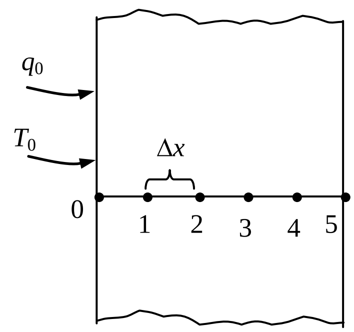
5-29 A plate is subjected to specified heat flux and specified temperature on one side, and no conditions on the other. The finite difference formulation of this problem is to be obtained, and the temperature of the other side under steady conditions is to be determined.

Assumptions **1** Heat transfer through the plate is given to be steady and one-dimensional. **2** There is no heat generation in the plate.

Properties The thermal conductivity is given to be k = 2.5 W/m·°C.



Analysis The nodal spacing is given to be Δx=0.06 m.

Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{0.3\,\text{m}}{0.06\,\text{m}} + 1 = 6$$

Nodes 1, 2, 3, and 4 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{g}_m}{k} = 0 \to T_{m+1} - 2T_m + T_{m-1} = 0 \quad (\text{since } \dot{g} = 0), \quad \text{for } m = 1, 2, 3, \text{and } 4$$

The finite difference equation for node 0 on the left surface is obtained by applying an energy balance on the half volume element about node 0 and taking the direction of all heat transfers to be towards the node under consideration,

$$\dot{q}_0 + k\frac{T_1 - T_0}{\Delta x} = 0 \quad \longrightarrow \quad 700\,\text{W/m}^2 + (2.5\,\text{W/m}\cdot{}^\circ\text{C})\frac{T_1 - 60^\circ\text{C}}{0.06\,\text{m}} = 0 \quad \longrightarrow \quad T_1 = 43.2^\circ\text{C}$$

Other nodal temperatures are determined from the general interior node relation as follows:

$$m = 1: \qquad T_2 = 2T_1 - T_0 = 2 \times 43.2 - 60 = 26.4^\circ\text{C}$$
$$m = 2: \qquad T_3 = 2T_2 - T_1 = 2 \times 26.4 - 43.2 = 9.6^\circ\text{C}$$
$$m = 3: \qquad T_4 = 2T_3 - T_2 = 2 \times 9.6 - 26.4 = -7.2^\circ\text{C}$$
$$m = 4: \qquad T_5 = 2T_4 - T_3 = 2 \times (-7.2) - 9.6 = \mathbf{-24^\circ C}$$

Therefore, the temperature of the other surface will be –24°C

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

5-30E A large plate lying on the ground is subjected to convection and radiation. Finite difference formulation is to be obtained, and the top and bottom surface temperatures under steady conditions are to be determined.

Assumptions **1** Heat transfer through the plate is given to be steady and one-dimensional. **2** There is no heat generation in the plate and the soil. **3** Thermal contact resistance at plate-soil interface is negligible.

Properties The thermal conductivity of the plate and the soil are given to be k_{plate} = 7.2 Btu/h·ft·°F and k_{soil} = 0.49 Btu/h·ft·°F.

Analysis The nodal spacing is given to be Δx_1=1 in. in the plate, and be Δx_2=0.6 ft in the soil. Then the number of nodes becomes

$$M = \left(\frac{L}{\Delta x}\right)_{\text{plate}} + \left(\frac{L}{\Delta x}\right)_{\text{soil}} + 1 = \frac{5\text{ in}}{1\text{ in}} + \frac{3\text{ ft}}{0.6\text{ ft}} + 1 = 11$$

The temperature at node 10 (bottom of thee soil) is given to be T_{10} =50°F. Nodes 1, 2, 3, and 4 in the plate and 6, 7, 8, and 9 in the soil are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{g}_m}{k} = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} = 0 \quad (\text{since } \dot{g} = 0)$$

The finite difference equation for node 0 on the left surface and node 5 at the interface are obtained by applying an energy balance on their respective volume elements and taking the direction of all heat transfers to be towards the node under consideration:

Node 0 (top surface): $h(T_\infty - T_0) + \varepsilon\sigma[T_{sky}^4 - (T_0 + 460)^4] + k_{\text{plate}} \dfrac{T_1 - T_0}{\Delta x_1} = 0$

Node 1 (interior): $T_0 - 2T_1 + T_2 = 0$

Node 2 (interior): $T_1 - 2T_2 + T_3 = 0$

Node 3 (interior): $T_2 - 2T_3 + T_4 = 0$

Node 4 (interior): $T_3 - 2T_4 + T_5 = 0$

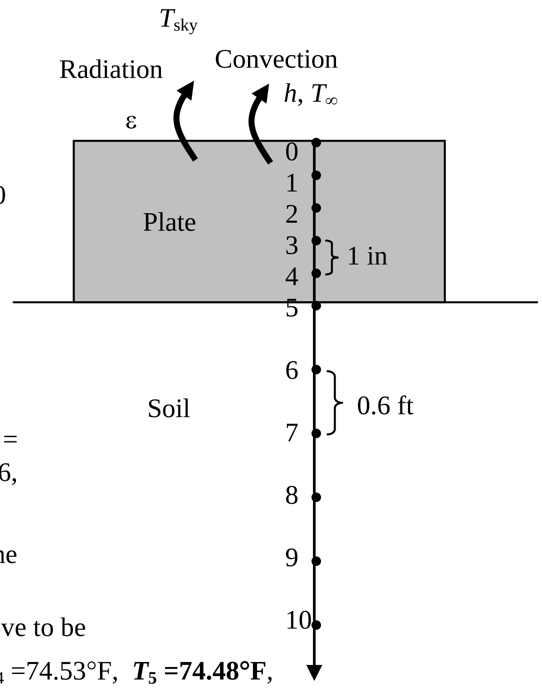
Node 5 (interface): $k_{\text{plate}} \dfrac{T_4 - T_5}{\Delta x_1} + k_{\text{soil}} \dfrac{T_6 - T_5}{\Delta x_2} = 0$

Node 6 (interior): $T_5 - 2T_6 + T_7 = 0$

Node 7 (interior): $T_6 - 2T_7 + T_8 = 0$

Node 8 (interior): $T_7 - 2T_8 + T_9 = 0$

Node 9 (interior): $T_8 - 2T_9 + T_{10} = 0$



where Δx_1=1/12 ft, Δx_2=0.6 ft, k_{plate} = 7.2 Btu/h·ft·°F, k_{soil} = 0.49 Btu/h·ft·°F, h = 3.5 Btu/h·ft^2·°F, T_{sky} =510 R, ε = 0.6, $T_\infty = 80$°F, and T_{10} =50°F.

This system of 10 equations with 10 unknowns constitute the finite difference formulation of the problem.

(*b*) The temperatures are determined by solving equations above to be

T_0 = 74.71°F, T_1 =74.67°F, T_2 =74.62°F, T_3 =74.58°F, T_4 =74.53°F, **T_5 =74.48°F**,

T_6 =69.6°F, T_7 =64.7°F, T_8 =59.8°F, T_9 =54.9°F

Discussion Note that the plate is essentially isothermal at about 74.6°F. Also, the temperature in each layer varies linearly and thus we could solve this problem by considering 3 nodes only (one at the interface and two at the boundaries).

5-31E A large plate lying on the ground is subjected to convection from its exposed surface. The finite difference formulation of this problem is to be obtained, and the top and bottom surface temperatures under steady conditions are to be determined.

Assumptions **1** Heat transfer through the plate is given to be steady and one-dimensional. **2** There is no heat generation in the plate and the soil. **3** The thermal contact resistance at the plate-soil interface is negligible. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity of the plate and the soil are given to be k_{plate} = 7.2 Btu/h·ft·°F and k_{soil} = 0.49 Btu/h·ft·°F.

Analysis The nodal spacing is given to be Δx_1=1 in. in the plate, and be Δx_2=0.6 ft in the soil. Then the number of nodes becomes

$$M = \left(\frac{L}{\Delta x}\right)_{\text{plate}} + \left(\frac{L}{\Delta x}\right)_{\text{soil}} + 1 = \frac{5\text{ in}}{1\text{ in}} + \frac{3\text{ ft}}{0.6\text{ ft}} + 1 = 11$$

The temperature at node 10 (bottom of thee soil) is given to be T_{10} =50°F. Nodes 1, 2, 3, and 4 in the plate and 6, 7, 8, and 9 in the soil are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{g}_m}{k} = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} = 0 \quad (\text{since } \dot{g} = 0)$$

The finite difference equation for node 0 on the left surface and node 5 at the interface are obtained by applying an energy balance on their respective volume elements and taking the direction of all heat transfers to be towards the node under consideration:

Node 0 (top surface): $h(T_\infty - T_0) + k_{\text{plate}} \dfrac{T_1 - T_0}{\Delta x_1} = 0$

Node 1 (interior): $T_0 - 2T_1 + T_2 = 0$

Node 2 (interior): $T_1 - 2T_2 + T_3 = 0$

Node 3 (interior): $T_2 - 2T_3 + T_4 = 0$

Node 4 (interior): $T3 - 2T_4 + T_5 = 0$

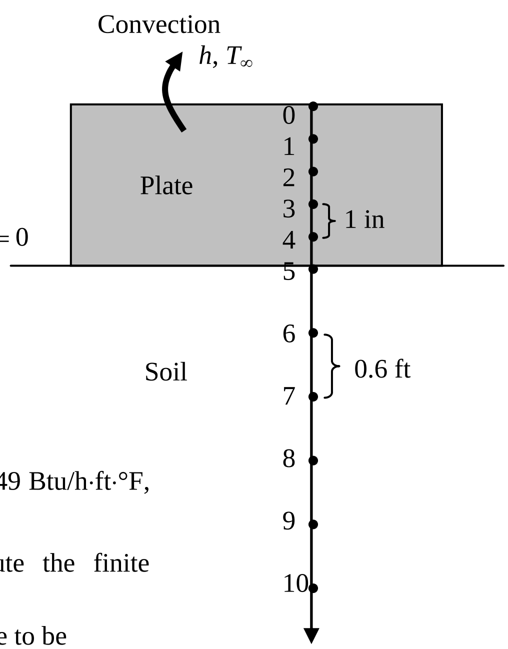
Node 5 (interface): $k_{\text{plate}} \dfrac{T_4 - T_5}{\Delta x_1} + k_{\text{soil}} \dfrac{T_6 - T_5}{\Delta x_2} = 0$

Node 6 (interior): $T_5 - 2T_6 + T_7 = 0$

Node 7 (interior): $T_6 - 2T_7 + T_8 = 0$

Node 8 (interior): $T_7 - 2T_8 + T_9 = 0$

Node 9 (interior): $T_8 - 2T_9 + T_{10} = 0$



where Δx_1=1/12 ft, Δx_2=0.6 ft, k_{plate} = 7.2 Btu/h·ft·°F, k_{soil} = 0.49 Btu/h·ft·°F, h = 3.5 Btu/h·ft^2·°F, $T_\infty = 80$°F, and T_{10} =50°F.

This system of 10 equations with 10 unknowns constitute the finite difference formulation of the problem.

(*b*) The temperatures are determined by solving equations above to be

$\boldsymbol{T_0}$ **=78.67°F**, T_1 =78.62°F, T_2 =78.57°F, T_3 =78.51°F, T_4 =78.46°F, $\boldsymbol{T_5}$ **=78.41°F**,

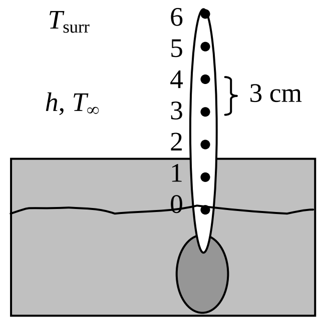
T_6 =72.7°F, T_7 =67.0°F, T_8 =61.4°F, T_9 =55.7°F

Discussion Note that the plate is essentially isothermal at about 78.6°F. Also, the temperature in each layer varies linearly and thus we could solve this problem by considering 3 nodes only (one at the interface and two at the boundaries).

5-32 The handle of a stainless steel spoon partially immersed in boiling water loses heat by convection and radiation. The finite difference formulation of the problem is to be obtained, and the tip temperature of the spoon as well as the rate of heat transfer from the exposed surfaces are to be determined.

Assumptions **1** Heat transfer through the handle of the spoon is given to be steady and one-dimensional. **2** Thermal conductivity and emissivity are constant. **3** Convection heat transfer coefficient is constant and uniform.

Properties The thermal conductivity and emissivity are given to be k = 15.1 W/m·°C and ε = 0.8.



Analysis The nodal spacing is given to be Δx=3 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{18\,\text{cm}}{3\,\text{cm}} + 1 = 7$$

The base temperature at node 0 is given to be T_0 = 95°C. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Nodes 1, 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) + \varepsilon\sigma(p\Delta x)[T_{surr}^4 - (T_m + 273)^4] = 0$$

or $T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{surr}^4 - (T_m + 273)^4] = 0$, m = 1,2,3,4,5

The finite difference equation for node 6 at the fin tip is obtained by applying an energy balance on the half volume element about node 6. Then,

$$m=1:\ T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{surr}^4 - (T_1 + 273)^4] = 0$$

$$m=2:\ T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{surr}^4 - (T_2 + 273)^4] = 0$$

$$m=3:\ T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{surr}^4 - (T_3 + 273)^4] = 0$$

$$m=4:\ T_3 - 2T_4 + T_5 + h(p\Delta x^2 / kA)(T_\infty - T_4) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{surr}^4 - (T_4 + 273)^4] = 0$$

$$m=5:\ T_4 - 2T_5 + T_6 + h(p\Delta x^2 / kA)(T_\infty - T_5) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{surr}^4 - (T_5 + 273)^4] = 0$$

$$\text{Node 6: } kA\frac{T_5 - T_6}{\Delta x} + h(p\Delta x/2 + A)(T_\infty - T_6) + \varepsilon\sigma(p\Delta x/2 + A)[T_{surr}^4 - (T_6 + 273)^4] = 0$$

where $\Delta x = 0.03\,\text{m},\ k = 15.1\,\text{W/m}\cdot°\text{C},\ \varepsilon = 0.6,\ T_\infty = 25°\text{C}, T_0 = 95°\text{C},\ T_{surr} = 295\,\text{K},\ h = 13\,\text{W/m}^2\cdot°\text{C}$

and $A = (1\,\text{cm})(0.2\,\text{cm}) = 0.2\,\text{cm}^2 = 0.2\times10^{-4}\ \text{m}^2$ and $p = 2(1 + 0.2\,\text{cm}) = 2.4\,\text{cm} = 0.024\,\text{m}$

The system of 6 equations with 6 unknowns constitute the finite difference formulation of the problem.

(b) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_1 =49.0°C, T_2 = 33.0°C, T_3 =27.4°C, T_4 =25.5°C, T_5 =24.8°C, and $\boldsymbol{T_6}$ **=24.6°C**,

(c) The total rate of heat transfer from the spoon handle is simply the sum of the heat transfer from each nodal element, and is determined from

$$\dot{Q}_{fin} = \sum_{m=0}^{6}\dot{Q}_{element,m} = \sum_{m=0}^{6} hA_{surface,m}(T_m - T_\infty) + \sum_{m=0}^{6}\varepsilon\sigma A_{surface,m}[(T_m + 273)^4 - T_{surr}^4] = \mathbf{0.92\ W}$$

where $A_{surface,\,m}$ =$p\Delta x$/2 for node 0, $A_{surface,\,m}$ =$p\Delta x$/2+A for node 6, and $A_{surface,\,m}$ =$p\Delta x$ for other nodes.

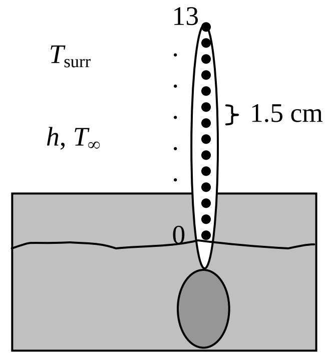
5-33 The handle of a stainless steel spoon partially immersed in boiling water loses heat by convection and radiation. The finite difference formulation of the problem for all nodes is to be obtained, and the temperature of the tip of the spoon as well as the rate of heat transfer from the exposed surfaces of the spoon are to be determined.

Assumptions **1** Heat transfer through the handle of the spoon is given to be steady and one-dimensional. **2** The thermal conductivity and emissivity are constant. **3** Heat transfer coefficient is constant and uniform.

Properties The thermal conductivity and emissivity are given to be k = 15.1 W/m·°C and ε = 0.8.

Analysis The nodal spacing is given to be Δx=1.5 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{18\,\text{cm}}{1.5\,\text{cm}} + 1 = 13$$



The base temperature at node 0 is given to be T_0 = 95°C. This problem involves 12 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Nodes 1 through 12 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) + \varepsilon\sigma(p\Delta x)[T_{\text{surr}}^4 - (T_m + 273)^4] = 0$$

or $$T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_m + 273)^4] = 0, \quad m = 1\text{-}12$$

The finite difference equation for node 6 at the fin tip is obtained by applying an energy balance on the half volume element about node 13. Then,

$$m=1:\ T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_1 + 273)^4] = 0$$

$$m=2:\ T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_2 + 273)^4] = 0$$

$$m=3:\ T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_3 + 273)^4] = 0$$

$$m=4:\ T_3 - 2T_4 + T_5 + h(p\Delta x^2 / kA)(T_\infty - T_4) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_4 + 273)^4] = 0$$

$$m=5:\ T_4 - 2T_5 + T_6 + h(p\Delta x^2 / kA)(T_\infty - T_5) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_5 + 273)^4] = 0$$

$$m=6:\ T_5 - 2T_6 + T_7 + h(p\Delta x^2 / kA)(T_\infty - T_6) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_6 + 273)^4] = 0$$

$$m=7:\ T_6 - 2T_7 + T_8 + h(p\Delta x^2 / kA)(T_\infty - T_7) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_7 + 273)^4] = 0$$

$$m=8:\ T_7 - 2T_8 + T_9 + h(p\Delta x^2 / kA)(T_\infty - T_8) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_8 + 273)^4] = 0$$

$$m=9:\ T_8 - 2T_9 + T_{10} + h(p\Delta x^2 / kA)(T_\infty - T_9) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_9 + 273)^4] = 0$$

$$m=10:\ T_9 - 2T_{10} + T_{11} + h(p\Delta x^2 / kA)(T_\infty - T_{10}) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_{10} + 273)^4] = 0$$

$$m=11:\ T_{10} - 2T_{11} + T_{12} + h(p\Delta x^2 / kA)(T_\infty - T_{11}) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_{11} + 273)^4] = 0$$

$$m=12:\ T_{11} - 2T_{12} + T_{13} + h(p\Delta x^2 / kA)(T_\infty - T_{12}) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_{12} + 273)^4] = 0$$

$$\text{Node 13:}\ kA\frac{T_{12} - T_{13}}{\Delta x} + h(p\Delta x / 2 + A)(T_\infty - T_{13}) + \varepsilon\sigma(p\Delta x / 2 + A)[T_{\text{surr}}^4 - (T_{13} + 273)^4] = 0$$

where $\Delta x = 0.03\,\text{m},\ k = 15.1\,\text{W/m}\cdot°\text{C},\ \varepsilon = 0.6,\ T_\infty = 25°\text{C},\ T_0 = 95°\text{C},\ T_{surr} = 295\,\text{K},\ h = 13\,\text{W/m}^2\cdot°\text{C}$

$$A = (1\,\text{cm})(0.2\,\text{cm}) = 0.2\,\text{cm}^2 = 0.2\times10^{-4}\ \text{m}^2 \quad \text{and} \quad p = 2(1 + 0.2\,\text{cm}) = 2.4\,\text{cm} = 0.024\,\text{m}$$

(b) The nodal temperatures under steady conditions are determined by solving the equations above to be

T_1 =65.2°C, T_2 = 48.1°C, T_3 =38.2°C, T_4 =32.4°C, T_5 =29.1°C, T_6 =27.1°C, T_7 =26.0°C,

T_8 =25.3°C, T_9 = 24.9°C, T_{10} =24.7°C, T_{11} =24.6°C, T_{12} =24.5°C, and $\boldsymbol{T_{13}}$ **=24.5°C**, (*c*) The total rate of heat transfer from the spoon handle is the sum of the heat transfer from each element,

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{13} \dot{Q}_{\text{element},m} = \sum_{m=0}^{13} hA_{\text{surface},m}(T_m - T_\infty) + \sum_{m=0}^{13} \varepsilon\sigma A_{\text{surface},m}[(T_m + 273)^4 - T_{\text{surr}}^4] = \mathbf{0.83\ W}$$

where $A_{\text{surface, m}} = p\Delta x/2$ for node 0, $A_{\text{surface, m}} = p\Delta x/2 + \text{A}$ for node 13, and $A_{\text{surface, m}} = p\Delta x$ for other nodes.

5-34 "!PROBLEM 5-34"

```
"GIVEN"
k=15.1 "[W/m-C], parameter to be varied"
"epsilon=0.6 parameter to be varied"
T_0=95 "[C]"
T_infinity=25 "[C]"
w=0.002 "[m]"
s=0.01 "[m]"
L=0.18 "[m]"
h=13 "[W/m^2-C]"
T_surr=295 "[K]"
DELTAx=0.015 "[m]"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
"(b)"
M=L/DELTAx+1 "Number of nodes"
A=w*s
p=2*(w+s)
"Using the finite difference method, the five equations for the unknown temperatures at 12 nodes
are determined to be"
T_0-2*T_1+T_2+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_1)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_1+273)^4)=0 "mode 1"
T_1-2*T_2+T_3+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_2)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_2+273)^4)=0 "mode 2"
T_2-2*T_3+T_4+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_3)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_3+273)^4)=0 "mode 3"
T_3-2*T_4+T_5+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_4)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_4+273)^4)=0 "mode 4"
T_4-2*T_5+T_6+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_5)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_5+273)^4)=0 "mode 5"
T_5-2*T_6+T_7+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_6)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_6+273)^4)=0 "mode 6"
T_6-2*T_7+T_8+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_7)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_7+273)^4)=0 "mode 7"
T_7-2*T_8+T_9+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_8)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_8+273)^4)=0 "mode 8"
T_8-2*T_9+T_10+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_9)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_9+273)^4)=0 "mode 9"
T_9-2*T_10+T_11+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_10)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_10+273)^4)=0 "mode 10"
T_10-2*T_11+T_12+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_11)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_11+273)^4)=0 "mode 11"
T_11-2*T_12+T_13+h*(p*DELTAx^2)/(k*A)*(T_infinity-
T_12)+epsilon*sigma*(p*DELTAx^2)/(k*A)*(T_surr^4-(T_12+273)^4)=0 "mode 12"

k*A*(T_12-T_13)/DELTAx+h*(p*DELTAx/2+A)*(T_infinity-
T_13)+epsilon*sigma*(p*DELTAx/2+A)*(T_surr^4-(T_13+273)^4)=0 "mode 13"

T_tip=T_13
"(c)"
A_s_0=p*DELTAx/2
A_s_13=p*DELTAx/2+A
A_s=p*DELTAx
Q_dot=Q_dot_0+Q_dot_1+Q_dot_2+Q_dot_3+Q_dot_4+Q_dot_5+Q_dot_6+Q_dot_7+Q_dot_8+
Q_dot_9+Q_dot_10+Q_dot_11+Q_dot_12+Q_dot_13 "where"
Q_dot_0=h*A_s_0*(T_0-T_infinity)+epsilon*sigma*A_s_0*((T_0+273)^4-T_surr^4)
```

```
Q_dot_1=h*A_s*(T_1-T_infinity)+epsilon*sigma*A_s*((T_1+273)^4-T_surr^4)
Q_dot_2=h*A_s*(T_2-T_infinity)+epsilon*sigma*A_s*((T_2+273)^4-T_surr^4)
Q_dot_3=h*A_s*(T_3-T_infinity)+epsilon*sigma*A_s*((T_3+273)^4-T_surr^4)
Q_dot_4=h*A_s*(T_4-T_infinity)+epsilon*sigma*A_s*((T_4+273)^4-T_surr^4)
Q_dot_5=h*A_s*(T_5-T_infinity)+epsilon*sigma*A_s*((T_5+273)^4-T_surr^4)
Q_dot_6=h*A_s*(T_6-T_infinity)+epsilon*sigma*A_s*((T_6+273)^4-T_surr^4)
Q_dot_7=h*A_s*(T_7-T_infinity)+epsilon*sigma*A_s*((T_7+273)^4-T_surr^4)
Q_dot_8=h*A_s*(T_8-T_infinity)+epsilon*sigma*A_s*((T_8+273)^4-T_surr^4)
Q_dot_9=h*A_s*(T_9-T_infinity)+epsilon*sigma*A_s*((T_9+273)^4-T_surr^4)
Q_dot_10=h*A_s*(T_10-T_infinity)+epsilon*sigma*A_s*((T_10+273)^4-T_surr^4)
Q_dot_11=h*A_s*(T_11-T_infinity)+epsilon*sigma*A_s*((T_11+273)^4-T_surr^4)
Q_dot_12=h*A_s*(T_12-T_infinity)+epsilon*sigma*A_s*((T_12+273)^4-T_surr^4)
Q_dot_13=h*A_s_13*(T_13-T_infinity)+epsilon*sigma*A_s_13*((T_13+273)^4-T_surr^4)
```

k [W/m.C]	T_{tip} [C]	Q [W]
10	24.38	0.6889
30.53	25.32	1.156
51.05	27.28	1.482
71.58	29.65	1.745
92.11	32.1	1.969
112.6	34.51	2.166
133.2	36.82	2.341
153.7	39	2.498
174.2	41.06	2.641
194.7	42.98	2.772
215.3	44.79	2.892
235.8	46.48	3.003
256.3	48.07	3.106
276.8	49.56	3.202
297.4	50.96	3.291
317.9	52.28	3.374
338.4	53.52	3.452
358.9	54.69	3.526
379.5	55.8	3.595
400	56.86	3.66

ε	**T_{tip} [C]**	**Q [W]**
0.1	25.11	0.722
0.15	25.03	0.7333
0.2	24.96	0.7445
0.25	24.89	0.7555
0.3	24.82	0.7665
0.35	24.76	0.7773
0.4	24.7	0.7881
0.45	24.64	0.7987
0.5	24.59	0.8092
0.55	24.53	0.8197
0.6	24.48	0.83
0.65	24.43	0.8403
0.7	24.39	0.8504
0.75	24.34	0.8605
0.8	24.3	0.8705
0.85	24.26	0.8805
0.9	24.22	0.8904
0.95	24.18	0.9001
1	24.14	0.9099

5-35 One side of a hot vertical plate is to be cooled by attaching aluminum fins of rectangular profile. The finite difference formulation of the problem for all nodes is to be obtained, and the nodal temperatures, the rate of heat transfer from a single fin and from the entire surface of the plate are to be determined.

Assumptions **1** Heat transfer along the fin is given to be steady and one-dimensional. **2** The thermal conductivity is constant. **3** Combined convection and radiation heat transfer coefficient is constant and uniform.

Properties The thermal conductivity is given to be k = 237 W/m·°C.

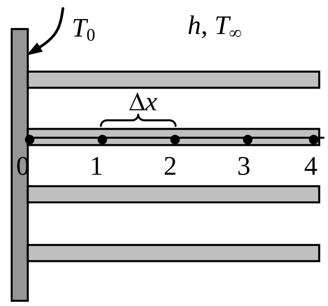
Analysis (*a*) The nodal spacing is given to be Δx=0.5 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{2\,\text{cm}}{0.5\,\text{cm}} + 1 = 5$$

The base temperature at node 0 is given to be T_0 = 130°C. This problem involves 4 unknown nodal temperatures, and thus we need to have 4 equations to determine them uniquely. Nodes 1, 2, and 3 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \;\rightarrow\; T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) = 0$$

The finite difference equation for node 4 at the fin tip is obtained by applying an energy balance on the half volume element about that node. Then,



$$m = 1:\; T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) = 0$$

$$m = 2:\; T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) = 0$$

$$m = 3:\; T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) = 0$$

$$\text{Node 4:}\quad kA\frac{T_3 - T_4}{\Delta x} + h(p\Delta x / 2 + A)(T_\infty - T_4) = 0$$

where $\Delta x = 0.005\,\text{m},\; k = 237\,\text{W/m}\cdot°\text{C},\; T_\infty = 35°\text{C}, T_0 = 130°\text{C},\; h = 30\,\text{W/m}^2\cdot°\text{C}$

and $A = (3\,\text{m})(0.003\text{m}) = 0.009\,\text{m}^2$ and $p = 2(3 + 0.003\,\text{m}) = 6.006\,\text{m}$.

This system of 4 equations with 4 unknowns constitute the finite difference formulation of the problem.

(*b*) The nodal temperatures under steady conditions are determined by solving the 4 equations above simultaneously with an equation solver to be

T_1 =129.2°C, T_2 =128.7°C, T_3 =128.3°C, T_4 =128.2°C

(*c*) The rate of heat transfer from a single fin is simply the sum of the heat transfer from each nodal element,

$$\begin{aligned}\dot{Q}_{\text{fin}} &= \sum_{m=0}^{4}\dot{Q}_{\text{element},m} = \sum_{m=0}^{4} hA_{\text{surface},m}(T_m - T_\infty)\\ &= hp(\Delta x/2)(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 - 3T_\infty) + h(p\Delta x/2 + A)(T_4 - T_\infty) = \mathbf{363\ W}\end{aligned}$$

(*d*) The number of fins on the surface is

$$\text{No. of fins} = \frac{\text{Plate height}}{\text{Fin thickness} + \text{fin spacing}} = \frac{2\,\text{m}}{(0.003 + 0.004)\text{m}} = 286\,\text{fins}$$

Then the rate of heat tranfer from the fins, the unfinned portion, and the entire finned surface become

$$\dot{Q}_{\text{fin, total}} = (\text{No. of fins})\dot{Q}_{\text{fin}} = 286(363\,\text{W}) = 103{,}818\text{W}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_0 - T_\infty) = (30\,\text{W/m}^2\cdot°\text{C})(286\times 3\,\text{m}\times 0.004\,\text{m})(130\text{-}35)°\text{C} = 9781\text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin, total}} + \dot{Q}_{\text{unfinned}} = 103{,}818 + 9781 = \mathbf{113{,}600W} \cong \mathbf{114\ kW}$$

5-36 One side of a hot vertical plate is to be cooled by attaching aluminum pin fins. The finite difference formulation of the problem for all nodes is to be obtained, and the nodal temperatures, the rate of heat transfer from a single fin and from the entire surface of the plate are to be determined.

Assumptions **1** Heat transfer along the fin is given to be steady and one-dimensional. **2** The thermal conductivity is constant. **3** Combined convection and radiation heat transfer coefficient is constant and uniform.

Properties The thermal conductivity is given to be k = 237 W/m·°C.

Analysis (*a*) The nodal spacing is given to be Δx=0.5 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{3\,\text{cm}}{0.5\,\text{cm}} + 1 = 7$$

The base temperature at node 0 is given to be T_0 = 100°C. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Nodes 1, 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \;\rightarrow\; T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) = 0$$

The finite difference equation for node 6 at the fin tip is obtained by applying an energy balance on the half volume element about that node. Then,

$$m=1:\; T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) = 0$$

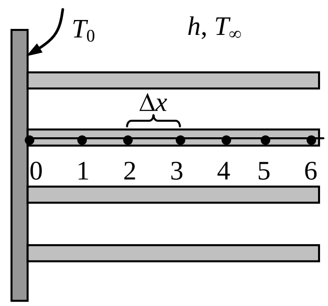
$$m=2:\; T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) = 0$$

$$m=3:\; T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) = 0$$

$$m=4:\; T_3 - 2T_4 + T_5 + h(p\Delta x^2 / kA)(T_\infty - T_4) = 0$$

$$m=5:\; T_4 - 2T_5 + T_6 + h(p\Delta x^2 / kA)(T_\infty - T_5) = 0$$

$$\text{Node 6:}\; kA\frac{T_5 - T_6}{\Delta x} + h(p\Delta x / 2 + A)(T_\infty - T_6) = 0$$



where $\Delta x = 0.005\,\text{m}$, $k = 237\,\text{W/m}\cdot°\text{C}$, $T_\infty = 30°\text{C}$, $T_0 = 100°\text{C}$, $h = 35\,\text{W/m}^2\cdot°\text{C}$

and
$$A = \pi D^2/4 = \pi(0.25\,\text{cm})^2/4 = 0.0491\,\text{cm}^2 = 0.0491\times10^{-4}\,\text{m}^2$$
$$p = \pi D = \pi(0.0025\,\text{m}) = 0.00785\,\text{m}$$

(*b*) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_1 **=97.9°C,** T_2 **=96.1°C,** T_3 **=94.7°C,** T_4 **=93.8°C,** T_5 **=93.1°C,** T_6 **=92.9°C**

(*c*) The rate of heat transfer from a single fin is simply the sum of the heat transfer from the nodal elements,

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{6}\dot{Q}_{\text{element},m} = \sum_{m=0}^{6} hA_{\text{surface},m}(T_m - T_\infty)$$
$$= hp\Delta x/2(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 + T_4 + T_5 - 5T_\infty) + h(p\Delta x/2 + A)(T_6 - T_\infty) = \mathbf{0.5496\ W}$$

(*d*) The number of fins on the surface is $\text{No. of fins} = \dfrac{1\,\text{m}^2}{(0.006\text{m})(0.006\text{m})} = 27{,}778\,\text{fins}$

Then the rate of heat tranfer from the fins, the unfinned portion, and the entire finned surface become

$$\dot{Q}_{\text{fin, total}} = (\text{No. of fins})\dot{Q}_{\text{fin}} = 27{,}778(0.5496\,\text{W}) = 15{,}267\,\text{W}$$
$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_0 - T_\infty) = (35\,\text{W/m}^2\cdot°\text{C})(1 - 27{,}778\times0.0491\times10^{-4}\,\text{m}^2)(100-30)°\text{C} = 2116\,\text{W}$$
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin, total}} + \dot{Q}_{\text{unfinned}} = 15{,}267 + 2116 = \mathbf{17{,}383\,W} \cong \mathbf{17.4\ kW}$$

5-37 One side of a hot vertical plate is to be cooled by attaching copper pin fins. The finite difference formulation of the problem for all nodes is to be obtained, and the nodal temperatures, the rate of heat transfer from a single fin and from the entire surface of the plate are to be determined.

Assumptions **1** Heat transfer along the fin is given to be steady and one-dimensional. **2** The thermal conductivity is constant. **3** Combined convection and radiation heat transfer coefficient is constant and uniform.

Properties The thermal conductivity is given to be k = 386 W/m·°C.

Analysis (a) The nodal spacing is given to be Δx=0.5 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{3\,\text{cm}}{0.5\,\text{cm}} + 1 = 7$$

The base temperature at node 0 is given to be T_0 = 100°C. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Nodes 1, 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \;\rightarrow\; T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) = 0$$

The finite difference equation for node 6 at the fin tip is obtained by applying an energy balance on the half volume element about that node. Then,

m= 1: $T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) = 0$

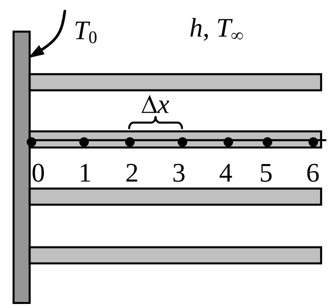
m= 2: $T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) = 0$

m= 3: $T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) = 0$

m= 4: $T_3 - 2T_4 + T_5 + h(p\Delta x^2 / kA)(T_\infty - T_4) = 0$

m= 5: $T_4 - 2T_5 + T_6 + h(p\Delta x^2 / kA)(T_\infty - T_5) = 0$

Node 6: $kA\dfrac{T_5 - T_6}{\Delta x} + h(p\Delta x / 2 + A)(T_\infty - T_6) = 0$



where $\Delta x = 0.005\,\text{m},\; k = 386\,\text{W/m}\cdot°\text{C},\; T_\infty = 30°\text{C}, T_0 = 100°\text{C},\; h = 35\,\text{W/m}^2\cdot°\text{C}$

and $A = \pi D^2 / 4 = \pi(0.25\,\text{cm})^2/4 = 0.0491\text{cm}^2 = 0.0491\times10^{-4}\,\text{m}^2$

$p = \pi D = \pi(0.0025\text{m}) = 0.00785\text{m}$

(b) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

$\boldsymbol{T_1}$ **=98.6°C,** $\boldsymbol{T_2}$ **=97.5°C,** $\boldsymbol{T_3}$ **=96.7°C,** $\boldsymbol{T_4}$ **=96.0°C,** $\boldsymbol{T_5}$ **=95.7°C,** $\boldsymbol{T_6}$ **=95.5°C**

(c) The rate of heat transfer from a single fin is simply the sum of the heat transfer from the nodal elements,

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{6} \dot{Q}_{\text{element},m} = \sum_{m=0}^{6} hA_{\text{surface},m}(T_m - T_\infty)$$
$$= hp\Delta x / 2(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 + T_4 + T_5 - 5T_\infty) + h(p\Delta x/2 + A)(T_6 - T_\infty) = \mathbf{0.5641\ W}$$

(d) The number of fins on the surface is $\quad \text{No. of fins} = \dfrac{1\,\text{m}^2}{(0.006\text{m})(0.006\text{m})} = 27{,}778\,\text{fins}$

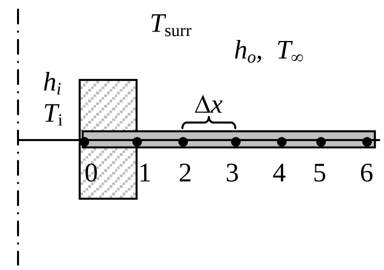
Then the rate of heat tranfer from the fins, the unfinned portion, and the entire finned surface become

$$\dot{Q}_{\text{fin, total}} = (\text{No. of fins})\dot{Q}_{\text{fin}} = 27{,}778(0.5641\text{W}) = 15{,}670\text{W}$$
$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_0 - T_\infty) = (35\,\text{W/m}^2\cdot°\text{C})(1\text{-}27{,}778\times0.0491\times10^{-4}\,\text{m}^2)(100\text{-}30)°\text{C} = 2116\,\text{W}$$
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin, total}} + \dot{Q}_{\text{unfinned}} = 15{,}670 + 2116 = \mathbf{17{,}786\,W} \cong \mathbf{17.8\,kW}$$

5-38 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges, and heat is lost from the flanges by convection and radiation. The finite difference formulation of the problem for all nodes is to be obtained, and the temperature of the tip of the flange as well as the rate of heat transfer from the exposed surfaces of the flange are to be determined.

Assumptions **1** Heat transfer through the flange is stated to be steady and one-dimensional. **2** The thermal conductivity and emissivity are constants. **3** Convection heat transfer coefficient is constant and uniform.

Properties The thermal conductivity and emissivity are given to be k = 52 W/m·°C and ε = 0.8.



Analysis (*a*) The distance between nodes 0 and 1 is the thickness of the pipe, Δx_1=0.4 cm=0.004 m. The nodal spacing along the flange is given to be Δx_2=1 cm = 0.01 m. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 2 = \frac{5\,\text{cm}}{1\,\text{cm}} + 2 = 7$$

This problem involves 7 unknown nodal temperatures, and thus we need to have 7 equations to determine them uniquely. Noting that the total thickness of the flange is t = 0.02 m, the heat conduction area at any location along the flange is $A_{\text{cond}} = 2\pi rt$ where the values of radii at the nodes and between the nodes (the mid points) are

r_0=0.046 m, r_1=0.05 m, r_2=0.06 m, r_3=0.07 m, r_4=0.08 m, r_5=0.09 m, r_6=0.10 m

r_{01}=0.048 m, r_{12}=0.055 m, r_{23}=0.065 m, r_{34}=0.075 m, r_{45}=0.085 m, r_{56}=0.095 m

Then the finite difference equations for each node are obtained from the energy balance to be as follows:

Node 0: $$h_i(2\pi t r_0)(T_i - T_0) + k(2\pi t r_{01})\frac{T_1 - T_0}{\Delta x_1} = 0$$

Node 1:

$$k(2\pi t r_{01})\frac{T_0 - T_1}{\Delta x_1} + k(2\pi t r_{12})\frac{T_2 - T_1}{\Delta x_2} + 2[2\pi t(r_1 + r_{12})/2)](\Delta x_2/2)\{h(T_\infty - T_1) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_1 + 273)^4]\} = 0$$

Node 2: $$k(2\pi t r_{12})\frac{T_1 - T_2}{\Delta x_2} + k(2\pi t r_{23})\frac{T_3 - T_2}{\Delta x_2} + 2(2\pi t r_2 \Delta x_2)\{h(T_\infty - T_2) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_2 + 273)^4]\} = 0$$

Node 3: $$k(2\pi t r_{23})\frac{T_2 - T_3}{\Delta x_2} + k(2\pi t r_{34})\frac{T_4 - T_3}{\Delta x_2} + 2(2\pi t r_3 \Delta x_2)\{h(T_\infty - T_3) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_3 + 273)^4]\} = 0$$

Node 4: $$k(2\pi t r_{34})\frac{T_3 - T_4}{\Delta x_2} + k(2\pi t r_{45})\frac{T_5 - T_4}{\Delta x_2} + 2(2\pi t r_4 \Delta x_2)\{h(T_\infty - T_4) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_4 + 273)^4]\} = 0$$

Node 5: $$k(2\pi t r_{45})\frac{T_4 - T_5}{\Delta x_2} + k(2\pi t r_{56})\frac{T_6 - T_5}{\Delta x_2} + 2(2\pi t r_5 \Delta x_2)\{h(T_\infty - T_5) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_5 + 273)^4]\} = 0$$

Node 6: $$k(2\pi t r_{56})\frac{T_5 - T_6}{\Delta x_2} + 2[2\pi t(\Delta x_2/2)(r_{56} + r_6)/2 + 2\pi r_6 t]\{h(T_\infty - T_6) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_6 + 273)^4]\} = 0$$

where $\Delta x_1 = 0.004\,\text{m}$, $\Delta x_2 = 0.01\,\text{m}$, $k = 52\,\text{W/m}\cdot°\text{C}$, $\varepsilon = 0.8$, $T_\infty = 8°\text{C}$, $T_{in} = 200°\text{C}$, $T_{surr} = 290\,\text{K}$ and $h = 25\,\text{W/m}^2\cdot°\text{C}$, $h_i = 180\,\text{W/m}^2\cdot°\text{C}$, $\sigma = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4$.

The system of 7 equations with 7 unknowns constitutes the finite difference formulation of the problem.

(*b*) The nodal temperatures under steady conditions are determined by solving the 7 equations above simultaneously with an equation solver to be

T_0 =119.7°C, T_1 =118.6°C, T_2 = 116.3°C, T_3 =114.3°C, T_4 =112.7°C, T_5 =111.2°C, and $\mathbf{T_6}$ **= 109.9°C** (*c*) Knowing the inner surface temperature, the rate of heat transfer from the flange under steady conditions is simply the rate of heat transfer from the steam to the pipe at flange section

$$\dot{Q}_{\text{fin}} = \sum_{m=1}^{6} \dot{Q}_{\text{element},m} = \sum_{m=1}^{6} hA_{\text{surface},m}(T_m - T_\infty) + \sum_{m=1}^{6} \varepsilon\sigma A_{\text{surface},m}[(T_m + 273)^4 - T_{\text{surr}}^4] = \mathbf{83.6\ W}$$

where $A_{\text{surface, m}}$ are as given above for different nodes.

5-39 "!PROBLEM 5-39"

```
"GIVEN"
t_pipe=0.004 "[m]"
k=52 "[W/m-C]"
epsilon=0.8
D_o_pipe=0.10 "[m]"
t_flange=0.01 "[m]"
D_o_flange=0.20 "[m]"
T_steam=200 "[C], parameter to be varied"
h_i=180 "[W/m^2-C]"
T_infinity=8 "[C]"
"h=25 [W/m^2-C], parameter to be varied"
T_surr=290 "[K]"
DELTAx=0.01 "[m]"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
"(b)"
DELTAx_1=t_pipe "the distance between nodes 0 and 1"
DELTAx_2=t_flange "nodal spacing along the flange"
L=(D_o_flange-D_o_pipe)/2
M=L/DELTAx_2+2 "Number of nodes"
t=2*t_flange "total thixkness of the flange"
"The values of radii at the nodes and between the nodes /-(the midpoints) are"
r_0=0.046 "[m]"
r_1=0.05 "[m]"
r_2=0.06 "[m]"
r_3=0.07 "[m]"
r_4=0.08 "[m]"
r_5=0.09 "[m]"
r_6=0.10 "[m]"
r_01=0.048 "[m]"
r_12=0.055 "[m]"
r_23=0.065 "[m]"
r_34=0.075 "[m]"
r_45=0.085 "[m]"
r_56=0.095 "[m]"
"Using the finite difference method, the five equations for the unknown temperatures at 7 nodes
are determined to be"
h_i*(2*pi*t*r_0)*(T_steam-T_0)+k*(2*pi*t*r_01)*(T_1-T_0)/DELTAx_1=0 "node 0"
k*(2*pi*t*r_01)*(T_0-T_1)/DELTAx_1+k*(2*pi*t*r_12)*(T_2-
T_1)/DELTAx_2+2*2*pi*t*(r_1+r_12)/2*(DELTAx_2/2)*(h*(T_infinity-
T_1)+epsilon*sigma*(T_surr^4-(T_1+273)^4))=0 "node 1"
k*(2*pi*t*r_12)*(T_1-T_2)/DELTAx_2+k*(2*pi*t*r_23)*(T_3-
T_2)/DELTAx_2+2*2*pi*t*r_2*DELTAx_2*(h*(T_infinity-T_2)+epsilon*sigma*(T_surr^4-
(T_2+273)^4))=0 "node 2"
k*(2*pi*t*r_23)*(T_2-T_3)/DELTAx_2+k*(2*pi*t*r_34)*(T_4-
T_3)/DELTAx_2+2*2*pi*t*r_3*DELTAx_2*(h*(T_infinity-T_3)+epsilon*sigma*(T_surr^4-
(T_3+273)^4))=0 "node 3"
k*(2*pi*t*r_34)*(T_3-T_4)/DELTAx_2+k*(2*pi*t*r_45)*(T_5-
T_4)/DELTAx_2+2*2*pi*t*r_4*DELTAx_2*(h*(T_infinity-T_4)+epsilon*sigma*(T_surr^4-
(T_4+273)^4))=0 "node 4"
k*(2*pi*t*r_45)*(T_4-T_5)/DELTAx_2+k*(2*pi*t*r_56)*(T_6-
T_5)/DELTAx_2+2*2*pi*t*r_5*DELTAx_2*(h*(T_infinity-T_5)+epsilon*sigma*(T_surr^4-
(T_5+273)^4))=0 "node 5"
```

```
k*(2*pi*t*r_56)*(T_5-
T_6)/DELTAx_2+2*(2*pi*t*(r_56+r_6)/2*(DELTAx_2/2)+2*pi*t*r_6)*(h*(T_infinity-
T_6)+epsilon*sigma*(T_surr^4-(T_6+273)^4))=0 "node 6"
T_tip=T_6
"(c)"
Q_dot=Q_dot_1+Q_dot_2+Q_dot_3+Q_dot_4+Q_dot_5+Q_dot_6 "where"
Q_dot_1=h*2*2*pi*t*(r_1+r_12)/2*DELTAx_2/2*(T_1-
T_infinity)+epsilon*sigma*2*2*pi*t*(r_1+r_12)/2*DELTAx_2/2*((T_1+273)^4-T_surr^4)
Q_dot_2=h*2*2*pi*t*r_2*DELTAx_2*(T_2-
T_infinity)+epsilon*sigma*2*2*pi*t*r_2*DELTAx_2*((T_2+273)^4-T_surr^4)
Q_dot_3=h*2*2*pi*t*r_3*DELTAx_2*(T_3-
T_infinity)+epsilon*sigma*2*2*pi*t*r_3*DELTAx_2*((T_3+273)^4-T_surr^4)
Q_dot_4=h*2*2*pi*t*r_4*DELTAx_2*(T_4-
T_infinity)+epsilon*sigma*2*2*pi*t*r_4*DELTAx_2*((T_4+273)^4-T_surr^4)
Q_dot_5=h*2*2*pi*t*r_5*DELTAx_2*(T_5-
T_infinity)+epsilon*sigma*2*2*pi*t*r_5*DELTAx_2*((T_5+273)^4-T_surr^4)
Q_dot_6=h*2*(2*pi*t*(r_56+r_6)/2*(DELTAx_2/2)+2*pi*t*r_6)*(T_6-
T_infinity)+epsilon*sigma*2*(2*pi*t*(r_56+r_6)/2*(DELTAx_2/2)+2*pi*t*r_6)*((T_6+273)^4-
T_surr^4)
```

T_{steam} [C]	T_{tip} [C]	Q [W]
150	84.42	60.83
160	89.57	65.33
170	94.69	69.85
180	99.78	74.4
190	104.8	78.98
200	109.9	83.58
210	114.9	88.21
220	119.9	92.87
230	124.8	97.55
240	129.7	102.3
250	134.6	107
260	139.5	111.8
270	144.3	116.6
280	149.1	121.4
290	153.9	126.2
300	158.7	131.1

h [W/m^2.C]	T_{tip} [C]	Q [W]
15	126.5	68.18
20	117.6	76.42
25	109.9	83.58
30	103.1	89.85
35	97.17	95.38
40	91.89	100.3
45	87.17	104.7
50	82.95	108.6
55	79.14	112.1
60	75.69	115.3

5-40 Using an equation solver or an iteration method, the solutions of the following systems of algebraic equations are determined to be as follows:

(*a*)

$$3x_1 - x_2 + 3x_3 = 0$$
$$-x_1 + 2x_2 + x_3 = 3$$
$$2x_1 - x_2 - x_3 = 2$$

Solution: x_1=2, x_2=3, x_3=1

(*b*)

$$4x_1 - 2x_2^2 + 0.5x_3 = -2$$
$$x_1^3 - x_2 + x_3 = 11.964$$
$$x_1 + x_2 + x_3 = 3$$

Solution: x_1=2.532, x_2=2.364, x_3=-1.896

```
"ANALYSIS"
"(a)"
3*x_1a-x_2a+3*x_3a=0
-x_1a+2*x_2a+x_3a=3
2*x_1a-x_2a-x_3a=2
"(b)"
4*x_1b-2*x_2b^2+0.5*x_3b=-2
x_1b^3-x_2b+-x_3b=11.964
x_1b+x_2b+x_3b=3
```

5-41 Using an equation solver or an iteration method, the solutions of the following systems of algebraic equations are determined to be as follows:

(*a*)

$$3x_1 - 2x_2 - x_3 + x_4 = 6$$
$$x_1 + 2x_2 - x_4 = -3$$
$$-2x_1 + x_2 + 3x_3 + x_4 = 2$$
$$3x_2 + x_3 - 4x_4 = -6$$

Solution: x_1=13, x_2=-9, x_3=13, x_4= -2

(*b*)

$$3x_1 + x_2^2 + 2x_3 = 8$$
$$-x_1^2 + 3x_2 + 2x_3 = -6.293$$
$$2x_1 - x_2^4 + 4x_3 = -12$$

Solution: x_1=2.825, x_2=1.791, x_3=-1.841

```
"ANALYSIS"
"(a)"
3*x_1a+2*x_2a-x_3a+x_4a=6
x_1a+2*x_2a-x_4a=-3
-2*x_1a+x_2a+3*x_3a+x_4a=2
3*x_2a+x_3a-4*x_4a=-6
"(b)"
3*x_1b+x_2b^2+2*x_3b=8
-x_1b^2+3*x_2b+2*x_3b=-6.293
2*x_1b-x_2b^4+4*x_3b=-12
```

5-42 Using an equation solver or an iteration method, the solutions of the following systems of algebraic equations are determined to be as follows:

(*a*)

$$4x_1 - x_2 + 2x_3 + x_4 = -6$$
$$x_1 + 3x_2 - x_3 + 4x_4 = -1$$
$$-x_1 + 2x_2 + 5x_4 = 5$$
$$2x_2 - 4x_3 - 3x_4 = 2$$

Solution: x_1=-0.744, x_2=-8, x_3=-7.54, x_4= 4.05

(*b*)

$$2x_1 + x_2^4 - 2x_3 + x_4 = 1$$
$$x_1^2 + 4x_2 + 2x_3^2 - 2x_4 = -3$$
$$-x_1 + x_2^4 + 5x_3 = 10$$
$$3x_1 - x_3^2 + 8x_4 = 15$$

Solution: x_1=0.263, x_2=-1.15, x_3=1.70, x_4=2.14

```
"ANALYSIS"
"(a)"
4*x_1a-x_2a+2*x_3a+x_4a=-6
x_1a+3*x_2a-x_3a+4*x_4a=-1
-x_1a+2*x_2a+5*x_4a=5
2*x_2a-4*x_3a-3*x_4a=2
"(b)"
2*x_1b+x_2b^4-2*x_3b+x_4b=1
x_1b^2+4*x_2b+2*x_3b^2-2*x_4b=-3
-x_1b+x_2b^4+5*x_3b=10
3*x_1b-x_3b^2+8*x_4b=15
```