
Transient Heat Conduction in Multidimensional Systems

4-69C The product solution enables us to determine the dimensionless temperature of two- or three-dimensional heat transfer problems as the product of dimensionless temperatures of one-dimensional heat transfer problems. The dimensionless temperature for a two-dimensional problem is determined by determining the dimensionless temperatures in both directions, and taking their product.

4-70C The dimensionless temperature for a three-dimensional heat transfer is determined by determining the dimensionless temperatures of one-dimensional geometries whose intersection is the three dimensional geometry, and taking their product.

4-71C This short cylinder is physically formed by the intersection of a long cylinder and a plane wall. The dimensionless temperatures at the center of plane wall and at the center of the cylinder are determined first. Their product yields the dimensionless temperature at the center of the short cylinder.

4-72C The heat transfer in this short cylinder is one-dimensional since there is no heat transfer in the axial direction. The temperature will vary in the radial direction only.

4-73 A short cylinder is allowed to cool in atmospheric air. The temperatures at the centers of the cylinder and the top surface as well as the total heat transfer from the cylinder for 15 min of cooling are to be determined.

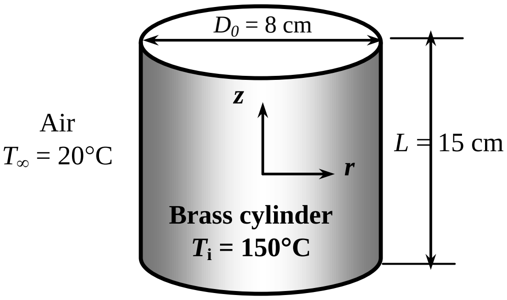
Assumptions **1** Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of brass are given to be $\rho = 8530\ \text{kg/m}^3, C_p = 0.389\ \text{kJ/kg}\cdot°\text{C}$, $k = 110\ \text{W/m}\cdot°\text{C}$, and $\alpha = 3.39\times10^{-5}\ \text{m}^2/\text{s}$.

Analysis This short cylinder can physically be formed by the intersection of a long cylinder of radius *D*/2 = 4 cm and a plane wall of thickness 2*L* = 15 cm. We measure *x* from the midplane.

(*a*) The Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.075\ \text{m})}{(110\ \text{W/m}.°\text{C})} = 0.02727$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 0.164 \quad \text{and} \quad A_1 = 1.0050$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.39\times10^{-5}\ \text{m}^2/\text{s})(15\ \text{min}\times 60\ \text{s/min})}{(0.075\ \text{m})^2} = 5.424 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0050)e^{-(0.164)^2(5.424)} = 0.869$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_0}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.04\ \text{m})}{(110\ \text{W/m}.°\text{C})} = 0.01455$$

$$\lambda_1 = 0.1704 \quad \text{and} \quad A_1 = 1.0038$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.39\times10^{-5}\ \text{m}^2/\text{s})(15\times 60\ \text{s})}{(0.04\ \text{m})^2} = 19.069 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0038)e^{-(0.1704)^2(19.069)} = 0.577$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta_{o,wall}\times\theta_{o,cyl} = 0.869\times 0.577 = 0.501$$

$$\frac{T(0,0,t) - 20}{150 - 20} = 0.501 \longrightarrow T(0,0,t) = \mathbf{85.1°C}$$

(*b*) The center of the top surface of the cylinder is still at the center of the long cylinder ($r = 0$), but at the outer surface of the plane wall ($x = L$). Therefore, we first need to determine the dimensionless temperature at the surface of the wall.

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 L / L) = (1.0050) e^{-(0.164)^2 (5.424)} \cos(0.164) = 0.857$$

Then the center temperature of the top surface of the cylinder becomes

$$\left[\frac{T(L,0,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{wall} \times \theta_{o,cyl} = 0.857 \times 0.577 = 0.494$$

$$\frac{T(L,0,t)-20}{150-20} = 0.494 \longrightarrow T(L,0,t) = \mathbf{84.2°C}$$

(*c*) We first need to determine the maximum heat can be transferred from the cylinder

$$m = \rho V = \rho \pi r_o{}^2 L = (8530\,\text{kg/m}^3)\left[\pi(0.04\,\text{m})^{2.}(0.15\,\text{m})\right] = 6.43\,\text{kg}$$
$$Q_{\max} = mC_p(T_i - T_\infty) = (6.43\,\text{kg})(0.389\,\text{kJ/kg.°C})(150-20)\text{°C} = 325\,\text{kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.869)\frac{\sin(0.164)}{0.164} = 0.135$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1-2(0.577)\frac{0.0846}{0.1704} = 0.427$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}}\left[1-\left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right] = 0.135+(0.427)(1-0.135) = 0.504$$

Then the total heat transfer from the short cylinder during the first 15 minutes of cooling becomes

$$Q = 0.503 Q_{\max} = (0.504)(325\,\text{kJ}) = \mathbf{164\ kJ}$$

4-74

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"!PROBLEM 4-74"

"GIVEN"
D=0.08 "[m]"
r_o=D/2
height=0.15 "[m]"
L=height/2
T_i=150 "[C]"
T_infinity=20 "[C]"
h=40 "[W/m^2-C]"
"time=15 [min], parameter to be varied"

"PROPERTIES"
k=110 "[W/m-C]"
rho=8530 "[kg/m^3]"
C_p=0.389 "[kJ/kg-C]"
alpha=3.39E-5 "[m^2/s]"

"ANALYSIS"
"(a)"
"This short cylinder can physically be formed by the intersection of a long cylinder of radius r_o and a plane wall of thickness 2L"
"For plane wall"
Bi_w=(h*L)/k
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1_w=0.2282 "w stands for wall"
A_1_w=1.0060
tau_w=(alpha*time*Convert(min, s))/L^2
theta_o_w=A_1_w*exp(-lambda_1_w^2*tau_w) "theta_o_w=(T_o_w-T_infinity)/(T_i-T_infinity)"
"For long cylinder"
Bi_c=(h*r_o)/k "c stands for cylinder"
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1_c=0.1704
A_1_c=1.0038
tau_c=(alpha*time*Convert(min, s))/r_o^2
theta_o_c=A_1_c*exp(-lambda_1_c^2*tau_c) "theta_o_c=(T_o_c-T_infinity)/(T_i-T_infinity)"
(T_o_o-T_infinity)/(T_i-T_infinity)=theta_o_w*theta_o_c "center temperature of short cylinder"
"(b)"
theta_L_w=A_1_w*exp(-lambda_1_w^2*tau_w)*Cos(lambda_1_w*L/L) "theta_L_w=(T_L_w-T_infinity)/(T_i-T_infinity)"
(T_L_o-T_infinity)/(T_i-T_infinity)=theta_L_w*theta_o_c "center temperature of the top surface"
"(c)"
V=pi*r_o^2*(2*L)
m=rho*V
Q_max=m*C_p*(T_i-T_infinity)
Q_w=1-theta_o_w*Sin(lambda_1_w)/lambda_1_w "Q_w=(Q/Q_max)_w"
Q_c=1-2*theta_o_c*J_1/lambda_1_c "Q_c=(Q/Q_max)_c"
J_1=0.0846 "From Table 4-2, at lambda_1_c"
Q/Q_max=Q_w+Q_c*(1-Q_w) "total heat transfer"
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time [min]	$T_{o,o}$ [C]	$T_{L,o}$ [C]	Q [kJ]
5	119.3	116.8	80.58
10	95.18	93.23	140.1
15	76.89	75.42	185.1
20	63.05	61.94	219.2
25	52.58	51.74	245
30	44.66	44.02	264.5
35	38.66	38.18	279.3
40	34.12	33.75	290.5
45	30.69	30.41	298.9
50	28.09	27.88	305.3
55	26.12	25.96	310.2
60	24.63	24.51	313.8

4-75 A semi-infinite aluminum cylinder is cooled by water. The temperature at the center of the cylinder 10 cm from the end surface is to be determined.

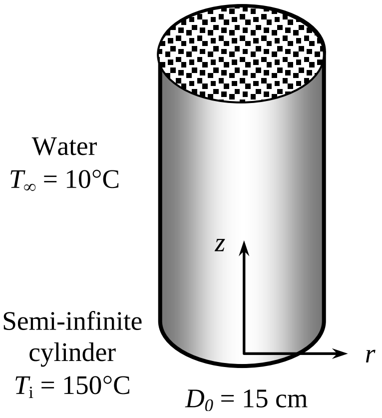
Assumptions **1** Heat conduction in the semi-infinite cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of aluminum are given to be $k = 237$ W/m.°C and $\alpha = 9.71\times10^{-5}\text{m}^2/\text{s}$.

Analysis This semi-infinite cylinder can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 7.5$ cm and a semi-infinite medium. The dimensionless temperature 5 cm from the surface of a semi-infinite medium is first determined from

$$\begin{aligned}\frac{T(x,t)-T_i}{T_\infty - T_i} &= erfc\left(\frac{x}{\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]\\ &= erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}\right) - \exp\left(\frac{(140)(0.05)}{237}+\frac{(140)^2(9.71\times10^{-5})(8\times60)}{(237)^2}\right)\\ &\quad\times\left[erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}+\frac{(140)\sqrt{(9.71\times10^{-5})(8\times60)}}{237}\right)\right]\\ &= erfc(0.1158) - \exp(0.0458)erfc(0.2433) = 0.8699-(1.0468)(0.7308) = 0.1049\end{aligned}$$

$$\theta_{semi-\inf} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = 1-0.1049 = 0.8951$$



The Biot number is calculated for the long cylinder to be

$$Bi = \frac{hr_o}{k} = \frac{(140\text{ W/m}^2.°\text{C})(0.075\text{ m})}{(237\text{ W/m.°C})} = 0.0443$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 0.2948 \quad\text{and}\quad A_1 = 1.0110$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.71\times10^{-5}\text{ m}^2/\text{s})(8\times60\text{ s})}{(0.075\text{ m})^2} = 8.286 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0110)e^{-(0.2948)^2(8.286)} = 0.4921$$

The center temperature of the semi-infinite cylinder then becomes

$$\left[\frac{T(x,0,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{\text{semi-infinite}\\ \text{cylinder}}} = \theta_{\text{semi-inf}}(x,t)\times\theta_{o,cyl} = 0.8951\times0.4921 = 0.4405$$

$$\left[\frac{T(x,0,t)-10}{150-10}\right]_{\substack{\text{semi-infinite}\\ \text{cylinder}}} = 0.4405 \longrightarrow T(x,0,t) = \mathbf{71.7°C}$$

4-76E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and also as an infinitely long cylinder.

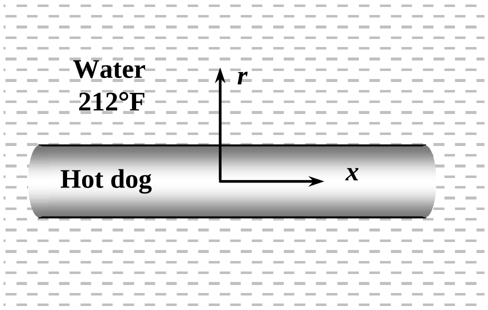
Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial *r*- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be $k = 0.44$ Btu/h.ft.°F, $\rho = 61.2$ lbm/ft^3 $C_p = 0.93$ Btu/lbm.°F, and $\alpha = 0.0077$ ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = (0.4/12)$ ft and a plane wall of thickness $2L = (5/12)$ ft. The distance *x* is measured from the midplane.

After 5 minutes

First the Biot number is calculated for the plane wall to be



$$Bi = \frac{hL}{k} = \frac{(120 \text{ Btu/h.ft}^2.°\text{F})(2.5/12 \text{ ft})}{(0.44 \text{ Btu/h.ft.}°\text{F})} = 56.8$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.5421 \text{ and } A_1 = 1.2728$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(5/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728) e^{-(1.5421)^2 (0.015)} \cong 1$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120 \text{ Btu/h.ft}^2.°\text{F})(0.4/12 \text{ ft})}{(0.44 \text{ Btu/h.ft.}°\text{F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(5/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 0.578 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618) e^{-(2.1589)^2 (0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.106 = 0.106$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.106 \longrightarrow T(0,0,t) = \mathbf{194°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(10/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.03)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(10/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 1.156 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.156)} = 0.007$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.007 = 0.007$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.007 \longrightarrow T(0,0,t) = \mathbf{211°F}$$

After 15 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(15/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.045)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(15/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 1.734 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.734)} = 0.0005$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.0005 = 0.0005$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.0005 \longrightarrow T(0,0,t) = \mathbf{212°F}$$

(*b*) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

4-77E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and an infinitely long cylinder.

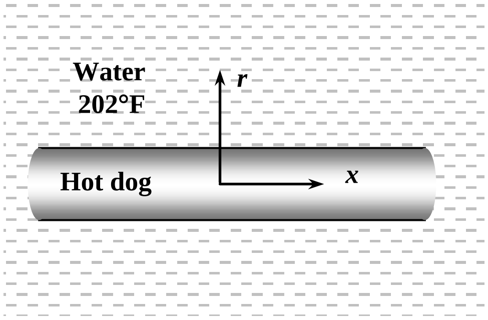
Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial *r*- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be k = 0.44 Btu/h.ft.°F, ρ = 61.2 lbm/ft^3 C_p = 0.93 Btu/lbm.°F, and α = 0.0077 ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2$ = (0.4/12) ft and a plane wall of thickness $2L$ = (5/12) ft. The distance x is measured from the midplane.

After 5 minutes

First the Biot number is calculated for the plane wall to be



$$Bi = \frac{hL}{k} = \frac{(120\ \text{Btu/h.ft}^2.{}^\circ\text{F})(2.5/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.}{}^\circ\text{F})} = 56.8$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.5421 \text{ and } A_1 = 1.2728$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.015)} \cong 1$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120\ \text{Btu/h.ft}^2.{}^\circ\text{F})(0.4/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.}{}^\circ\text{F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 0.578 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.106 = 0.106$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.106 \longrightarrow T(0,0,t) = \mathbf{185°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.03)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.156 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.156)} = 0.007$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.007 = 0.007$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.007 \longrightarrow T(0,0,t) = \mathbf{201°F}$$

After 15 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.045)} \cong 1$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.734 > 0.2$$

$$\theta_{o,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.734)} = 0.0005$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1 \times 0.0005 = 0.0005$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.0005 \longrightarrow T(0,0,t) = \mathbf{202°F}$$

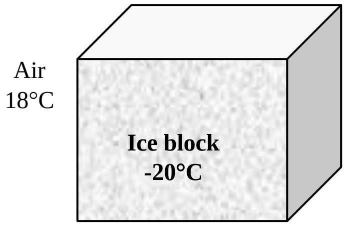
(b) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

4-78 A rectangular ice block is placed on a table. The time the ice block starts melting is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *y*- directions. **2** The thermal properties of the ice block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be k = 2.22 W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.

Analysis This rectangular ice block can be treated as a short rectangular block that can physically be formed by the intersection of two infinite plane wall of thickness 2L = 4 cm and an infinite plane wall of thickness 2L = 10 cm. We measure x from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness 2L = 10 cm. Since the melting starts at the corner of the top surface, we need to determine the time required to melt ice block which will happen when the temperature drops below $0°\text{C}$ at this location. The Biot numbers and the corresponding constants are first determined to be



$$Bi_{\text{wall},1} = \frac{hL_1}{k} = \frac{(12\ \text{W/m}^2.°\text{C})(0.02\ \text{m})}{(2.22\ \text{W/m.°C})} = 0.1081 \longrightarrow \lambda_1 = 0.3208 \quad \text{and} \quad A_1 = 1.0173$$

$$Bi_{\text{wall},3} = \frac{hL_3}{k} = \frac{(12\ \text{W/m}^2.°\text{C})(0.05\ \text{m})}{(2.22\ \text{W/m.°C})} = 0.2703 \longrightarrow \lambda_1 = 0.4951 \quad \text{and} \quad A_1 = 1.0408$$

The ice will start melting at the corners because of the maximum exposed surface area there. Noting that $\tau = \alpha t / L^2$ and assuming that τ > 0.2 in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as

$$\theta(L_1, L_2, L_3, t)_{block} = \theta(L_1, t)_{\text{wall},1}{}^2\, \theta(L_3, t)_{\text{wall},2}$$

$$\frac{0-18}{-20-18} = \left[A_1 e^{-\lambda_1{}^2\tau}\cos(\lambda_1 L_1 / L_1)\right]^2 \left[A_1 e^{-\lambda_1{}^2\tau}\cos(\lambda_1 L_3 / L_3)\right]$$

$$0.4737 = \left\{(1.0173)\exp\left[-(0.3208)^2\frac{(0.124\times10^{-7})t}{(0.02)^2}\right]\cos(0.3208)\right\}^2$$

$$\times\left\{(1.0408)\exp\left[-(0.4951)^2\frac{(0.124\times10^{-7})t}{(0.05)^2}\right]\cos(0.4951)\right\}$$

$$\longrightarrow t = 108{,}135\ \text{s} = \mathbf{30.04\ hours}$$

Therefore, the ice will start melting in about 30 hours.

Discussion Note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\ \text{m}^2\text{/s})(1108{,}135\ \text{s/h})}{(0.05\ \text{m})^2} = 0.536 > 0.2$$

and thus the assumption of τ > 0.2 for the applicability of the one-term approximate solution is verified.

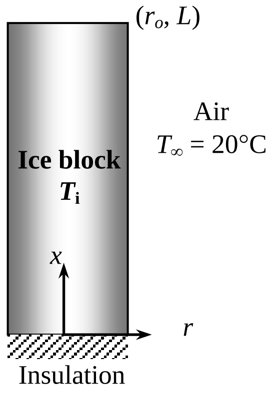
4-79

```
"!PROBLEM 4-79"
"GIVEN"
2*L_1=0.04 "[m]"
L_2=L_1
2*L_3=0.10 "[m]"
"T_i=-20 [C], parameter to be varied"
T_infinity=18 "[C]"
h=12 "[W/m^2-C]"
T_L1_L2_L3=0 "[C]"

"PROPERTIES"
k=2.22 "[W/m-C]"
alpha=0.124E-7 "[m^2/s]"

"ANALYSIS"
"This block can physically be formed by the intersection of two infinite plane wall of thickness
2L=4 cm and an infinite plane wall of thickness 2L=10 cm"
"For the two plane walls"
Bi_w1=(h*L_1)/k
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1_w1=0.3208 "w stands for wall"
A_1_w1=1.0173
time*Convert(min, s)=tau_w1*L_1^2/alpha
"For the third plane wall"
Bi_w3=(h*L_3)/k
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1_w3=0.4951
A_1_w3=1.0408
time*Convert(min, s)=tau_w3*L_3^2/alpha
theta_L_w1=A_1_w1*exp(-lambda_1_w1^2*tau_w1)*Cos(lambda_1_w1*L_1/L_1)
"theta_L_w1=(T_L_w1-T_infinity)/(T_i-T_infinity)"
theta_L_w3=A_1_w3*exp(-lambda_1_w3^2*tau_w3)*Cos(lambda_1_w3*L_3/L_3)
"theta_L_w3=(T_L_w3-T_infinity)/(T_i-T_infinity)"
(T_L1_L2_L3-T_infinity)/(T_i-T_infinity)=theta_L_w1^2*theta_L_w3 "corner temperature"
```

T_i [C]	time [min]
-26	1614
-24	1512
-22	1405
-20	1292
-18	1173
-16	1048
-14	914.9
-12	773.3
-10	621.9
-8	459.4
-6	283.7
-4	92.84

4-80 A cylindrical ice block is placed on a table. The initial temperature of the ice block to avoid melting for 2 h is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *r*- directions. **2** Heat transfer from the base of the ice block to the table is negligible. **3** The thermal properties of the ice block are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).



Properties The thermal properties of the ice are given to be $k = 2.22$ W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.

Analysis This cylindrical ice block can be treated as a short cylinder that can physically be formed by the intersection of a long cylinder of diameter $D = 2$ cm and an infinite plane wall of thickness $2L = 4$ cm. We measure *x* from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L = 4$ cm. The melting starts at the outer surfaces of the top surface when the temperature drops below $0°\text{C}$ at this location. The Biot numbers, the corresponding constants, and the Fourier numbers are

$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(13\,\text{W/m}^2.°\text{C})(0.02\,\text{m})}{(2.22\,\text{W/m.°C})} = 0.1171 \longrightarrow \lambda_1 = 0.3318 \quad \text{and} \quad A_1 = 1.0187$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(13\,\text{W/m}^2.°\text{C})(0.01\,\text{m})}{(2.22\,\text{W/m.°C})} = 0.05856 \longrightarrow \lambda_1 = 0.3407 \quad \text{and} \quad A_1 = 1.0146$$

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\ \text{m}^2/\text{s})(2\ \text{h}\times3600\ \text{s/h})}{(0.02\ \text{m})^2} = 0.2232 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^{\,2}} = \frac{(0.124\times10^{-7}\ \text{m}^2/\text{s})(2\ \text{h}\times3600\ \text{s/h})}{(0.01\ \text{m})^2} = 0.8928 > 0.2$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable. The product solution for this problem can be written as

$$\theta(L,r_o,t)_{\text{block}} = \theta(L,t)_{\text{wall}}\,\theta(r_o,t)_{\text{cyl}}$$

$$\frac{0-20}{T_i-20} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L)\right]\left[A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o/r_o)\right]$$

$$\frac{0-20}{T_i-20} = \left[(1.0187)e^{-(0.3318)^2(0.2232)}\cos(0.3318)\right]\left[(1.0146)e^{-(0.3407)^2(0.8928)}(0.9707)\right]$$

which gives $\quad T_i = \mathbf{-4°\,C}$

Therefore, the ice will not start melting for at least 2 hours if its initial temperature is -4°C or below.