens while keeping the refrigeration time to a minimum is to be determined.

Assumptions **1** The chicken is a homogeneous spherical object. **2** Heat conduction in the chicken is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the chicken are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the chicken are given to be k = 0.26 Btu/h.ft.°F, ρ = 74.9 lbm/ft^3, C_p = 0.98 Btu/lbm.°F, and α = 0.0035 ft^2/h.

Analysis The radius of the chicken is determined to be



Refrigerator
T_∞ = 5°F

$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{5\,\text{lbm}}{74.9\,\text{lbm/ft}^3} = 0.06676\,\text{ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.06676\,\text{ft}^3)}{4\pi}} = 0.2517\,\text{ft}$$

From Fig. 4-15b we have

$$\left.\begin{aligned} \frac{T - T_\infty}{T_o - T_\infty} &= \frac{35-5}{45-5} = 0.75 \\ \frac{x}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 1.75$$

Then the heat transfer coefficients becomes

$$h = \frac{k}{1.75 r_o} = \frac{(0.26\,\text{Btu/h.ft.°F})}{1.75(0.2517\,\text{ft})} = \mathbf{0.590\ \ Btu/h.ft^2.°F}$$

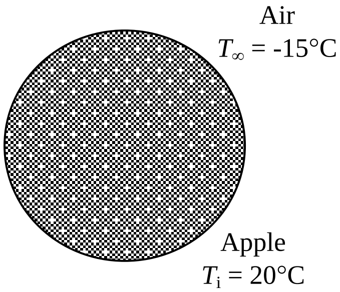
4-50 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined.

Assumptions **1** The apples are spherical in shape with a diameter of 9 cm. **2** Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the apples are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the apples are given to be $k = 0.418$ W/m.°C, $\rho = 840$ kg/m^3, $C_p = 3.81$ kJ/kg.°C, and $\alpha = 1.3\times10^{-7}$ m^2/s.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8\text{ W/m}^2.°\text{C})(0.045\text{ m})}{(0.418\text{ W/m.°C})} = 0.861$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.476 \quad\text{and}\quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_0^2} = \frac{(1.3\times10^{-7}\text{ m}^2/\text{s})(1\text{ h}\times 3600\text{ s/h})}{(0.045\text{ m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{o,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{T_0 - (-15)}{20-(-15)} = (1.239)e^{-(1.476)^2(0.231)} = 0.749 \longrightarrow T_0 = \mathbf{11.2°C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.239)e^{-(1.476)^2(0.231)}\frac{\sin(1.476\text{ rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t)-(-15)}{20-(-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{2.7°C}$$

The maximum possible heat transfer is

$$m = \rho V = \rho\frac{4}{3}\pi r_o^3 = (840\text{ kg/m}^3)\left[\frac{4}{3}\pi(0.045\text{ m})^{3.}\right] = 0.3206\text{ kg}$$

$$Q_{\max} = mC_p(T_i - T_\infty) = (0.3206\text{ kg})(3.81\text{ kJ/kg.°C})[20-(-15)]°\text{C} = 42.76\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.749)\frac{\sin(1.476\text{ rad}) - (1.476)\cos(1.476\text{ rad})}{(1.476)^3} = 0.402$$

$$Q = 0.402Q_{\max} = (0.402)(42.76\text{ kJ}) = \mathbf{17.2\text{ kJ}}$$

4-51

```
"!PROBLEM 4-51"

"GIVEN"
T_infinity=-15 "[C]"
"T_i=20 [C], parameter to be varied"
h=8 "[W/m^2-C]"
r_o=0.09/2 "[m]"
time=1*3600 "[s]"

"PROPERTIES"
k=0.513 "[W/m-C]"
rho=840 "[kg/m^3]"
C_p=3.6 "[kJ/kg-C]"
alpha=1.3E-7 "[m^2/s]"

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1=1.3525
A_1=1.1978
tau=(alpha*time)/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)

(T_r-T_infinity)/(T_i-T_infinity)=A_1*exp(-
lambda_1^2*tau)*Sin(lambda_1*r_o/r_o)/(lambda_1*r_o/r_o)

V=4/3*pi*r_o^3
m=rho*V
Q_max=m*C_p*(T_i-T_infinity)
Q/Q_max=1-3*(T_o-T_infinity)/(T_i-T_infinity)*(Sin(lambda_1)-
lambda_1*Cos(lambda_1))/lambda_1^3
```

T_i [C]	**T_o [C]**	**T_r [C]**	**Q [kJ]**
2	-1.658	-5.369	6.861
4	-0.08803	-4.236	7.668
6	1.482	-3.103	8.476
8	3.051	-1.97	9.283
10	4.621	-0.8371	10.09
12	6.191	0.296	10.9
14	7.76	1.429	11.7
16	9.33	2.562	12.51
18	10.9	3.695	13.32
20	12.47	4.828	14.13
22	14.04	5.961	14.93
24	15.61	7.094	15.74
26	17.18	8.227	16.55
28	18.75	9.36	17.35
30	20.32	10.49	18.16

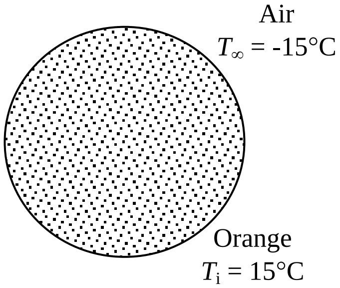
4-52 An orange is exposed to very cold ambient air. It is to be determined whether the orange will freeze in 4 h in subfreezing temperatures.

Assumptions **1** The orange is spherical in shape with a diameter of 8 cm. **2** Heat conduction in the orange is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the orange are constant, and are those of water. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the orange are approximated by those of water at the average temperature of about 5°C, k = 0.571 W/m.°C and $\alpha = k/\rho C_p = 0.571/(1000 \times 4205) = 0.136 \times 10^{-6}\ \text{m}^2/\text{s}$ (Table A-9).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(15\ \text{W/m}^2.^\circ\text{C})(0.04\ \text{m})}{(0.571\ \text{W/m}.^\circ\text{C})} = 1.051 \approx 1.0$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.5708 \quad \text{and} \quad A_1 = 1.2732$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.136\times10^{-6}\ \text{m}^2/\text{s})(4\ \text{h}\times 3600\,\text{s/h})}{(0.04\,\text{m})^2} = 1.224 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the oranges becomes

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.2732)e^{-(1.5708)^2(1.224)}\frac{\sin(1.5708\,\text{rad})}{1.5708} = 0.0396$$

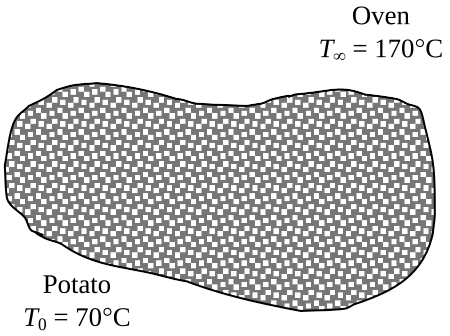
$$\frac{T(r_o,t)-(-6)}{15-(-6)} = 0.0396 \longrightarrow T(r_o,t) = \mathbf{-\ 5.2\ ^\circ C}$$

which is less than 0°C. Therefore, the oranges will freeze.

4-53 A hot baked potato is taken out of the oven and wrapped so that no heat is lost from it. The time the potato is baked in the oven and the final equilibrium temperature of the potato after it is wrapped are to be determined.

Assumptions **1** The potato is spherical in shape with a diameter of 8 cm. **2** Heat conduction in the potato is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the potato are given to be $k = 0.6$ W/m.°C, $\rho = 1100$ kg/m^3, $C_p = 3.9$ kJ/kg.°C, and $\alpha = 1.4\times10^{-7}$ m^2/s.



Analysis (*a*) The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(25\ \mathrm{W/m^2.°C})(0.04\ \mathrm{m})}{(0.6\ \mathrm{W/m.°C})} = 1.67$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.8777 \quad \text{and} \quad A_1 = 1.4113$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{70-170}{25-170} = 0.69 = (1.4113)e^{-(1.8777)^2\tau} \longrightarrow \tau = 0.203 > 0.2$$

The baking time of the potatoes is determined to be

$$t = \frac{\tau r_o^{\,2}}{\alpha} = \frac{(0.203)(0.04\ \mathrm{m})^2}{(1.4\times10^{-7}\ \mathrm{m^2/s})} = 2320\ \mathrm{s} = \mathbf{38.7\ min}$$

(*b*) The maximum amount of heat transfer is

$$m = \rho V = \rho\frac{4}{3}\pi r_o^{\,3} = (1100\,\mathrm{kg/m^3})\left[\frac{4}{3}\pi(0.04\,\mathrm{m})^{3.}\right] = 0.295\,\mathrm{kg}$$

$$Q_{\max} = mC_p(T_\infty - T_i) = (0.295\,\mathrm{kg})(3.900\,\mathrm{kJ/kg.°C})(170-25)°\mathrm{C} = 166.76\mathrm{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^{\,3}} = 1 - 3(0.69)\frac{\sin(1.8777) - (1.8777)\cos(1.8777)}{(1.8777)^3} = 0.525$$

$$Q = 0.525 Q_{\max} = (0.525)(166.76\ \mathrm{kJ}) = \mathbf{87.5\,kJ}$$

The final equilibrium temperature of the potato after it is wrapped is

$$Q = mC_p(T_{eqv} - T_i) \longrightarrow T_{eqv} = T_i + \frac{Q}{mC_p} = 25°\mathrm{C} + \frac{87.5\,\mathrm{kJ}}{(0.295\,\mathrm{kg})(3.9\,\mathrm{kJ/kg.°C})} = \mathbf{101°C}$$

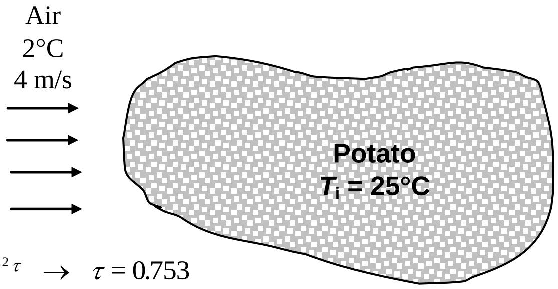
4-54 The center temperature of potatoes is to be lowered to 6°C during cooling. The cooling time and if any part of the potatoes will suffer chilling injury during this cooling process are to be determined.

Assumptions **1** The potatoes are spherical in shape with a radius of r_0 = 3 cm. **2** Heat conduction in the potato is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of potatoes are given to be k = 0.50 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:

$$\text{Bi} = \frac{hr_0}{k} = \frac{(19\text{ W/m}^2.^\circ\text{C})(0.03\text{ m})}{0.5\text{ W/m}.^\circ\text{C}} = 1.14$$



From Table 4-1 we read, for a sphere, λ_1 = 1.635 and A_1 = 1.302. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{6-2}{25-2} = 1.302 e^{-(1.635)^2 \tau} \rightarrow \tau = 0.753$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_0^2} \longrightarrow t = \frac{\tau r_0^2}{\alpha} = \frac{(0.753)(0.03\,\text{m})^2}{0.13\times10^{-6}\text{ m}^2/\text{s}} = 5213\text{s} = \mathbf{1.45h}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0} \rightarrow \frac{T(r_0)-T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_0/r_0)}{\lambda_1 r_0/r_0} = \frac{T_0 - T_\infty}{T_i - T_\infty} \frac{\sin(\lambda_1 r_0/r_0)}{\lambda_1 r_0/r_0}$$

Substituting, $$\frac{T(r_0)-2}{25-2} = \left(\frac{6-2}{25-2}\right)\frac{\sin(1.635\,\text{rad})}{1.635} \longrightarrow T(r_0) = 4.44^\circ\text{C}$$

which is above the temperature range of 3 to 4 °C for chilling injury for potatoes. Therefore, **no part** of the potatoes will experience chilling injury during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} &\frac{1}{\text{Bi}} = \frac{k}{hr_o} = \frac{0.50\text{W/m.}^\circ\text{C}}{(19\text{W/m}^2.^\circ\text{C})(0.03\text{m})} = 0.877 \\ &\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{6-2}{25-2} = 0.174 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o{}^2} = 0.75 \quad (\text{Fig. 4-15a})$$

Therefore, $$t = \frac{\tau\, r_0{}^2}{\alpha} = \frac{(0.75)(0.03)^2}{0.13\times10^{-6}\text{ m}^2/\text{s}} = 5192\,\text{s} \cong \mathbf{1.44\ h}$$

The surface temperature is determined from

$$\left.\begin{aligned} &\frac{1}{\text{Bi}} = \frac{k}{hr_0} = 0.877 \\ &\frac{r}{r_0} = 1 \end{aligned}\right\} \frac{T(r) - T_\infty}{T_0 - T_\infty} = 0.6 \ (\text{Fig. 4-15b})$$

which gives $T_{surface} = T_\infty + 0.6(T_o - T_\infty) = 2 + 0.6(6-2) = 4.4^\circ\text{C}$

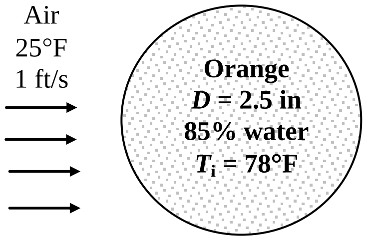
The slight difference between the two results is due to the reading error of the charts.

4-55E The center temperature of oranges is to be lowered to 40°F during cooling. The cooling time and if any part of the oranges will freeze during this cooling process are to be determined.

Assumptions **1** The oranges are spherical in shape with a radius of r_0 =1.25 in = 0.1042 ft. **2** Heat conduction in the orange is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the orange are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of oranges are given to be $k = 0.26$ Btu/h·ft·°F and $\alpha = 1.4\times10^{-6}$ ft²/s.

Analysis First we find the Biot number:



$$\mathrm{Bi} = \frac{hr_0}{k} = \frac{(4.6\ \mathrm{Btu/h.ft^2.°F})(1.25/12\ \mathrm{ft})}{0.26\ \mathrm{Btu/h.ft.°C}} = 1.843$$

From Table 4-1 we read, for a sphere, λ_1 = 1.9569 and A_1 = 1.447. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{40-25}{78-25} = 1.447e^{-(1.9569)^2\tau} \rightarrow \tau = 0.426$$

which is greater than 0.2 and thus the one-term solution is applicable.
Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_0^2} \rightarrow t = \frac{\tau r_0^2}{\alpha} = \frac{(0.426)(1.25/12\ \mathrm{ft})^2}{1.4\times10^{-6}\ \mathrm{ft^2/s}} = 3302\ \mathrm{s} = \mathbf{55.0\ min}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0} \rightarrow \frac{T(r_0)-T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_0/r_0)}{\lambda_1 r_0/r_0} = \frac{T_0 - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_0/r_0)}{\lambda_1 r_0/r_0}$$

Substituting,
$$\frac{T(r_0)-25}{78-25} = \left(\frac{40-25}{78-25}\right)\frac{\sin(1.9569\,\mathrm{rad})}{1.9569} \longrightarrow T(r_0) = 32.1°\mathrm{F}$$

which is above the freezing temperature of 31 °C for oranges . Therefore, no part of the oranges will freeze during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} &\frac{1}{Bi} = \frac{k}{hr_o} = \frac{0.26\ \ \mathrm{Btu/h.ft.°F}}{(4.6\,\mathrm{Btu/h.ft^2.°F})(1.25/12\mathrm{ft})} = 0.543 \\ &\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{40-25}{78-25} = 0.283 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o{}^2} = 0.43 \quad (\mathrm{Fig.4\text{-}15a})$$

Therefore,
$$t = \frac{\tau r_o{}^2}{\alpha} = \frac{(0.43)(1.25/12\mathrm{ft})^2}{1.4\times10^{-6}\,\mathrm{ft^2/s}} = 3333\mathrm{s} = 55.5\,\mathrm{min}$$

The lowest temperature during cooling will occur on the surface (r/r_0 =1) of the oranges is determined to be

$$\left.\begin{aligned} &\frac{1}{Bi} = \frac{k}{hr_o} = 0.543 \\ &\frac{r}{r_o} = 1 \end{aligned}\right\} \frac{T(r)-T_\infty}{T_o - T_\infty} = 0.45 \quad (\mathrm{Fig.}\,4-15\mathrm{b})$$

which gives $T_{surface} = T_\infty + 0.45(T_o - T_\infty) = 25 + 0.45(40-25) = 31.8°\mathrm{F}$

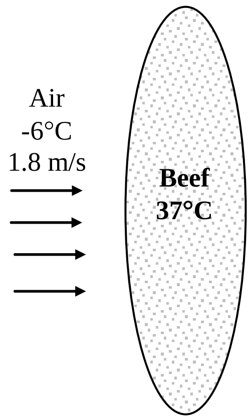
The slight difference between the two results is due to the reading error of the charts.

4-56 The center temperature of a beef carcass is to be lowered to 4°C during cooling. The cooling time and if any part of the carcass will suffer freezing injury during this cooling process are to be determined.

Assumptions **1** The beef carcass can be approximated as a cylinder with insulated top and base surfaces having a radius of r_0 = 12 cm and a height of H = 1.4 m. **2** Heat conduction in the carcass is one-dimensional in the radial direction because of the symmetry about the centerline. **3** The thermal properties of the carcass are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of carcass are given to be $k = 0.47$ W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_0}{k} = \frac{(22\ \text{W/m}^2.^\circ\text{C})(0.12\ \text{m})}{0.47\ \text{W/m}.^\circ\text{C}} = 5.62$$

From Table 4-1 we read, for a cylinder, $\lambda_1 = 2.027$ and $A_1 = 1.517$. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{4-(-6)}{37-(-6)} = 1.517 e^{-(2.027)^2 \tau} \rightarrow \tau = 0.456$$

which is greater than 0.2 and thus the one-term solution is applicable.
Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_0^2} \rightarrow t = \frac{\tau r_0^2}{\alpha} = \frac{(0.456)(0.12\,\text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 50{,}558\,\text{s} = \mathbf{14.0\,h}$$

The lowest temperature during cooling will occur on the surface ($r/r_0 = 1$), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_0) \rightarrow \frac{T(r_0)-T_\infty}{T_i - T_\infty} = \theta_0 J_0(\lambda_1 r/r_0) = \frac{T_0 - T_\infty}{T_i - T_\infty} J_0(\lambda_1 r_0/r_0)$$

Substituting, $$\frac{T(r_0)-(-6)}{37-(-6)} = \left(\frac{4-(-6)}{37-(-6)}\right) J_0(\lambda_1) = 0.2326\times0.2084 = 0.0485 \longrightarrow T(r_0) = \text{-}3.9^\circ\text{C}$$

which is below the freezing temperature of -1.7 °C. Therefore, the outer part of the beef carcass will freeze during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.47\text{W/m.}^\circ\text{C}}{(22\ \text{W/m}^2.^\circ\text{C})(0.12\,\text{m})} = 0.178 \\ \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{4-(-6)}{37-(-6)} = 0.23 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.4 \quad (\text{Fig.}4-14\text{a})$$

Therefore, $$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.4)(0.12\text{m})^2}{0.13\times10^{-6}\,\text{m}^2/\text{s}} = 44{,}308\text{s} \cong 12.3\text{h}$$

The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.178 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r) - T_\infty}{T_o - T_\infty} = 0.17 \quad (\text{Fig.}4-14\text{b})$$

which gives $T_{surface} = T_\infty + 0.17(T_o - T_\infty) = -6 + 0.17[4-(-6)] = -4.3^\circ C$

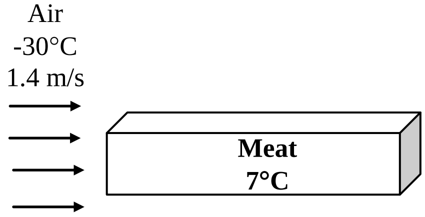
The difference between the two results is due to the reading error of the charts.

4-57 The center temperature of meat slabs is to be lowered to -18°C during cooling. The cooling time and the surface temperature of the slabs at the end of the cooling process are to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 11.5 cm. **2** Heat conduction in the meat slabs is one-dimensional because of the symmetry about the centerplane. **3** The thermal properties of the meat slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual cooling time will be much longer than the value determined.

Properties The thermal conductivity and thermal diffusivity of meat slabs are given to be k = 0.47 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s. These properties will be used for both fresh and frozen meat.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_0}{k} = \frac{(20\ \text{W/m}^2.^\circ\text{C})(0.115\ \text{m})}{0.47\ \text{W/m.}^\circ\text{C}} = 4.89$$

From Table 4-1 we read, for a plane wall, λ_1 = 1.308 and A_1=1.239. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{-18-(-30)}{7-(-30)} = 1.239 e^{-(1.308)^2 \tau} \rightarrow \tau = 0.783$$

which is greater than 0.2 and thus the one-term solution is applicable.
Then the cooling time becomes

$$\tau = \frac{\alpha t}{L^2} \rightarrow t = \frac{\tau L^2}{\alpha} = \frac{(0.783)(0.115\ \text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 79{,}650\ \text{s} = \mathbf{22.1\ h}$$

The lowest temperature during cooling will occur on the surface (x/L = 1), and is determined to be

$$\frac{T(x)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L) \rightarrow \frac{T(L)-T_\infty}{T_i - T_\infty} = \theta_0 \cos(\lambda_1 L/L) = \frac{T_0 - T_\infty}{T_i - T_\infty}\cos(\lambda_1)$$

Substituting,

$$\frac{T(L)-(-30)}{7-(-30)} = \left(\frac{-18-(-30)}{7-(-30)}\right)\cos(\lambda_1) = 0.3243\times0.2598 = 0.08425 \longrightarrow T(L) = -\mathbf{26.9^\circ C}$$

which is close the temperature of the refrigerated air.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hL} = \frac{0.47\text{W/m.}^\circ\text{C}}{(20\ \text{W/m}^2.^\circ\text{C})(0.115\text{m})} = 0.204 \\ \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{-18-(-30)}{7-(-30)} = 0.324 \end{aligned}\right\} \tau = \frac{\alpha t}{L^2} = 0.75 \quad (\text{Fig.4-13a})$$

Therefore, $t = \dfrac{\tau r_o{}^2}{\alpha} = \dfrac{(0.75)(0.115\ \text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 76{,}300\,\text{s} \cong 21.2\ \text{h}$

The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hL} = 0.204 \\ \frac{x}{L} &= 1 \end{aligned}\right\} \frac{T(x)-T_\infty}{T_o - T_\infty} = 0.22 \quad (\text{Fig. 4-13b})$$

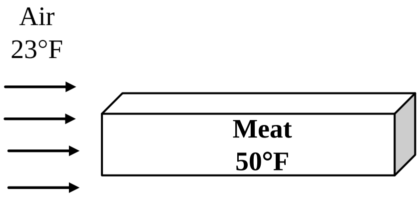
which gives $T_{surface} = T_\infty + 0.22(T_o - T_\infty) = -30 + 0.22[-18-(-30)] = -27.4^\circ\text{C}$

The slight difference between the two results is due to the reading error of the charts.

4-58E The center temperature of meat slabs is to be lowered to 36°F during 12-h of cooling. The average heat transfer coefficient during this cooling process is to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 3-in. **2** Heat conduction in the meat slabs is one-dimensional because of symmetry about the centerplane. **3** The thermal properties of the meat slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of meat slabs are given to be k = 0.26 Btu/h·ft·°F and $\alpha=1.4\times10^{-6}$ ft^2/s.



Analysis The average heat transfer coefficient during this cooling process is determined from the transient temperature charts for a flat plate as follows:

$$\left.\begin{aligned}\tau &= \frac{\alpha t}{L^2} = \frac{(1.4\times10^{-6}\ \text{ft}^2/\text{s})(12\times3600\ \text{s})}{(3/12\ \text{ft})^2} = 0.968\\ \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{36-23}{50-23} = 0.481\end{aligned}\right\}\ \frac{1}{Bi} = 0.7 \quad (\text{Fig.}4-13\text{a})$$

Therefore,

$$h = \frac{kBi}{L} = \frac{(0.26\ \text{Btu/h.ft.°F})(1/0.7)}{(3/12)\ \text{ft}} = \mathbf{1.5\ Btu/h.ft^2.°F}$$

Discussion We could avoid the uncertainty associated with the reading of the charts and obtain a more accurate result by using the one-term solution relation for an infinite plane wall, but it would require a trial and error approach since the Bi number is not known.

4-59 Chickens are to be chilled by holding them in agitated brine for 2.5 h. The center and surface temperatures of the chickens are to be determined, and if any part of the chickens will freeze during this cooling process is to be assessed.

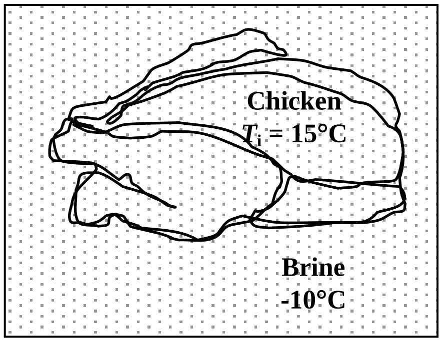
Assumptions **1** The chickens are spherical in shape. **2** Heat conduction in the chickens is one-dimensional in the radial direction because of symmetry about the midpoint. **3** The thermal properties of the chickens are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual the temperatures will be much higher than the values determined since a considerable part of the cooling process will occur during phase change (freezing of chicken).

Properties The thermal conductivity, thermal diffusivity, and density of chickens are given to be $k = 0.45$ W/m·°C, $\alpha = 0.13\times10^{-6}$ m^2/s, and $\rho = 950$ kg/ m^3. These properties will be used for both fresh and frozen chicken.

Analysis We first find the volume and equivalent radius of the chickens:

$$V = m/\rho = 1700\text{g}/(0.95\text{g/cm}^3) = 1789\text{cm}^3$$

$$r_o = \left(\frac{3}{4\pi}V\right)^{1/3} = \left(\frac{3}{4\pi}1789\,\text{cm}^3\right)^{1/3} = 7.53\,\text{cm} = 0.0753\,\text{m}$$



Then the Biot and Fourier numbers become

$$\text{Bi} = \frac{hr_0}{k} = \frac{(440\,\text{W/m}^2.°\text{C})(0.0753\text{m})}{0.45\,\text{W/m.°C}} = 73.6$$

$$\tau = \frac{\alpha t}{r_0^2} = \frac{(0.13\times10^{-6}\,\text{m}^2\text{/s})(2.5\times3600\text{s})}{(0.0753\text{m})^2} = 0.2063$$

Note that $\tau = 0.207 > 0.2$, and thus the one-term solution is applicable. From Table 4-1 we read, for a sphere, $\lambda_1 = 3.094$ and $A_1 = 1.998$. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{T_0 - (-10)}{15-(-10)} = 1.998 e^{-(3.094)^2(0.2063)} = 0.277 \longrightarrow T_0 = \mathbf{-3.1°C}$$

The lowest temperature during cooling will occur on the surface ($r/r_0 = 1$), and is determined to be

$$\frac{T(r) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0} \rightarrow \frac{T(r_0) - T_\infty}{T_i - T_\infty} = \theta_0\frac{\sin(\lambda_1 r_0/r_0)}{\lambda_1 r_0/r_0} = \frac{T_0 - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_0/r_0)}{\lambda_1 r_0/r_0}$$

Substituting,
$$\frac{T(r_0) - (-10)}{15-(-10)} = 0.277\frac{\sin(3.094\,\text{rad})}{3.094} \rightarrow T(r_0) = \mathbf{-9.9°C}$$

The entire chicken will freeze during this process since the freezing point of chicken is -2.8°C, and even the center temperature of chicken is below this value.

Discussion We could also solve this problem using transient temperature charts, but the data in this case falls at a point on the chart which is very difficult to read:

$$\left.\begin{aligned}\tau &= \frac{\alpha t}{r_0^2} = \frac{(0.13\times10^{-6}\,\text{m}^2\text{/s})(2.5\times3600\text{s})}{(0.0753\text{m})^2} = 0.206\\ \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.45\ \ \text{W/m.°C}}{(440\text{W/m}^2.°\text{C})(0.0753\text{m})} = 0.0136\end{aligned}\right\}\ \frac{T_o - T_\infty}{T_i - T_\infty} = 0.15....0.30\ ??\qquad(\text{Fig.}4-15)$$

Transient Heat Conduction in Semi-Infinite Solids

4-60C A semi-infinite medium is an idealized body which has a single exposed plane surface and extends to infinity in all directions. The earth and thick walls can be considered to be semi-infinite media.

4-61C A thick plane wall can be treated as a semi-infinite medium if all we are interested in is the variation of temperature in a region near one of the surfaces for a time period during which the temperature in the mid section of the wall does not experience any change.

4-62C The total amount of heat transfer from a semi-infinite solid up to a specified time t_0 can be determined by integration from

$$Q = \int_0^{t_o} Ah[T(0,t) - T_\infty]dt$$

where the surface temperature $T(0,t)$ is obtained from Eq. 4-22 by substituting $x = 0$.

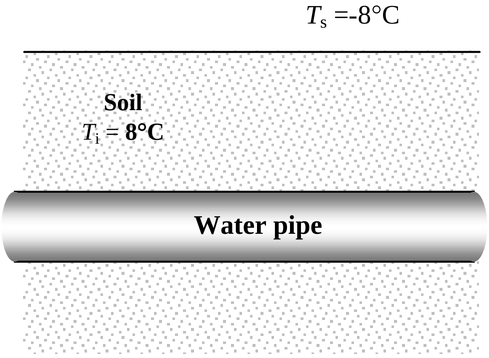
4-63 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be $k = 0.35$ W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis The length of time the snow pack stays on the ground is

$$t = (60 \text{ days})(24 \text{ hr / days})(3600 \text{ s / hr}) = 5.184\times10^6 \text{ s}$$



The surface is kept at -18°C at all times. The depth at which freezing at 0°C occurs can be determined from the analytical solution,

$$\frac{T(x,t) - T_i}{T_s - T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$

$$\frac{0-8}{-8-8} = erfc\left(\frac{x}{2\sqrt{(0.15\times10^{-6} \text{ m}^2\text{/s})(5.184\times10^6 \text{ s})}}\right)$$

$$\longrightarrow 0.444 = erfc\left(\frac{x}{1.7636}\right)$$

Then from Table 4-3 we get $\frac{x}{1.7636} = 0.5297 \longrightarrow x = \mathbf{0.934 m}$

Discussion The solution could also be determined using the chart, but it would be subject to reading error.

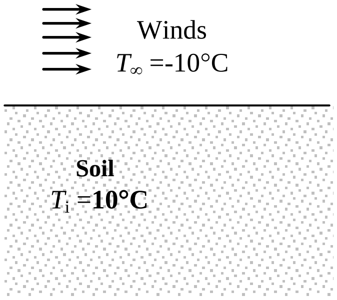
4-64 An area is subjected to cold air for a 10-h period. The soil temperatures at distances 0, 10, 20, and 50 cm from the earth's surface are to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.9 W/m.°C and $\alpha = 1.6\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the ground can be determined from

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(40\text{ W/m}^2.°\text{C})\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\times3600\text{s})}}{0.9\text{ W/m.°C}} = 33.7$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 33.7^2 = 1138$$

Then we conclude that the last term in the temperature distribution relation above must be zero regardless of x despite the exponential term tending to infinity since (1) $erfc(\xi)\to 0$ for $\xi > 4$ (see Table 4-3) and (2) the term has to remain less than 1 to have physically meaningful solutions. That is,

$$\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right] = \exp\left(\frac{hx}{k}+1138\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+33.3\right)\right] \cong 0$$

Therefore, the temperature distribution relation simplifies to

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) \to T(x,t) = T_i + (T_\infty - T_i)erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Then the temperatures at 0, 10, 20, and 50 cm depth from the ground surface become

x = 0: $$T(0,10\text{ h}) = T_i + (T_\infty - T_i)erfc\left(\frac{0}{2\sqrt{\alpha t}}\right) = T_i + (T_\infty - T_i)erfc(0) = T_i + (T_\infty - T_i)\times 1 = T_\infty = \mathbf{-10°C}$$

x = 0.1m: $$T(0.1\text{ m},10\text{ h}) = 10 + (-10-10)erfc\left(\frac{0.1\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{s/h})}}\right)$$
$$= 10 - 20erfc(0.066) = 10 - 20\times0.9257 = \mathbf{-8.5°C}$$

x = 0.2 m: $$T(0.2\text{ m},10\text{ h}) = 10 + (-10-10)erfc\left(\frac{0.2\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{s/h})}}\right)$$
$$= 10 - 20erfc(0.132) = 10 - 20\times0.8519 = \mathbf{-7.0°C}$$

x = 0.5 m: $$T(0.5\text{ m},10\text{ h}) = 10 + (-10-10)erfc\left(\frac{0.5\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{s/h})}}\right)$$
$$= 10 - 20erfc(0.329) = 10 - 20\times0.6418 = \mathbf{-2.8°C}$$

4-65

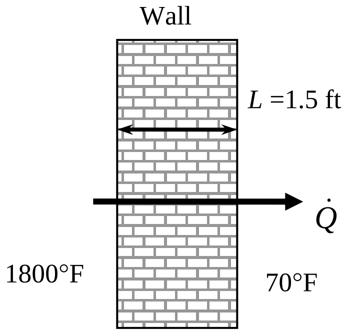
```
"!PROBLEM 4-65"

"GIVEN"
T_i=10 "[C]"
T_infinity=-10 "[C]"
h=40 "[W/m^2-C]"
time=10*3600 "[s]"
"x=0.1 [m], parameter to be varied"

"PROPERTIES"
k=0.9 "[W/m-C]"
alpha=1.6E-5 "[m^2/s]"

"ANALYSIS"
(T_x-T_i)/(T_infinity-T_i)=erfc(x/(2*sqrt(alpha*time)))-
exp((h*x)/k+(h^2*alpha*time)/k^2)*erfc(x/(2*sqrt(alpha*time))+(h*sqrt(alpha*time)/k))
```

x [m]	**T_x [C]**
0	-9.666
0.05	-8.923
0.1	-8.183
0.15	-7.447
0.2	-6.716
0.25	-5.993
0.3	-5.277
0.35	-4.572
0.4	-3.878
0.45	-3.197
0.5	-2.529
0.55	-1.877
0.6	-1.24
0.65	-0.6207
0.7	-0.01894
0.75	0.5643
0.8	1.128
0.85	1.672
0.9	2.196
0.95	2.7
1	3.183

4-66 The walls of a furnace made of concrete are exposed to hot gases at the inner surfaces. The time it will take for the temperature of the outer surface of the furnace to change is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 1800°F. **2** The thermal properties of the concrete wall are constant.

Properties The thermal properties of the concrete are given to be k = 0.64 Btu/h.ft.°F and α = 0.023 ft^2/h.



Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

But,

$$\frac{T(x,t)-T_i}{T_s-T_i}=\frac{70.1-70}{1800-70}=0.00006 \rightarrow 0.00006=erfc(2.85) \quad \text{(Table 4-3)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}}=2.85 \longrightarrow t=\frac{x^2}{4\times(2.85)^2\alpha}=\frac{(1.5\,\text{ft})^2}{4\times(2.85)^2(0.023\,\text{ft}^2/\text{h})}=3.01\,\text{h}=\mathbf{181\,min}$$

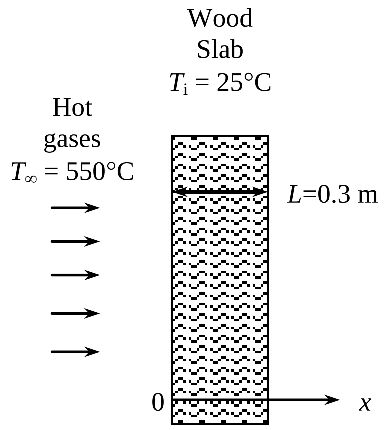
4-67 A thick wood slab is exposed to hot gases for a period of 5 minutes. It is to be determined whether the wood will ignite.

Assumptions **1** The wood slab is treated as a semi-infinite medium subjected to convection at the exposed surface. **2** The thermal properties of the wood slab are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The thermal properties of the wood are k = 0.17 W/m.°C and $\alpha = 1.28\times10^{-7}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wood can be determined from

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(35\ \text{W/m}^2.°\text{C})\sqrt{(1.28\times10^{-7}\ \text{m}^2/\text{s})(5\times60\,\text{s})}}{0.17\ \text{W/m}.°\text{C}} = 1.276$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 1.276^2 = 1.628$$

Noting that x = 0 at the surface and using Table 4-3 for *erfc* values,

$$\begin{aligned}\frac{T(x,t)-25}{550-25} &= erfc(0) - \exp(0+1.628)erfc(0+1.276)\\ &= 1-(5.0937)(0.0727)\\ &= 0.630\end{aligned}$$

Solving for $T(x, t)$ gives

$$T(x,t) = \mathbf{356°C}$$

which is less than the ignition temperature of 450°C. Therefore, the wood will not ignite.

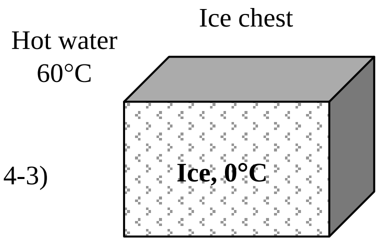
4-68 The outer surfaces of a large cast iron container filled with ice are exposed to hot water. The time before the ice starts melting and the rate of heat transfer to the ice are to be determined.

Assumptions **1** The temperature in the container walls is affected by the thermal conditions at outer surfaces only and the convection heat transfer coefficient outside inside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the wall are constant.

Properties The thermal properties of the cast iron are given to be k = 52 W/m.°C and $\alpha = 1.70\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



But,

$$\frac{T(x,t)-T_i}{T_s-T_i}=\frac{0.1-0}{60-0}=0.00167\rightarrow\ 0.00167=erfc(2.225)\ \ \text{(Table 4-3)}$$

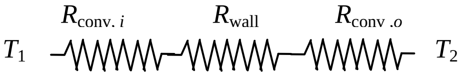
Therefore,

$$\frac{x}{2\sqrt{\alpha t}}=2.225\longrightarrow t=\frac{x^2}{4\times(2.225)^2\alpha}=\frac{(0.05\,\text{m})^2}{4(2.225)^2(1.7\times10^{-5}\ \text{m}^2/\text{s})}=\mathbf{7.4\,s}$$

The rate of heat transfer to the ice when steady operation conditions are reached can be determined by applying the thermal resistance network concept as

$$R_{conv,i}=\frac{1}{h_iA}=\frac{1}{(250\,\text{W/m}^2.°\text{C})(1.2\times2\,\text{m}^2)}=0.00167°\text{C/W}$$

$$R_{wall}=\frac{L}{kA}=\frac{0.05\,\text{m}}{(52\,\text{W/m.°C})(1.2\times2\,\text{m}^2)}=0.00040°\text{C/W}$$



$$R_{conv,0}=\frac{1}{h_oA}=\frac{1}{(\infty)(1.2\times2\,\text{m}^2)}\cong0°\text{C/W}$$

$$R_{total}=R_{conv,1}+R_{wall}+R_{conv,2}=0.00167+0.00040+0=0.00207°\text{C/W}$$

$$\dot{Q}=\frac{T_2-T_1}{R_{total}}=\frac{(60-0)°\text{C}}{0.00207°\ \text{C/W}}=\mathbf{28{,}990\,W}$$