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سایت آموزش مهندسی مکانیک

**4-81** A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

**Assumptions** **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all  $x$ -,  $y$ , and  $z$ - directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial  $x$ - and radial  $r$ - directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is  $\tau > 0.2$  so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

**Properties** The thermal properties of the granite are given to be  $k = 2.5 \text{ W/m}\cdot\text{°C}$  and  $\alpha = 1.15 \times 10^{-6} \text{ m}^2/\text{s}$ .

**Analysis:**

**Cubic block:** This cubic block can physically be formed by the intersection of three infinite plane walls of thickness  $2L = 5 \text{ cm}$ .

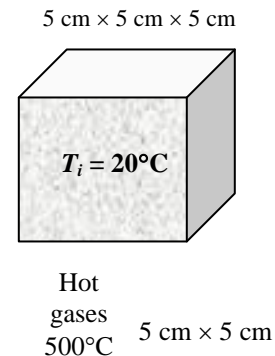
After 10 minutes: The Biot number, the corresponding constants, and the Fourier number are

$$Bi = \frac{hL}{k} = \frac{(40 \text{ W/m}^2 \cdot \text{°C})(0.025 \text{ m})}{(2.5 \text{ W/m}\cdot\text{°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15 \times 10^{-6} \text{ m}^2/\text{s})(10 \text{ min} \times 60 \text{ s/min})}{(0.025 \text{ m})^2} = 1.104 > 0.2$$

To determine the center temperature, the product solution can be written as

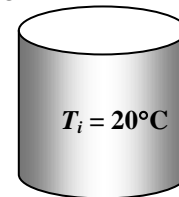
$$\begin{aligned} \theta(0,0,0,t)_{\text{block}} &= [\theta(0,t)_{\text{wall}}]^3 \\ \frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} &= \left( A_1 e^{-\lambda_1^2 \tau} \right)^3 \\ \frac{T(0,0,0,t) - 500}{20 - 500} &= \left\{ (1.0580) e^{-(0.5932)^2 (1.104)} \right\}^3 = 0.369 \\ T(0,0,0,t) &= \mathbf{323^\circ\text{C}} \end{aligned}$$



After 20 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15 \times 10^{-6} \text{ m}^2/\text{s})(20 \text{ min} \times 60 \text{ s/min})}{(0.025 \text{ m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t) - 500}{20 - 500} = \left\{ (1.0580) e^{-(0.5932)^2 (2.208)} \right\}^3 = 0.115 \longrightarrow T(0,0,0,t) = \mathbf{445^\circ\text{C}}$$



After 60 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15 \times 10^{-6} \text{ m}^2/\text{s})(60 \text{ min} \times 60 \text{ s/min})}{(0.025 \text{ m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t) - 500}{20 - 500} = \left\{ (1.0580) e^{-(0.5932)^2 (6.624)} \right\}^3 = 0.00109 \longrightarrow T(0,0,0,t) = \mathbf{500^\circ\text{C}}$$

Note that  $\tau > 0.2$  in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

**Cylinder:** This cylindrical block can physically be formed by the intersection of a long cylinder of radius  $r_o = D/2 = 2.5$  cm and a plane wall of thickness  $2L = 5$  cm.

After 10 minutes: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40 \text{ W/m}^2 \cdot \text{°C})(0.025 \text{ m})}{(2.5 \text{ W/m} \cdot \text{°C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution can be written as

$$\begin{aligned} \theta(0,0,t)_{block} &= [\theta(0,t)_{wall}] [\theta(0,t)_{cyl}] \\ \frac{T(0,0,t) - T_\infty}{T_i - T_\infty} &= \left( A_1 e^{-\lambda_1^2 \tau} \right)_{wall} \left( A_1 e^{-\lambda_1^2 \tau} \right)_{cyl} \\ \frac{T(0,0,t) - 500}{20 - 500} &= \left\{ (1.0580) e^{-(0.5932)^2 (1.104)} \right\} \left\{ (1.0931) e^{-(0.8516)^2 (1.104)} \right\} = 0.352 \longrightarrow T(0,0,t) = \mathbf{331^\circ\text{C}} \end{aligned}$$

After 20 minutes

$$\frac{T(0,0,t) - 500}{20 - 500} = \left\{ (1.0580) e^{-(0.5932)^2 (2.208)} \right\} \left\{ (1.0931) e^{-(0.8516)^2 (2.208)} \right\} = 0.107 \longrightarrow T(0,0,t) = \mathbf{449^\circ\text{C}}$$

After 60 minutes

$$\frac{T(0,0,t) - 500}{20 - 500} = \left\{ (1.0580) e^{-(0.5932)^2 (6.624)} \right\} \left\{ (1.0931) e^{-(0.8516)^2 (6.624)} \right\} = 0.00092 \longrightarrow T(0,0,t) = \mathbf{500^\circ\text{C}}$$

Note that  $\tau > 0.2$  in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

**4-82** A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

**Assumptions 1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all  $x$ -,  $y$ , and  $z$ - directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial  $x$ - and radial  $r$ - directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is  $\tau > 0.2$  so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

**Properties** The thermal properties of the granite are  $k = 2.5 \text{ W/m}\cdot\text{°C}$  and  $\alpha = 1.15 \times 10^{-6} \text{ m}^2/\text{s}$ .

**Analysis:**

**Cubic block:** This cubic block can physically be formed by the intersection of three infinite plane wall of thickness  $2L = 5 \text{ cm}$ . Two infinite plane walls are exposed to the hot gases with a heat transfer coefficient of  $h = 40 \text{ W/m}^2\cdot\text{°C}$  and one with  $h = 80 \text{ W/m}^2\cdot\text{°C}$ .

After 10 minutes: The Biot number and the corresponding constants for  $h = 40 \text{ W/m}^2\cdot\text{°C}$  are

$$Bi = \frac{hL}{k} = \frac{(40 \text{ W/m}^2\cdot\text{°C})(0.025 \text{ m})}{(2.5 \text{ W/m}\cdot\text{°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

The Biot number and the corresponding constants for  $h = 80 \text{ W/m}^2\cdot\text{°C}$  are

$$Bi = \frac{hL}{k} = \frac{(80 \text{ W/m}^2\cdot\text{°C})(0.025 \text{ m})}{(2.5 \text{ W/m}\cdot\text{°C})} = 0.800$$

$$\longrightarrow \lambda_1 = 0.7910 \quad \text{and} \quad A_1 = 1.1016$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15 \times 10^{-6} \text{ m}^2/\text{s})(10 \text{ min} \times 60 \text{ s/min})}{(0.025 \text{ m})^2} = 1.104 > 0.2$$

To determine the center temperature, the product solution method can be written as

$$\begin{aligned} \theta(0,0,0,t)_{\text{block}} &= [\theta(0,t)_{\text{wall}}]^2 [\theta(0,t)_{\text{wall}}] \\ \frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} &= \left( A_1 e^{-\lambda_1^2 \tau} \right)^2 \left( A_1 e^{-\lambda_1^2 \tau} \right) \\ \frac{T(0,0,0,t) - 500}{20 - 500} &= \left\{ (1.0580) e^{-(0.5932)^2 (1.104)} \right\}^2 \left\{ (1.1016) e^{-(0.7910)^2 (1.104)} \right\} = 0.284 \end{aligned}$$

$$T(0,0,0,t) = 364^\circ\text{C}$$

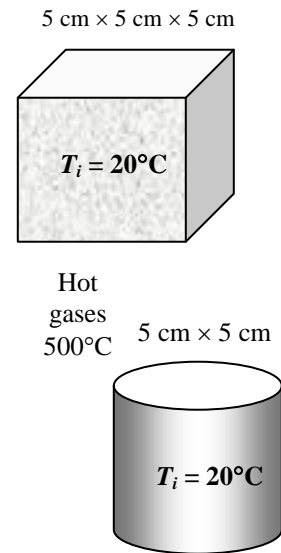
After 20 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15 \times 10^{-6} \text{ m}^2/\text{s})(20 \text{ min} \times 60 \text{ s/min})}{(0.025 \text{ m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t) - 500}{20 - 500} = \left\{ (1.0580) e^{-(0.5932)^2 (2.208)} \right\}^2 \left\{ (1.1016) e^{-(0.7910)^2 (2.208)} \right\} = 0.0654$$

$$\longrightarrow T(0,0,0,t) = 469^\circ\text{C}$$

After 60 minutes



$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15 \times 10^{-6} \text{ m}^2/\text{s})(60 \text{ min} \times 60 \text{ s/min})}{(0.025 \text{ m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t) - 500}{20 - 500} = \left\{ (1.0580)e^{-(0.5932)^2(6.624)} \right\}^2 \left\{ (1.1016)e^{-(0.7910)^2(6.624)} \right\} = 0.000186$$

$$\longrightarrow T(0,0,0,t) = 500^\circ\text{C}$$

Note that  $\tau > 0.2$  in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

**Cylinder:** This cylindrical block can physically be formed by the intersection of a long cylinder of radius  $r_o = D/2 = 2.5 \text{ cm}$  exposed to the hot gases with a heat transfer coefficient of  $h = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$  and a plane wall of thickness  $2L = 5 \text{ cm}$  exposed to the hot gases with  $h = 80 \text{ W/m}^2 \cdot ^\circ\text{C}$ .

After 10 minutes: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40 \text{ W/m}^2 \cdot ^\circ\text{C})(0.025 \text{ m})}{(2.5 \text{ W/m} \cdot ^\circ\text{C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution method can be written as

$$\begin{aligned} \theta(0,0,t)_{\text{block}} &= [\theta(0,t)_{\text{wall}}][\theta(0,t)_{\text{cyl}}] \\ \frac{T(0,0,t) - T_\infty}{T_i - T_\infty} &= \left( A_1 e^{-\lambda_1^2 \tau} \right)_{\text{wall}} \left( A_1 e^{-\lambda_1^2 \tau} \right)_{\text{cyl}} \\ \frac{T(0,0,t) - 500}{20 - 500} &= \left\{ (1.1016)e^{-(0.7910)^2(1.104)} \right\} \left\{ (1.0931)e^{-(0.8516)^2(1.104)} \right\} = 0.271 \end{aligned}$$

$$T(0,0,t) = 370^\circ\text{C}$$

After 20 minutes

$$\frac{T(0,0,t) - 500}{20 - 500} = \left\{ (1.1016)e^{-(0.7910)^2(2.208)} \right\} \left\{ (1.0931)e^{-(0.8516)^2(2.208)} \right\} = 0.06094 \longrightarrow T(0,0,t) = 471^\circ\text{C}$$

After 60 minutes

$$\frac{T(0,0,t) - 500}{20 - 500} = \left\{ (1.1016)e^{-(0.7910)^2(6.624)} \right\} \left\{ (1.0931)e^{-(0.8516)^2(6.624)} \right\} = 0.0001568 \longrightarrow T(0,0,t) = 500^\circ\text{C}$$

Note that  $\tau > 0.2$  in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

**4-83** A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transfer to the block are to be determined.

**Assumptions** 1 Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial  $x$ - and radial  $r$ - directions. 2 The thermal properties of the aluminum are constant. 3 The heat transfer coefficient is constant and uniform over the entire surface. 4 The Fourier number is  $\tau > 0.2$  so that the one-term approximate solutions (or the transient temperature charts) are applicable (it will be verified).

**Properties** The thermal properties of the aluminum block are given to be  $k = 236 \text{ W/m}\cdot^\circ\text{C}$ ,  $\rho = 2702 \text{ kg/m}^3$ ,  $C_p = 0.896 \text{ kJ/kg}\cdot^\circ\text{C}$ , and  $\alpha = 9.75 \times 10^{-5} \text{ m}^2/\text{s}$ .

**Analysis** This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness  $2L = 20 \text{ cm}$ , and a long cylinder of radius  $r_o = D/2 = 7.5 \text{ cm}$ . The Biot numbers and the corresponding constants are first determined to be

$$Bi = \frac{hL}{k} = \frac{(80 \text{ W/m}^2 \cdot ^\circ\text{C})(0.1 \text{ m})}{(236 \text{ W/m}\cdot^\circ\text{C})} = 0.0339 \quad \longrightarrow \lambda_1 = 0.1811 \text{ and } A_1 = 1.0056$$

$$Bi = \frac{hr_o}{k} = \frac{(80 \text{ W/m}^2 \cdot ^\circ\text{C})(0.075 \text{ m})}{236 \text{ W/m}\cdot^\circ\text{C}} = 0.0254 \quad \longrightarrow \lambda_1 = 0.2217 \text{ and } A_1 = 1.0063$$

Noting that  $\tau = \alpha t / L^2$  and assuming  $\tau > 0.2$  in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}} \theta(0,t)_{\text{cyl}} = \left( A_1 e^{-\lambda_1^2 \tau} \right)_{\text{wall}} \left( A_1 e^{-\lambda_1^2 \tau} \right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{ (1.0056) \exp \left[ - (0.1811)^2 \frac{(9.75 \times 10^{-5}) t}{(0.1)^2} \right] \right\} \left\{ (1.0063) \exp \left[ - (0.2217)^2 \frac{(9.75 \times 10^{-5}) t}{(0.075)^2} \right] \right\} = 0.7627$$

Solving for the time  $t$  gives  $t = 241 \text{ s} = \mathbf{4.0 \text{ min}}$ . We note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(9.75 \times 10^{-5} \text{ m}^2/\text{s})(241 \text{ s})}{(0.1 \text{ m})^2} = 2.35 > 0.2$$

and thus the assumption of  $\tau > 0.2$  for the applicability of the one-term approximate solution is verified.

The maximum amount of heat transfer is

$$m = \rho V = \rho \pi r_o^2 L = (2702 \text{ kg/m}^3) \left[ \pi (0.075 \text{ m})^2 (0.2 \text{ m}) \right] = 9.550 \text{ kg}$$

$$Q_{\text{max}} = m C_p (T_i - T_\infty) = (9.550 \text{ kg})(0.896 \text{ kJ/kg}\cdot^\circ\text{C})(20 - 1200)^\circ\text{C} = 10,100 \text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left( \frac{Q}{Q_{\text{max}}} \right)_{\text{wall}} = 1 - \theta_{o,\text{wall}} \frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.7627) \frac{\sin(0.1811)}{0.1811} = 0.2415$$

$$\left( \frac{Q}{Q_{\text{max}}} \right)_{\text{cyl}} = 1 - 2\theta_{o,\text{cyl}} \frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7627) \frac{0.1101}{0.2217} = 0.2425$$

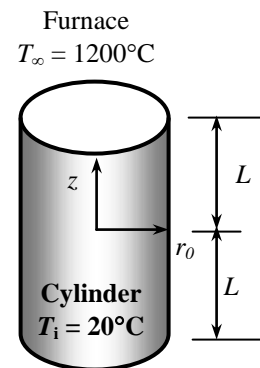
The heat transfer ratio for the short cylinder is

$$\left( \frac{Q}{Q_{\text{max}}} \right)_{\text{short cylinder}} = \left( \frac{Q}{Q_{\text{max}}} \right)_{\text{plane wall}} + \left( \frac{Q}{Q_{\text{max}}} \right)_{\text{long cylinder}} \left[ 1 - \left( \frac{Q}{Q_{\text{max}}} \right)_{\text{plane wall}} \right] = 0.2415 + (0.2425)(1 - 0.2415) = 0.4254$$

Then the total heat transfer from the short cylinder as it is cooled from  $300^\circ\text{C}$  at the center to  $20^\circ\text{C}$  becomes

$$Q = 0.4236 Q_{\text{max}} = (0.4254)(10,100 \text{ kJ}) = \mathbf{4297 \text{ kJ}}$$

which is identical to the heat transfer to the cylinder as the cylinder at  $20^\circ\text{C}$  is heated to  $300^\circ\text{C}$  at the center.



**4-84** A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transferred to the block are to be determined.

**Assumptions** 1 Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial  $x$ - and radial  $r$ - directions. 2 Heat transfer from the bottom surface of the block is negligible. 3 The thermal properties of the aluminum are constant. 4 The heat transfer coefficient is constant and uniform over the entire surface. 5 The Fourier number is  $\tau > 0.2$  so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

**Properties** The thermal properties of the aluminum block are given to be  $k = 236 \text{ W/m}\cdot\text{°C}$ ,  $\rho = 2702 \text{ kg/m}^3$ ,  $C_p = 0.896 \text{ kJ/kg}\cdot\text{°C}$ , and  $\alpha = 9.75 \times 10^{-5} \text{ m}^2/\text{s}$ .

**Analysis** This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness  $2L = 40 \text{ cm}$  and a long cylinder of radius  $r_0 = D/2 = 7.5 \text{ cm}$ . Note that the height of the short cylinder represents the half thickness of the infinite plane wall where the bottom surface of the short cylinder is adiabatic. The Biot numbers and corresponding constants are first determined to be

$$Bi = \frac{hL}{k} = \frac{(80 \text{ W/m}^2\cdot\text{°C})(0.2 \text{ m})}{(236 \text{ W/m}\cdot\text{°C})} = 0.0678 \quad \longrightarrow \lambda_1 = 0.2568 \quad \text{and} \quad A_1 = 1.0110$$

$$Bi = \frac{hr_0}{k} = \frac{(80 \text{ W/m}^2\cdot\text{°C})(0.075 \text{ m})}{(236 \text{ W/m}\cdot\text{°C})} = 0.0254 \quad \longrightarrow \lambda_1 = 0.2217 \quad \text{and} \quad A_1 = 1.0063$$

Noting that  $\tau = \alpha t / L^2$  and assuming  $\tau > 0.2$  in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}} \theta(0,t)_{\text{cyl}} = \left( A_1 e^{-\lambda_1^2 \tau} \right)_{\text{wall}} \left( A_1 e^{-\lambda_1^2 \tau} \right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{ (1.0110) \exp \left[ - (0.2568)^2 \frac{(9.75 \times 10^{-5}) t}{(0.2)^2} \right] \right\} \left\{ (1.0063) \exp \left[ - (0.2217)^2 \frac{(9.75 \times 10^{-5}) t}{(0.075)^2} \right] \right\} = 0.7627$$

Solving for the time  $t$  gives  $t = 285 \text{ s} = \mathbf{4.7 \text{ min}}$ . We note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(9.75 \times 10^{-5} \text{ m}^2/\text{s})(285 \text{ s})}{(0.2 \text{ m})^2} = 0.69 > 0.2$$

and thus the assumption of  $\tau > 0.2$  for the applicability of the one-term approximate solution is verified.

The maximum amount of heat transfer is

$$m = \rho V = \rho \pi r_0 L = (2702 \text{ kg/m}^3) [\pi (0.075 \text{ m})^2 (0.2 \text{ m})] = 9.55 \text{ kg}$$

$$Q_{\text{max}} = m C_p (T_i - T_\infty) = (9.55 \text{ kg})(0.896 \text{ kJ/kg}\cdot\text{°C})(20 - 1200)^\circ\text{C} = 10,100 \text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left( \frac{Q}{Q_{\text{max}}} \right)_{\text{wall}} = 1 - \theta_{o,\text{wall}} \frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.7627) \frac{\sin(0.2568)}{0.2568} = 0.2457$$

$$\left( \frac{Q}{Q_{\text{max}}} \right)_{\text{cyl}} = 1 - 2\theta_{o,\text{cyl}} \frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7627) \frac{0.1101}{0.2217} = 0.2425$$

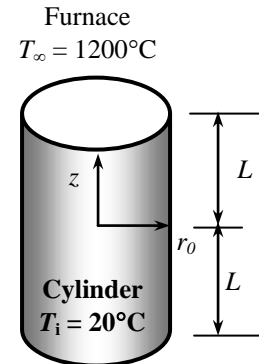
The heat transfer ratio for the short cylinder is

$$\left( \frac{Q}{Q_{\text{max}}} \right)_{\text{short cylinder}} = \left( \frac{Q}{Q_{\text{max}}} \right)_{\text{plane wall}} + \left( \frac{Q}{Q_{\text{max}}} \right)_{\text{long cylinder}} \left[ 1 - \left( \frac{Q}{Q_{\text{max}}} \right)_{\text{plane wall}} \right] = 0.2457 + (0.2425)(1 - 0.2457) = 0.4286$$

Then the total heat transfer from the short cylinder as it is cooled from  $300^\circ\text{C}$  at the center to  $20^\circ\text{C}$  becomes

$$Q = 0.4255 Q_{\text{max}} = (0.4286)(10,100 \text{ kJ}) = \mathbf{4239 \text{ kJ}}$$

which is identical to the heat transfer to the cylinder as the cylinder at  $20^\circ\text{C}$  is heated to  $300^\circ\text{C}$  at the center.



4-85

"!PROBLEM 4-85"

"GIVEN"

$$2*L=0.20 \text{ "[m]"}$$

$$2*r_o=0.15 \text{ "[m]"}$$

$$T_i=20 \text{ "[C]"}$$

$$T_{\infty}=1200 \text{ "[C]"}$$

$$T_{o_o}=300 \text{ [C], parameter to be varied"}$$

$$h=80 \text{ "[W/m^2-C]"}$$

"PROPERTIES"

$$k=236 \text{ "[W/m-C]"}$$

$$\rho=2702 \text{ "[kg/m^3]"}$$

$$C_p=0.896 \text{ "[kJ/kg-C]"}$$

$$\alpha=9.75E-5 \text{ "[m^2/s]"}$$

"ANALYSIS"

"This short cylinder can physically be formed by the intersection of a long cylinder of radius  $r_o$  and a plane wall of thickness  $2L$ "

"For plane wall"

$$Bi_w=(h*L)/k$$

"From Table 4-1 corresponding to this Bi number, we read"

$$\lambda_{1_w}=0.1439 \text{ "w stands for wall"}$$

$$A_{1_w}=1.0035$$

$$\tau_w=(\alpha*time)/L^2$$

$$\theta_{o_w}=A_{1_w}*exp(-\lambda_{1_w}^2*\tau_w) \text{ "}\theta_{o_w}=(T_{o_w}-T_{\infty})/(T_i-T_{\infty})\text{"}$$

"For long cylinder"

$$Bi_c=(h*r_o)/k \text{ "c stands for cylinder"}$$

"From Table 4-1 corresponding to this Bi number, we read"

$$\lambda_{1_c}=0.1762$$

$$A_{1_c}=1.0040$$

$$\tau_c=(\alpha*time)/r_o^2$$

$$\theta_{o_c}=A_{1_c}*exp(-\lambda_{1_c}^2*\tau_c) \text{ "}\theta_{o_c}=(T_{o_c}-T_{\infty})/(T_i-T_{\infty})\text{"}$$

$$(T_{o_o}-T_{\infty})/(T_i-T_{\infty})=\theta_{o_w}*\theta_{o_c} \text{ "center temperature of cylinder"}$$

$$V=\pi*r_o^2*(2*L)$$

$$m=\rho*V$$

$$Q_{\max}=m*C_p*(T_{\infty}-T_i)$$

$$Q_w=1-\theta_{o_w}*J_1(\lambda_{1_w})/\lambda_{1_w} \text{ "Q_w=(Q/Q_max)_w"}$$

$$Q_c=1-2*\theta_{o_c}*J_1(\lambda_{1_c})/\lambda_{1_c} \text{ "Q_c=(Q/Q_max)_c"}$$

$$J_1=0.0876 \text{ "From Table 4-2, at } \lambda_{1_c}\text{"}$$

$$Q/Q_{\max}=Q_w+Q_c*(1-Q_w) \text{ "total heat transfer"}$$

<b>T<sub>∞</sub> [C]</b>	<b>time [s]</b>	<b>Q [kJ]</b>
50	44.91	346.3
100	105	770.2
150	167.8	1194
200	233.8	1618
250	303.1	2042
300	376.1	2466
350	453.4	2890
400	535.3	3314
450	622.5	3738
500	715.7	4162
550	815.9	4586
600	924	5010
650	1042	5433
700	1170	5857
750	1313	6281
800	1472	6705
850	1652	7129
900	1861	7553
950	2107	7977
1000	2409	8401

**Special Topic: Refrigeration and Freezing of Foods**

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**4-86C** The common kinds of microorganisms are bacteria, yeasts, molds, and viruses. The undesirable changes caused by microorganisms are off-flavors and colors, slime production, changes in the texture and appearances, and the spoilage of foods.

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**4-87C** Microorganisms are the prime cause for the spoilage of foods. Refrigeration prevents or delays the spoilage of foods by reducing the rate of growth of microorganisms. Freezing extends the storage life of foods for months by preventing the growths of microorganisms.

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**4-88C** The environmental factors that affect of the growth rate of microorganisms are the temperature, the relative humidity, the oxygen level of the environment, and air motion.

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**4-89C** Cooking kills the microorganisms in foods, and thus prevents spoilage of foods. It is important to raise the internal temperature of a roast in an oven above 70°C since most microorganisms, including some that cause diseases, may survive temperatures below 70°C.

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**4-90C** The contamination of foods with microorganisms can be prevented or minimized by (1) preventing contamination by following strict sanitation practices such as washing hands and using fine filters in ventilation systems, (2) inhibiting growth by altering the environmental conditions, and (3) destroying the organisms by heat treatment or chemicals.

The growth of microorganisms in foods can be retarded by keeping the temperature below 4°C and relative humidity below 60 percent. Microorganisms can be destroyed by heat treatment, chemicals, ultraviolet light, and solar radiation.

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**4-91C** (a) High air motion retards the growth of microorganisms in foods by keeping the food surfaces dry, and creating an undesirable environment for the microorganisms. (b) Low relative humidity (dry) environments also retard the growth of microorganisms by depriving them of water that they need to grow. Moist air supplies the microorganisms with the water they need, and thus encourages their growth. Relative humidities below 60 percent prevent the growth rate of most microorganisms on food surfaces.

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**4-92C** Cooling the carcass with refrigerated air is at -10°C would certainly reduce the cooling time, but this proposal should be rejected since it will cause the outer parts of the carcasses to freeze, which is undesirable. Also, the refrigeration unit will consume more power to reduce the temperature to -10°C, and thus it will have a lower efficiency.

**4-93C** The freezing time could be decreased by (a) lowering the temperature of the refrigerated air, (b) increasing the velocity of air, (c) increasing the capacity of the refrigeration system, and (d) decreasing the size of the meat boxes.

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**4-94C** The rate of freezing can affect color, tenderness, and drip. Rapid freezing increases tenderness and reduces the tissue damage and the amount of drip after thawing.

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**4-95C** This claim is reasonable since the lower the storage temperature, the longer the storage life of beef. This is because some water remains unfrozen even at subfreezing temperatures, and the lower the temperature, the smaller the unfrozen water content of the beef.

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**4-96C** A refrigerated shipping dock is a refrigerated space where the orders are assembled and shipped out. Such docks save valuable storage space from being used for shipping purpose, and provide a more acceptable working environment for the employees. The refrigerated shipping docks are usually maintained at 1.5°C, and therefore the air that flows into the freezer during shipping is already cooled to about 1.5°C. This reduces the refrigeration load of the cold storage rooms.

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**4-97C** (a) The heat transfer coefficient during immersion cooling is much higher, and thus the cooling time during immersion chilling is much lower than that during forced air chilling. (b) The cool air chilling can cause a moisture loss of 1 to 2 percent while water immersion chilling can actually cause moisture absorption of 4 to 15 percent. (c) The chilled water circulated during immersion cooling encourages microbial growth, and thus immersion chilling is associated with more microbial growth. The problem can be minimized by adding chloride to the water.

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**4-98C** The proper storage temperature of frozen poultry is about -18°C or below. The primary freezing methods of poultry are the air blast tunnel freezing, cold plates, immersion freezing, and cryogenic cooling.

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**4-99C** The factors, which affect the quality of frozen, fish are the condition of the fish before freezing, the freezing method, and the temperature and humidity during storage and transportation, and the length of storage time.

**4-100** The chilling room of a meat plant with a capacity of 350 beef carcasses is considered. The cooling load, the air flow rate, and the heat transfer area of the evaporator are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Specific heats of beef carcass and air are constant.

**Properties** The density and specific heat of air at 0°C are given to be 1.28 kg/m<sup>3</sup> and 1.0 kJ/kg·°C. The specific heat of beef carcass is given to be 3.14 kJ/kg·°C.

**Analysis** (a) The amount of beef mass that needs to be cooled per unit time is

$$\begin{aligned} \dot{m}_{beef} &= (\text{Totalbeef mass cooled})/(\text{coolingtime}) \\ &= (350 \times 280 \text{ kg/carcass}) / (12 \text{ h} \times 3600 \text{ s}) = 2.27 \text{ kg/s} \end{aligned}$$

The product refrigeration load can be viewed as the energy that needs to be removed from the beef carcass as it is cooled from 35 to 16°C at a rate of 2.27 kg/s, and is determined to be

$$\begin{aligned} \dot{Q}_{beef} &= (\dot{m} C_p \Delta T)_{beef} \\ &= (2.27 \text{ kg/s})(3.14 \text{ kJ/kg} \cdot ^\circ\text{C})(35 - 16)^\circ\text{C} = 135 \text{ kW} \end{aligned}$$

Then the total refrigeration load of the chilling room becomes

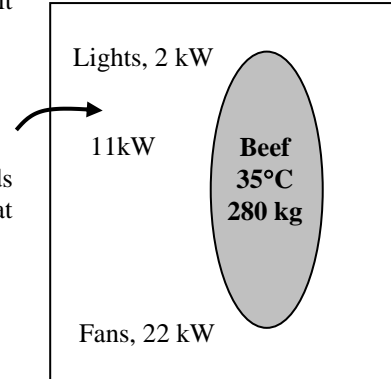
$$\dot{Q}_{\text{total, chilling room}} = \dot{Q}_{beef} + \dot{Q}_{fan} + \dot{Q}_{lights} + \dot{Q}_{\text{heat gain}} = 135 + 22 + 2 + 11 = \mathbf{170 \text{ kW}}$$

(b) Heat is transferred to air at the rate determined above, and the temperature of air rises from -2.2°C to 0.5°C as a result. Therefore, the mass flow rate of air is

$$\dot{m}_{air} = \frac{\dot{Q}_{air}}{(C_p \Delta T)_{air}} = \frac{170 \text{ kW}}{(1.0 \text{ kJ/kg} \cdot ^\circ\text{C})[0.5 - (-2.2)^\circ\text{C}]} = 63.0 \text{ kg/s}$$

Then the volume flow rate of air becomes

$$\dot{V}_{air} = \frac{\dot{m}_{air}}{\rho_{air}} = \frac{63.0 \text{ kg/s}}{1.28 \text{ kg/m}^3} = \mathbf{49.2 \text{ m}^3/\text{s}}$$

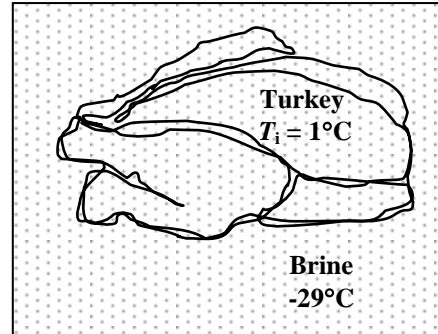


**4-101** Turkeys are to be frozen by submerging them into brine at  $-29^{\circ}\text{C}$ . The time it will take to reduce the temperature of turkey breast at a depth of 3.8 cm to  $-18^{\circ}\text{C}$  and the amount of heat transfer per turkey are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The thermal properties of turkeys are constant.

**Properties** It is given that the specific heats of turkey are 2.98 and 1.65 kJ/kg $\cdot^{\circ}\text{C}$  above and below the freezing point of  $-2.8^{\circ}\text{C}$ , respectively, and the latent heat of fusion of turkey is 214 kJ/kg.

**Analysis** The time required to freeze the turkeys from  $1^{\circ}\text{C}$  to  $-18^{\circ}\text{C}$  with brine at  $-29^{\circ}\text{C}$  can be determined directly from Fig. 4-45 to be



$$t \cong 180 \text{ min.} \cong \mathbf{3 \text{ hours}}$$

(a) Assuming the entire water content of turkey is frozen, the amount of heat that needs to be removed from the turkey as it is cooled from  $1^{\circ}\text{C}$  to  $-18^{\circ}\text{C}$  is

$$\text{Cooling to } -2.8^{\circ}\text{C}: Q_{\text{cooling, fresh}} = (mC \Delta T)_{\text{fresh}} = (7 \text{ kg})(2.98 \text{ kJ/kg}\cdot^{\circ}\text{C})[1 - (-2.8)^{\circ}\text{C}] = 79.3 \text{ kJ}$$

$$\text{Freezing at } -2.8^{\circ}\text{C}: Q_{\text{freezing}} = mh_{\text{latent}} = (7 \text{ kg})(214 \text{ kJ/kg}) = 1498 \text{ kJ}$$

$$\text{Cooling } -18^{\circ}\text{C}: Q_{\text{cooling, frozen}} = (mC \Delta T)_{\text{frozen}} = (7 \text{ kg})(1.65 \text{ kJ/kg}\cdot^{\circ}\text{C})[-2.8 - (-18)]^{\circ}\text{C} = 175.6 \text{ kJ}$$

Therefore, the total amount of heat removal per turkey is

$$Q_{\text{total}} = Q_{\text{cooling, fresh}} + Q_{\text{freezing}} + Q_{\text{cooling, frozen}} = 79.3 + 1498 + 175.6 \cong \mathbf{1753 \text{ kJ}}$$

(b) Assuming only 90 percent of the water content of turkey is frozen, the amount of heat that needs to be removed from the turkey as it is cooled from  $1^{\circ}\text{C}$  to  $-18^{\circ}\text{C}$  is

$$\text{Cooling to } -2.8^{\circ}\text{C}: Q_{\text{cooling, fresh}} = (mC \Delta T)_{\text{fresh}} = (7 \text{ kg})(2.98 \text{ kJ/kg}\cdot^{\circ}\text{C})[1 - (-2.98)^{\circ}\text{C}] = 79.3 \text{ kJ}$$

$$\text{Freezing at } -2.8^{\circ}\text{C}: Q_{\text{freezing}} = mh_{\text{latent}} = (7 \times 0.9 \text{ kg})(214 \text{ kJ/kg}) = 1,348 \text{ kJ}$$

$$\text{Cooling } -18^{\circ}\text{C}: Q_{\text{cooling, frozen}} = (mC \Delta T)_{\text{frozen}} = (7 \times 0.9 \text{ kg})(1.65 \text{ kJ/kg}\cdot^{\circ}\text{C})[-2.8 - (-18)]^{\circ}\text{C} = 158 \text{ kJ}$$

$$Q_{\text{cooling, unfrozen}} = (mC \Delta T)_{\text{fresh}} = (7 \times 0.1 \text{ kg})(2.98 \text{ kJ/kg}\cdot^{\circ}\text{C})[-2.8 - (-18)]^{\circ}\text{C} = 31.7 \text{ kJ}$$

Therefore, the total amount of heat removal per turkey is

$$Q_{\text{total}} = Q_{\text{cooling, fresh}} + Q_{\text{freezing}} + Q_{\text{cooling, frozen \& unfrozen}} = 79.3 + 1348 + 158 + 31.7 = \mathbf{1617 \text{ kJ}}$$

**4-102E** Chickens are to be frozen by refrigerated air. The cooling time of the chicken is to be determined for the cases of cooling air being at  $-40^{\circ}\text{F}$  and  $-80^{\circ}\text{F}$ .

**Assumptions** 1 Steady operating conditions exist. 2 The thermal properties of chickens are constant.

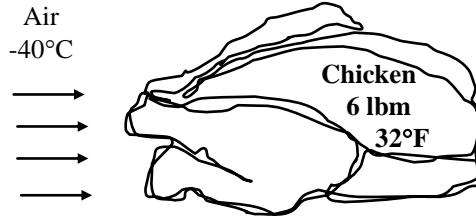
**Analysis** The time required to reduce the inner surface temperature of the chickens from  $32^{\circ}\text{F}$  to  $25^{\circ}\text{F}$  with refrigerated air at  $-40^{\circ}\text{F}$  is determined from Fig. 4-44 to be

$$t \cong 2.3 \text{ hours}$$

If the air temperature were  $-80^{\circ}\text{F}$ , the freezing time would be

$$t \cong 1.4 \text{ hours}$$

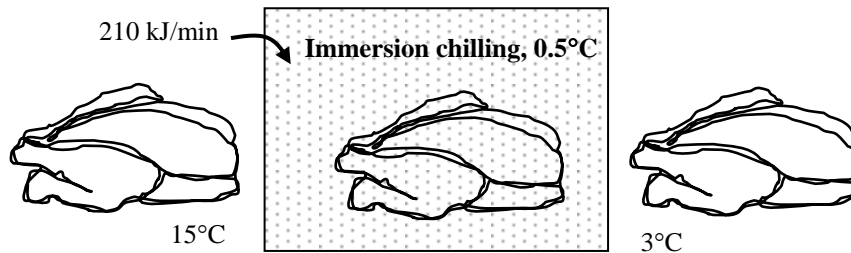
Therefore, the time required to cool the chickens to  $25^{\circ}\text{F}$  is reduced considerably when the refrigerated air temperature is decreased.



**4-103** Chickens are to be cooled by chilled water in an immersion chiller. The rate of heat removal from the chicken and the mass flow rate of water are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The thermal properties of chickens are constant.

**Properties** The specific heat of chicken are given to be  $3.54 \text{ kJ/kg}\cdot^{\circ}\text{C}$ . The specific heat of water is  $4.18 \text{ kJ/kg}\cdot^{\circ}\text{C}$  (Table A-9).



**Analysis** (a) Chickens are dropped into the chiller at a rate of 500 per hour. Therefore, chickens can be considered to flow steadily through the chiller at a mass flow rate of

$$\dot{m}_{\text{chicken}} = (500 \text{ chicken} / \text{h})(2.2 \text{ kg} / \text{chicken}) = 1100 \text{ kg} / \text{h} = 0.3056 \text{ kg} / \text{s}$$

Then the rate of heat removal from the chickens as they are cooled from  $15^{\circ}\text{C}$  to  $3^{\circ}\text{C}$  at this rate becomes

$$\dot{Q}_{\text{chicken}} = (\dot{m}C_p\Delta T)_{\text{chicken}} = (0.3056 \text{ kg/s})(3.54 \text{ kJ/kg}\cdot^{\circ}\text{C})(15 - 3)^{\circ}\text{C} = \mathbf{13.0 \text{ kW}}$$

(b) The chiller gains heat from the surroundings as a rate of  $210 \text{ kJ/min} = 3.5 \text{ kJ/s}$ . Then the total rate of heat gain by the water is

$$\dot{Q}_{\text{water}} = \dot{Q}_{\text{chicken}} + \dot{Q}_{\text{heat gain}} = 13.0 + 3.5 = 16.5 \text{ kW}$$

Noting that the temperature rise of water is not to exceed  $2^{\circ}\text{C}$  as it flows through the chiller, the mass flow rate of water must be at least

$$\dot{m}_{\text{water}} = \frac{\dot{Q}_{\text{water}}}{(C_p\Delta T)_{\text{water}}} = \frac{16.5 \text{ kW}}{(4.18 \text{ kJ/kg}\cdot^{\circ}\text{C})(2^{\circ}\text{C})} = \mathbf{1.97 \text{ kg/s}}$$

If the mass flow rate of water is less than this value, then the temperature rise of water will have to be more than  $2^{\circ}\text{C}$ .

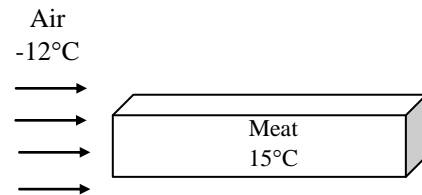
**4-104** The center temperature of meat slabs is to be lowered by chilled air to below 5°C while the surface temperature remains above -1°C to avoid freezing. The average heat transfer coefficient during this cooling process is to be determined.

**Assumptions** **1** The meat slabs can be approximated as very large plane walls of half-thickness  $L = 5\text{-cm}$ . **2** Heat conduction in the meat slabs is one-dimensional because of symmetry about the centerplane. **3** The thermal properties of the meat slab are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is  $\tau > 0.2$  so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

**Properties** The thermal properties of the beef slabs are given to be  $\rho = 1090\text{ kg/m}^3$ ,  $C_p = 3.54\text{ kJ/kg}\cdot^\circ\text{C}$ ,  $k = 0.47\text{ W/m}\cdot^\circ\text{C}$ , and  $\alpha = 0.13 \times 10^{-6}\text{ m}^2/\text{s}$ .

**Analysis** The lowest temperature in the steak will occur at the surfaces and the highest temperature at the center at a given time since the inner part of the steak will be last place to be cooled. In the limiting case, the surface temperature at  $x = L = 5\text{ cm}$  from the center will be  $-1^\circ\text{C}$  while the mid plane temperature is  $5^\circ\text{C}$  in an environment at  $-12^\circ\text{C}$ . Then from Fig. 4-13b we obtain

$$\left. \begin{aligned} \frac{x}{L} = \frac{5\text{ cm}}{5\text{ cm}} = 1 \\ \frac{T(L,t) - T_\infty}{T_o - T_\infty} = \frac{-1 - (-12)}{5 - (-12)} = 0.65 \end{aligned} \right\} \frac{1}{\text{Bi}} = \frac{k}{hL} = 0.95$$



which gives

$$h = \frac{k}{L} \text{Bi} = \frac{0.47\text{ W/m}\cdot^\circ\text{C}}{0.05\text{ m}} \left( \frac{1}{0.95} \right) = \mathbf{9.9\text{ W/m}^2\cdot^\circ\text{C}}$$

Therefore, the convection heat transfer coefficient should be kept below this value to satisfy the constraints on the temperature of the steak during refrigeration. We can also meet the constraints by using a lower heat transfer coefficient, but doing so would extend the refrigeration time unnecessarily.

**Discussion** We could avoid the uncertainty associated with the reading of the charts and obtain a more accurate result by using the one-term solution relation for an infinite plane wall, but it would require a trial and error approach since the Bi number is not known.