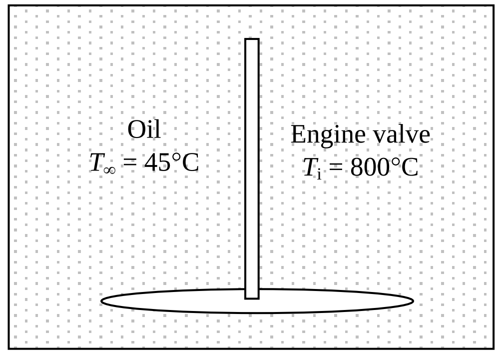
4-118 Internal combustion engine valves are quenched in a large oil bath. The time it takes for the valve temperature to drop to specified temperatures and the maximum heat transfer are to be determined.

Assumptions **1** The thermal properties of the valves are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface. **3** Depending on the size of the oil bath, the oil bath temperature will increase during quenching. However, an average canstant temperature as specified in the problem will be used. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be k = 48 W/m.°C, ρ = 7840 kg/m^3, and C_p = 440 J/kg.°C.



Analysis (*a*) The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{1.8(\pi D^2 L/4)}{2\pi DL} = \frac{1.8D}{8} = \frac{1.8(0.008\,\text{m})}{8} = 0.0018\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(650\,\text{W/m}^2.°\text{C})(0.0018\,\text{m})}{(48\,\text{W/m.°C})} = 0.024 < 0.1$$

Therefore, we can use lumped system analysis. Then the time for a final valve temperature of 400°C becomes

$$b = \frac{hA_s}{\rho C_p V} = \frac{8h}{1.8\rho C_p D} = \frac{8(650\,\text{W/m}^2.°\text{C})}{1.8(7840\,\text{kg/m}^3)(440\,\text{J/kg.°C})(0.008\,\text{m})} = 0.10468\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{400-45}{800-45} = e^{-(0.10468\,\text{s}^{-1})t} \longrightarrow t = \mathbf{7.2\,s}$$

(*b*) The time for a final valve temperature of 200°C is

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{200-45}{800-45} = e^{-(0.10468\,\text{s}^{-1})t} \longrightarrow t = \mathbf{15.1\,s}$$

(*c*) The time for a final valve temperature of 46°C is

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{46-45}{800-45} = e^{-(0.10468\,\text{s}^{-1})t} \longrightarrow t = \mathbf{63.3\,s}$$

(*d*) The maximum amount of heat transfer from a single valve is determined from

$$m = \rho V = \rho \frac{1.8\pi D^2 L}{4} = (7840\,\text{kg/m}^3)\frac{1.8\pi(0.008\,\text{m})^2(0.10\,\text{m})}{4} = 0.0709\,\text{kg}$$

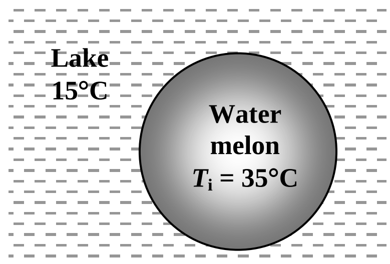
$$Q = mC_p[T_f - T_i] = (0.0709\,\text{kg})(440\,\text{J/kg.°C})(800-45)°\text{C} = 23{,}564\,\text{J} = \mathbf{23.56\,kJ}\ \text{(per valve)}$$

4-119 A watermelon is placed into a lake to cool it. The heat transfer coefficient at the surface of the watermelon and the temperature of the outer surface of the watermelon are to be determined.

Assumptions **1** The watermelon is a homogeneous spherical object. **2** Heat conduction in the watermelon is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the watermelon are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the watermelon are given to be k = 0.618 W/m.°C, $\alpha = 0.15\times10^{-6}$ m^2/s, ρ = 995 kg/m^3 and C_p = 4.18 kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.15\times10^{-6}\text{ m}^2/\text{s})\left[(4\times60+40\text{ min})\times60\text{ s/min}\right]}{(0.10\text{ m})^2} = 0.252$$

which is greater than 0.2. Then the one-term solution can be written in the form

$$\theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{20-15}{35-15} = 0.25 = A_1 e^{-\lambda_1^2(0.252)}$$

It is determined from Table 4-1 by trial and error that this equation is satisfied when Bi = 10, which corresponds to $\lambda_1 = 2.8363$ and $A_1 = 1.9249$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.618\text{ W/m.°C})(10)}{(0.10\text{ m})} = \mathbf{61.8\ W/m^2.°C}$$

The temperature at the surface of the watermelon is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9249)e^{-(2.8363)^2(0.252)}\frac{\sin(2.8363\text{ rad})}{2.8363}$$

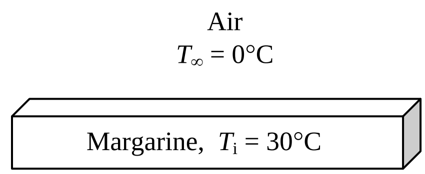
$$\frac{T(r_o,t)-15}{35-15} = 0.0269 \longrightarrow T(r_o,t) = \mathbf{15.5°C}$$

4-120 Large food slabs are cooled in a refrigeration room. Center temperatures are to be determined for different foods.

Assumptions **1** Heat conduction in the slabs is one-dimensional since the slab is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of foods are given to be k = 0.233 W/m.°C and $\alpha = 0.11\times10^{-6}$ m^2/s for margarine, k = 0.082 W/m.°C and $\alpha = 0.10\times10^{-6}$ m^2/s for white cake, and k = 0.106 W/m.°C and $\alpha = 0.12\times10^{-6}$ m^2/s for chocolate cake.

Analysis (*a*) In the case of margarine, the Biot number is



$$Bi = \frac{hL}{k} = \frac{(25\,\text{W/m}^2.°\text{C})(0.05\,\text{m})}{(0.233\,\text{W/m.°C})} = 5.365$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.3269 \text{ and } A_1 = 1.2431$$

The Fourier number is
$$\tau = \frac{\alpha t}{L^2} = \frac{(0.11\times10^{-6}\text{ m}^2/\text{s})(6\text{ h}\times3600\,\text{s/h})}{(0.05\,\text{m})^2} = 0.9504 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the center of the box if the box contains margarine becomes

$$\theta(0,t)_{wall} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.2431)e^{-(1.3269)^2(0.9504)}$$

$$\frac{T(0,t)-0}{30-0} = 0.233 \longrightarrow T(0,t) = \mathbf{7.0\,°C}$$

(*b*) Repeating the calculations for white cake,

$$Bi = \frac{hL}{k} = \frac{(25\,\text{W/m}^2.°\text{C})(0.05\,\text{m})}{(0.082\,\text{W/m.°C})} = 15.24 \longrightarrow \lambda_1 = 1.4641 \text{ and } A_1 = 1.2661$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.10\times10^{-6}\text{ m}^2/\text{s})(6\text{ h}\times3600\,\text{s/h})}{(0.05\,\text{m})^2} = 0.864 > 0.2$$

$$\theta(0,t)_{wall} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.2661)e^{-(1.4641)^2(0.864)}$$

$$\frac{T(0,t)-0}{30-0} = 0.199 \longrightarrow T(0,t) = \mathbf{6.0\,°C}$$

(*c*) Repeating the calculations for chocolate cake,

$$Bi = \frac{hL}{k} = \frac{(25\,\text{W/m}^2.°\text{C})(0.05\,\text{m})}{(0.106\,\text{W/m.°C})} = 11.79 \longrightarrow \lambda_1 = 1.4356 \text{ and } A_1 = 1.2634$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.12\times10^{-6}\text{ m}^2/\text{s})(6\text{ h}\times3600\,\text{s/h})}{(0.05\,\text{m})^2} = 1.0368 > 0.2$$

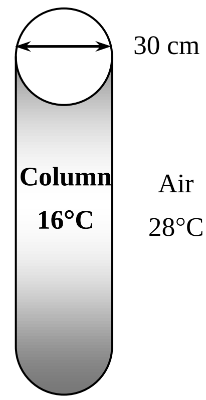
$$\theta(0,t)_{wall} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.2634)e^{-(1.4356)^2(1.0368)}$$

$$\frac{T(0,t)-0}{30-0} = 0.149 \longrightarrow T(0,t) = \mathbf{4.5\,°C}$$

4-121 A cold cylindrical concrete column is exposed to warm ambient air during the day. The time it will take for the surface temperature to rise to a specified value, the amounts of heat transfer for specified values of center and surface temperatures are to be determined.

Assumptions **1** Heat conduction in the column is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the column are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of concrete are given to be k = 0.79 W/m.°C, $\alpha = 5.94\times10^{-7}$ m^2/s, ρ = 1600 kg/m^3 and C_p = 0.84 kJ/kg.°C



Analysis (*a*) The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(14\ \text{W/m}^2.°\text{C})(0.15\ \text{m})}{(0.79\ \text{W/m.°C})} = 2.658$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-1,

$$\lambda_1 = 1.7240 \quad \text{and} \quad A_1 = 1.3915$$

Once the constant J_0 =0.3841 is determined from Table 4-2 corresponding to the constant λ_1, the Fourier number is determined to be

$$\frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o / r_o) \longrightarrow \frac{27-28}{16-28} = (1.3915)e^{-(1.7240)^2\tau}(0.3841) \longrightarrow \tau = 0.6253$$

which is above the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can be used. Then the time it will take for the column surface temperature to rise to 27°C becomes

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.6253)(0.15\ \text{m})^2}{(5.94\times10^{-7}\ \text{m}^2/\text{s})} = 23{,}685\ \text{s} = \mathbf{6.6\ hours}$$

(*b*) The heat transfer to the column will stop when the center temperature of column reaches to the ambient temperature, which is 28°C. That is, we are asked to determine the maximum heat transfer between the ambient air and the column.

$$m = \rho V = \rho\pi r_o^2 L = (1600\ \text{kg/m}^3)[\pi(0.15\ \text{m})^2(3.5\ \text{m})] = 395.8\ \text{kg}$$

$$Q_{\max} = mC_p[T_\infty - T_i] = (395.8\ \text{kg})(0.84\ \text{kJ/kg.°C})(28-16)°\text{C} = \mathbf{3990\ kJ}$$

(*c*) To determine the amount of heat transfer until the surface temperature reaches to 27°C, we first determine

$$\frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.3915)e^{-(1.7240)^2(0.6253)} = 0.2169$$

Once the constant J_1 = 0.5787 is determined from Table 4-2 corresponding to the constant λ_1, the amount of heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\times0.2169\times\frac{0.5787}{1.7240} = 0.854$$

$$Q = 0854 Q_{\max}$$

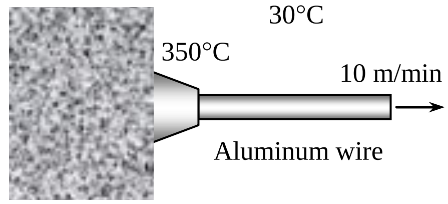
$$Q = 0.854(3990\ \text{kJ}) = \mathbf{3409\ kJ}$$

4-122 Long aluminum wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of aluminum are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, C_p = 0.896 kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\,\text{m}}{2} = 0.00075\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\,\text{W/m}^2.°\text{C})(0.00075\,\text{m})}{236\,\text{W/m.°C}} = 0.00011 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{35\,\text{W/m}^2.°\text{C}}{(2702\,\text{kg/m}^3)(896\,\text{J/kg.°C})(0.00075\,\text{m})} = 0.0193\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0193\,\text{s}^{-1})t} \longrightarrow t = \mathbf{144\,s}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \rightarrow \text{length} = (10/60\,\text{m/s})(144\,\text{s}) = \mathbf{24\,m}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho\dot{V} = \rho(\pi r_0^2/4)V = (2702\,\text{kg/m}^3)\pi(0.0015\,\text{m})^2(10\,\text{m/min}) = 0.191\,\text{kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

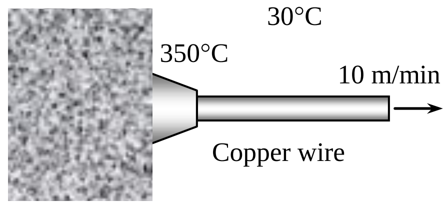
$$\dot{Q} = \dot{m}C_p[T(t)-T_\infty] = (0.191\,\text{kg/min})(0.896\,\text{kJ/kg.°C})(350-50)°\text{C} = 51.3\,\text{kJ/min} = \mathbf{856\,W}$$

4-123 Long copper wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the copper are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of copper are given to be k = 386 W/m.°C, ρ = 8950 kg/m^3, C_p = 0.383 kJ/kg.°C, and $\alpha = 1.13\times10^{-4}$ m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^{\,2} L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\,\text{m}}{2} = 0.00075\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\,\text{W/m}^2.°\text{C})(0.00075\,\text{m})}{386\,\text{W/m.°C}} = 0.000068 < 0.1$$

Since Bi < 0.1 the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{35\,\text{W/m}^2.°\text{C}}{(8950\,\text{kg/m}^3)(383\,\text{J/kg.°C})(0.00075\,\text{m})} = 0.0136\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0136\,\text{s}^{-1})t} \longrightarrow t = \mathbf{204\,s}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \longrightarrow \text{length} = \left(\frac{10\,\text{m/min}}{60\,\text{s/min}}\right)(204\,\text{s}) = \mathbf{34\,m}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho\dot{V} = \rho(\pi r_0^2/4)V = (8950\,\text{kg/m}^3)\pi(0.0015\,\text{m})^2(10\,\text{m/min}) = 0.633\,\text{kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

$$\dot{Q} = \dot{m}C_p[T(t)-T_\infty] = (0.633\,\text{kg/min})(0.383\,\text{kJ/kg.°C})(350-50)°\text{C} = 72.7\,\text{kJ/min} = \mathbf{1212\,W}$$

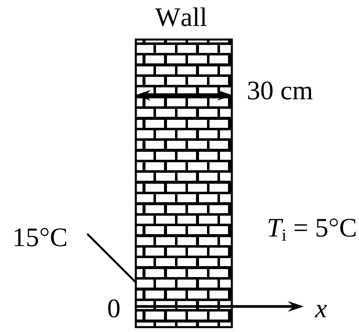
4-124 A brick house made of brick that was initially cold is exposed to warm atmospheric air at the outer surfaces. The time it will take for the temperature of the inner surfaces of the house to start changing is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at outer surfaces only, and thus the wall can be considered to be a semi-infinite medium with a specified outer surface temperature of 18°C. **2** The thermal properties of the brick wall are constant.

Properties The thermal properties of the brick are given to be k = 0.72 W/m.°C and $\alpha = 0.45\times10^{-4}$ m^2/s.

Analysis The exact analytical solution to this problem is

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$



Substituting,

$$\frac{5.1-5}{15-5} = 0.01 = erfc\left(\frac{0.3\,\text{m}}{2\sqrt{(0.45\times10^{-6}\ \text{m}^2/\text{s})t}}\right)$$

Noting from Table 4-3 that 0.01 = erfc(1.8215), the time is determined to be

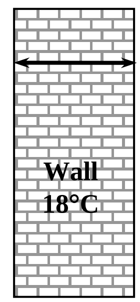
$$\left(\frac{0.3\,\text{m}}{2\sqrt{(0.45\times10^{-6}\ \text{m}^2/\text{s})t}}\right) = 1.8215 \longrightarrow t = 15{,}070\,\text{s} = \mathbf{251\,min}$$

4-125 A thick wall is exposed to cold outside air. The wall temperatures at distances 15, 30, and 40 cm from the outer surface at the end of 2-hour cooling period are to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at outer surfaces only. Therefore, the wall can be considered to be a semi-infinite medium **2** The thermal properties of the wall are constant.

Properties The thermal properties of the brick are given to be k = 0.72 W/m.°C and $\alpha = 1.6\times10^{-7}$ m^2/s.

Analysis For a 15 cm distance from the outer surface, from Fig. 4-23 we have



$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \mathrm{W/m^2.°C})\sqrt{(1.6\times10^{-6}\ \mathrm{m^2/s})(2\times3600\mathrm{s})}}{0.72\ \mathrm{W/m.°C}} = 2.98\\ \xi = \frac{x}{2\sqrt{\alpha t}} &= \frac{0.15\ \mathrm{m}}{2\sqrt{(1.6\times10^{-6}\ \mathrm{m^2/s})(2\times3600\mathrm{s})}} = 0.70\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.25$$

$$1-\frac{T-2}{18-2} = 0.25 \longrightarrow T = \mathbf{14.0°C}$$

For a 30 cm distance from the outer surface, from Fig. 4-23 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \mathrm{W/m^2.°C})\sqrt{(1.6\times10^{-6}\ \mathrm{m^2/s})(2\times3600\mathrm{s})}}{0.72\ \mathrm{W/m.°C}} = 2.98\\ \xi = \frac{x}{2\sqrt{\alpha t}} &= \frac{0.3\ \mathrm{m}}{2\sqrt{(1.6\times10^{-6}\ \mathrm{m^2/s})(2\times3600\mathrm{s})}} = 1.40\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.038$$

$$1-\frac{T-2}{18-2} = 0.038 \longrightarrow T = \mathbf{17.4°C}$$

For a 40 cm distance from the outer surface, that is for the inner surface, from Fig. 4-23 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \mathrm{W/m^2.°C})\sqrt{(1.6\times10^{-6}\ \mathrm{m^2/s})(2\times3600\mathrm{s})}}{0.72\ \mathrm{W/m.°C}} = 2.98\\ \xi = \frac{x}{2\sqrt{\alpha t}} &= \frac{0.4\ \mathrm{m}}{2\sqrt{(1.6\times10^{-6}\ \mathrm{m^2/s})(2\times3600\mathrm{s})}} = 1.87\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0$$

$$1-\frac{T-2}{18-2} = 0 \longrightarrow T = \mathbf{18.0°C}$$

Discussion This last result shows that the semi-infinite medium assumption is a valid one.

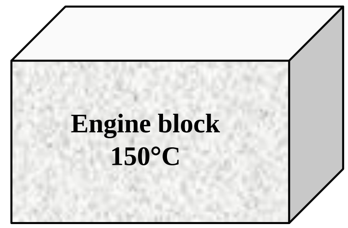
4-126 The engine block of a car is allowed to cool in atmospheric air. The temperatures at the center of the top surface and at the corner after a specified period of cooling are to be determined.

Assumptions **1** Heat conduction in the block is three-dimensional, and thus the temperature varies in all three directions. **2** The thermal properties of the block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of cast iron are given to be k = 52 W/m.°C and $\alpha = 1.7\times10^{-5}$ m^2/s.

Analysis This rectangular block can physically be formed by by the intersection of two infinite plane walls of thickness 2L = 40 cm (call planes A and B) and an infinite plane wall of thickness 2L = 80 cm (call plane C). We measure x from the center of the block.

(a) The Biot number is calculated for each of the plane wall to be



$$Bi_A = Bi_B = \frac{hL}{k} = \frac{(6\text{ W/m}^2.°\text{C})(0.2\text{ m})}{(52\text{ W/m.°C})} = 0.0231$$

$$Bi_C = \frac{hL}{k} = \frac{(6\text{ W/m}^2.°\text{C})(0.4\text{ m})}{(52\text{ W/m.°C})} = 0.0462$$

The constants λ_1 and A_1 corresponding to these Biot numbers are, from Table 4-1,

$$\lambda_{1(A,B)} = 0.150 \text{ and } A_{1(A,B)} = 1.0038$$

$$\lambda_{1(C)} = 0.212 \text{ and } A_{1(C)} = 1.0076$$

The Fourier numbers are

$$\tau_{A,B} = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5}\text{ m}^2\text{/s})(45\text{ min}\times 60\text{ s/min})}{(0.2\text{ m})^2} = 1.1475 > 0.2$$

$$\tau_C = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5}\text{ m}^2\text{/s})(45\text{ min}\times 60\text{ s/min})}{(0.4\text{ m})^2} = 0.2869 > 0.2$$

The center of the top surface of the block (whose sides are 80 cm and 40 cm) is at the center of the plane wall with 2L = 80 cm, at the center of the plane wall with 2L = 40 cm, and at the surface of the plane wall with 2L = 40 cm. The dimensionless temperatures are

$$\theta_{o,\text{wall(A)}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0038)e^{-(0.150)^2(1.1475)} = 0.9782$$

$$\theta(L,t)_{\text{wall(B)}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0038)e^{-(0.150)^2(1.1475)}\cos(0.150) = 0.9672$$

$$\theta_{o,\text{wall(C)}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0076)e^{-(0.212)^2(0.2869)} = 0.9947$$

Then the center temperature of the top surface of the cylinder becomes

$$\left[\frac{T(L,0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{\text{wall(B)}} \times \theta_{o,\text{wall(A)}} \times \theta_{o,\text{wall(C)}} = 0.9672\times0.9782\times0.9947 = 0.9411$$

$$\frac{T(L,0,0,t) - 17}{150 - 17} = 0.9411 \longrightarrow T(L,0,0,t) = \mathbf{142.2°C}$$

(b) The corner of the block is at the surface of each plane wall. The dimensionless temperature for the surface of the plane walls with 2L = 40 cm is determined in part (a). The dimensionless temperature for the surface of the plane wall with 2L = 80 cm is determined from

$$\theta(L,t)_{\text{wall(C)}} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L/L) = (1.0076)e^{-(0.212)^2(0.2869)}\cos(0.212) = 0.9724$$

Then the corner temperature of the block becomes

$$\left[\frac{T(L,L,L,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{\text{wall,C}} \times \theta(L,t)_{\text{wall,B}} \times \theta(L,t)_{\text{wall,A}} = 0.9724 \times 0.9672 \times 0.9672 = 0.9097$$

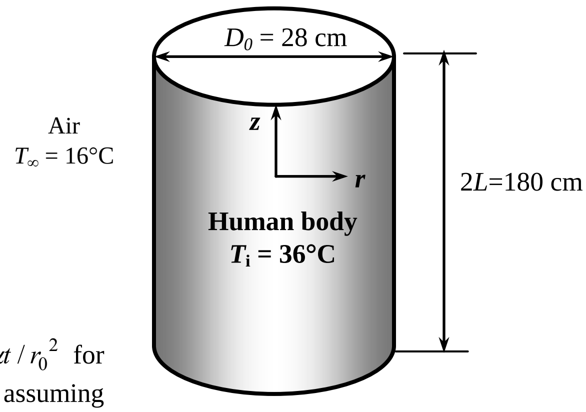
$$\frac{T(L,L,L,t)-17}{150-17} = 0.9097 \longrightarrow T(L,L,L,t) = \mathbf{138.0^\circ C}$$

4-127 A man is found dead in a room. The time passed since his death is to be estimated.

Assumptions **1** Heat conduction in the body is two-dimensional, and thus the temperature varies in both radial r- and x- directions. **2** The thermal properties of the body are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The human body is modeled as a cylinder. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of body are given to be k = 0.62 W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis A short cylinder can be formed by the intersection of a long cylinder of radius $D/2$ = 14 cm and a plane wall of thickness $2L$ = 180 cm. We measure x from the midplane. The temperature of the body is specified at a point that is at the center of the plane wall but at the surface of the cylinder. The Biot numbers and the corresponding constants are first determined to be



$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.90\ \text{m})}{(0.62\ \text{W/m.°C})} = 13.06$$

$$\longrightarrow \lambda_1 = 1.4495 \quad \text{and} \quad A_1 = 1.2644$$

$$Bi_{\text{cyl}} = \frac{hr_0}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.14\ \text{m})}{(0.62\ \text{W/m.°C})} = 2.03$$

$$\longrightarrow \lambda_1 = 1.6052 \quad \text{and} \quad A_1 = 1.3408$$

Noting that $\tau = \alpha t / L^2$ for the plane wall and $\tau = \alpha t / r_0^2$ for cylinder and J_0(1.6052)=0.4524 from Table 4-2, and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as

$$\theta(0, r_0, t)_{block} = \theta(0, t)_{\text{wall}}\theta(r_0, t)_{\text{cyl}}$$

$$\frac{23-16}{36-16} = (A_1 e^{-\lambda_1^2\tau})\left[A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r / r_0)\right]$$

$$0.40 = \left\{(1.2644)\exp\left[-(1.4495)^2 \frac{(0.15\times10^{-6})t}{(0.90)^2}\right]\right\} \times \left\{(1.3408)\exp\left[-(1.6052)^2 \frac{(0.15\times10^{-6})t}{(0.14)^2}\right](0.4524)\right\}$$

$$\longrightarrow t = 32{,}404\ \text{s} = \mathbf{9.0\ hours}$$

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